$$+ 3y^2I_6\} + i\frac{y}{a}\{3a^2(I_2 - I_1) + 3az_j(I_3 - I_4) - 3x^2I_7 + y^2I_5\}\Big]\Big\} - \frac{6c_{66}P_z}{a^4}G_0\Big\{fa[x\psi_1(\xi_0) + iy\psi_2(\xi_0)] - \bar{f}\Big[\frac{x}{a}\{3a^2(I_9 - I_2) + 3az_0(I_4 - I_8) - x^2I_{10} + 3y^2I_6\} + i\frac{y}{a}\{3a^2(I_2 - I_1) + 3az_0(I_3 - I_4) - 3x^2I_7 + y^2I_5\}\Big]\Big\},$$

$$\sigma_{zm} = \frac{6P_z}{a^3}\sum_{j=1}^{3} G_j\omega_{mj}[xf_x\psi_1(\xi_j) + yf_y\psi_2(\xi_j)],$$

$$\tau_{zm} = \frac{3P_z}{a^4}\sum_{j=1}^{3} G_js_j\omega_{mj}\{faz_j\psi_3(\xi_j) - \bar{f}[az_j\{\psi_1(\xi_j) - \psi_2(\xi_j)\} - a^2(I_{11} - I_{12}) + x^2I_8 - y^2I_3 + i2xyI_4]\} - \frac{3P_z}{a^4}G_0s_0\rho_m\{faz_0\psi_3(\xi_0) + \bar{f}[az_0\{\psi_1(\xi_0) - \psi_2(\xi_0)\} - a^2(I_{11} - I_{12}) + x^2I_8 - y^2I_3 + i2xyI_4]\} \quad (15)$$

where e, ξ_j $(j = 0, 1, 2, 3)$ and $F(\varphi_j, e)$ $(j = 0, 1, 2, 3)$ are the same as those in Eq. (11); ψ_k $(k = 1, 2, 3)$, I_{11}, I_{12}, and I_l $(l = 1, 2, 3, \ldots 12)$ are listed in Appendices A and B in Hanson and Puja (1997).

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Exact Eshelby Tensor for a Dynamic Circular Cylindrical Inclusion

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1 Introduction

This work is motivated by Mikata and Nemat-Nasser's (1990) study of dynamic transformation toughening of ceramics in which a typical dynamic problem of a spherical inclusion was solved. Mikata and Nemat-Nasser (1990, 1991), Mikata (1993), and Cheng and He (1996) have obtained exact analytic solutions for a dynamic spherical inclusion embedded in an infinite linear elastic and isotropic medium. However, the corresponding dynamic problem of a circular cylindrical inclusion has not been studied. Mura (1988) and Mura et al. (1996) have reviewed the literature on inclusion problems.

The time-harmonic elastic field caused by an infinitely long circular cylindrical inclusion is obtained in this paper, and a closed-form expression is derived for the dynamic Eshelby tensor. Unlike the static case, the Eshelby tensor for the dynamic problem is not uniform even at interior points within the circular cylinder. In the limit of quasi-static deformations the present solution reduces to Eshelby's results.

2 Analysis

Following Eshelby (1957, 1959) and Mura (1982), an inclusion is referred to a subset of a matrix that has a prescribed eigenstrain (or transformation strain) and has the same elastic properties as the matrix.

Consider the following time-harmonic eigenstrain

$$e^*_{kl}(\mathbf{x}, t) = e^{\Omega}_{kl}(\mathbf{x})\Lambda(\Omega)e^{-i\omega t},$$

$$\Lambda(\Omega) = \begin{cases} 1, & \mathbf{x} \in \Omega \\ 0, & \mathbf{x} \in R^3 - \Omega \end{cases} \quad (1)$$

where Ω is the region occupied by an inclusion that is embedded in an infinite (i.e., R^3) isotropic, linear elastic medium, and ω denotes an angular frequency. It is assumed that a time-harmonic eigenstrain will induce time-harmonic displacement, strain, and stress fields. Henceforth we omit the factor $\exp(-i\omega t)$. Also, a comma followed by a subscript i denotes a partial derivative with respect to the rectangular Cartesian coordinate x_i, a repeated index implies summation over the range of the index, Latin subscripts range over 1, 2, 3 and Greek subscripts over 1 and 2.

Equations for determining the displacement field in steady-state deformations of a linear elastic isotropic body are

$$\sigma_{ij,j} + \rho\omega^2u_i = 0, \quad \sigma_{ij} = C_{ijkl}[e_{kl} - e^{\Omega}_{kl}\Lambda(\Omega)],$$

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$$e_{kl} = \tfrac{1}{2}(u_{k,l} + u_{l,k}), \tag{2}$$

where ρ is the mass density,

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{3}$$

λ and μ the Lamé constants and δ_{ij} the Kronecker delta. The corresponding displacement field can be expressed as (Mura, 1982)

$$u_i(\mathbf{x}) = -\int_\Omega C_{jlmn} e^\Omega_{mn}(\mathbf{x}') g_{ij,l}(\mathbf{x} - \mathbf{x}') d\mathbf{x}', \tag{4}$$

where g_{ij} is the Green function defined by

$$g_{lm}(\mathbf{x} - \mathbf{x}') = \frac{1}{4\pi\rho\omega^2} \times \left[\beta^2 \delta_{lm} \frac{e^{i\beta r}}{r} - \frac{\partial^2}{\partial x_l \partial x_m} \left(\frac{e^{i\alpha r}}{r} - \frac{e^{i\beta r}}{r} \right) \right], \tag{5}$$

$$r^2 = (x_k - x'_k)(x_k - x'_k), \quad \alpha^2 = \frac{\rho\omega^2}{\lambda + 2\mu}, \quad \beta^2 = \frac{\rho\omega^2}{\mu}. \tag{6}$$

If $e^\Omega_{mn}(\mathbf{x})$ in Eq. (1) is constant over Ω, then the displacement and strain can be expressed as (Mikata and Nemat-Nasser, 1990)

$$u_i(\mathbf{x}) = J_{ikl}(\mathbf{x}) e^\Omega_{kl}, \quad e_{ij}(\mathbf{x}) = M_{ijkl}(\mathbf{x}) e^\Omega_{kl}, \tag{7}$$

for both inside and outside of the inclusion, where

$$M_{ijkl}(\mathbf{x}) = \tfrac{1}{2}[J_{ikl,j}(\mathbf{x}) + J_{jkl,i}(\mathbf{x})], \tag{8}$$

$$J_{ikl}(\mathbf{x}) = \frac{1}{4\pi\rho\omega^2} \{ \lambda\delta_{kl} f_{,imm}(\mathbf{x}, \alpha) + 2\mu[f_{,ikl}(\mathbf{x}, \alpha) - f_{,ikl}(\mathbf{x}, \beta)] - \mu\beta^2[\delta_{ik} f_{,l}(\mathbf{x}, \beta) + \delta_{il} f_{,k}(\mathbf{x}, \beta)] \}, \tag{9}$$

$$f(\mathbf{x}, k) = \int_\Omega \frac{e^{ikr}}{r} d\mathbf{x}'. \tag{10}$$

Mikata and Nemat-Nasser (1990) called $M_{ijkl}(\mathbf{x})$ in Eq. (8) the dynamic Eshelby tensor. The expression in Eq. (9) slightly differs from that given by Mikata and Nemat-Nasser (1990) since we have used

$$f_{,imm}(\mathbf{x}, \beta) + \beta^2 f_{,i}(\mathbf{x}, \beta) = 0 \tag{11}$$

to simplify (9). For a spherical inclusion, Mikata and Nemat-Nasser (1990) evaluated the integral (10) in closed form and hence computed the exact dynamic Eshelby tensor. Here we evaluate this integral for an infinitely long circular cylindrical inclusion Ω: $x_1^2 + x_2^2 < a^2$ and $-\infty < x_3 < \infty$, and then find the corresponding Eshelby tensor. To do this, we recall the following formulas (Gradshteyn and Ryzhik, 1965).

(a) Integral formula.

$$\int_{-\infty}^{\infty} \frac{e^{ikr}}{r} dx'_3 = i\pi H_0^{(1)}(kR), \tag{12}$$

$$R^2 = (x_\alpha - x'_\alpha)(x_\alpha - x'_\alpha) = z^2 + z'^2 - 2zz' \cos\theta, \tag{13}$$

$$z^2 = x_\alpha x_\alpha, \quad z'^2 = x'_\alpha x'_\alpha, \quad \cos\theta = \frac{x_\alpha x'_\alpha}{zz'}. \tag{14}$$

(b) Addition theorem.

$$H_0^{(1)}(kR) = \begin{cases} J_0(kz)H_0^{(1)}(kz') + 2\sum_{M=1}^{\infty} J_M(kz)H_M^{(1)}(kz') \cos M\theta, & (z < z') \\ J_0(kz')H_0^{(1)}(kz) + 2\sum_{M=1}^{\infty} J_M(kz')H_M^{(1)}(kz) \cos M\theta, & (z > z') \end{cases}. \tag{15}$$

(c) Recurrence relations

$$\frac{d}{dz}[zJ_1(z)] = zJ_0(z), \quad \frac{d}{dz}[zH_1^{(1)}(z)] = zH_0^{(1)}(z). \tag{16}$$

Here $J_M(z)$ is the Bessel function of order M and $H_M^{(1)}(z)$ is the Hankel function of the first kind of order M.

Based on the formulas (12), (15), and (16), the integral (10) can now be calculated for a circular cylindrical inclusion.

$$f(\mathbf{x}, k) = \begin{cases} \left(\int_0^z z'dz' + \int_z^a z'dz' \right) \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dx'_3 \frac{e^{ikr}}{r}, & \mathbf{x} \in \Omega \\ \int_0^a z'dz' \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dx'_3 \frac{e^{ikr}}{r}, & \mathbf{x} \in R^3 - \Omega \end{cases}$$

$$= \frac{2\pi^2 i}{k} \begin{cases} zJ_1(kz)H_0^{(1)}(kz) + aJ_0(kz)H_1^{(1)}(ka) - zJ_0(kz)H_1^{(1)}(kz), & \mathbf{x} \in \Omega \\ aJ_1(ka)H_0^{(1)}(kz), & \mathbf{x} \in R^3 - \Omega \end{cases}. \tag{17}$$

Furthermore, by using

$$J_0(kz)H_1^{(1)}(kz) - J_1(kz)H_0^{(1)}(kz) = -\frac{2i}{\pi kz}, \tag{18}$$

(17) can be simplified to

$$f(\mathbf{x}, k) \equiv N(z, k) = -4\pi \left[\frac{1}{k^2} \Lambda(\Omega) + \Phi(k)\Psi_0(kz) \right], \tag{19}$$

where

$$\Phi(k) = \begin{cases} -\frac{i\pi a}{2k} H_1^{(1)}(ka), & \mathbf{x} \in \Omega \\ -\frac{i\pi a}{2k} J_1(ka), & \mathbf{x} \in R^3 - \Omega \end{cases},$$

$$\Psi_0(kz) = \begin{cases} J_0(kz), & \mathbf{x} \in \Omega \\ H_0^{(1)}(kz), & \mathbf{x} \in R^3 - \Omega \end{cases}. \tag{20}$$

Thus, the exact steady-state Eshelby tensor for an infinite circular cylindrical inclusion is readily obtained from Eqs. (8), (9), and (19). As can be seen from these equations, unlike for the quasi-static problem (Eshelby, 1957), the dynamic Eshelby tensor varies even within the inclusion. The calculation of the dynamic Eshelby tensor (8) requires the following expressions for the derivatives of the potential function $f(\mathbf{x}, k)$.

$$f_{,3}(\mathbf{x}, k) = 0, \quad f_{,\alpha}(\mathbf{x}, k) = x_\alpha DN,$$

$$f_{,\alpha\beta}(\mathbf{x}, k) = \delta_{\alpha\beta} DN + x_\alpha x_\beta D^2 N,$$

$$f_{,\alpha\beta\omega}(\mathbf{x}, k) = (x_\alpha \delta_{\beta\omega} + x_\beta \delta_{\omega\alpha} + x_\omega \delta_{\alpha\beta}) D^2 N + x_\alpha x_\beta x_\omega D^3 N,$$

$$f_{,\alpha\beta\omega\rho}(\mathbf{x}, k) = (\delta_{\alpha\beta}\delta_{\omega\rho} + \delta_{\alpha\omega}\delta_{\beta\rho} + \delta_{\alpha\rho}\delta_{\beta\omega}) D^2 N$$

$$+ (x_\alpha x_\beta \delta_{\omega\rho} + x_\alpha x_\omega \delta_{\beta\rho} + x_\alpha x_\rho \delta_{\beta\omega} + x_\beta x_\omega \delta_{\alpha\rho} + x_\beta x_\rho \delta_{\alpha\omega} + x_\omega x_\rho \delta_{\alpha\beta}) D^3 N + x_\alpha x_\beta x_\omega x_\rho D^4 N, \quad (21)$$

where $D = d/(zdz)$ and

$$D^l N = -4\pi\left(-\frac{k}{z}\right)^l \Phi(k)\Psi_l(kz), \ (l \geq 1),$$

$$\Psi_l(kz) = \left(-\frac{z}{k}\right)^l D^l \Psi_0(kz), \ (l \geq 1),$$

$$\Psi_l(kz) = \begin{cases} J_l(kz), & \mathbf{x} \in \Omega \\ H_l^{(1)}(kz), & \mathbf{x} \in R^3 - \Omega \end{cases},$$

$$(l = 0, 1, 2, \ldots), \text{ no sum on } l. \quad (22)$$

3 Quasi-static Deformations

The classical Eshelby tensor S_{ijkl} for quasi-static deformations can be recovered from the present dynamic Eshelby tensor (8) by taking the limit $\omega \to 0$, i.e.,

$$S_{ijkl}(\mathbf{x}) = \lim_{\omega\to 0} M_{ijkl}(\mathbf{x}) = \tfrac{1}{2}[J^S_{ikl,j}(\mathbf{x}) + J^S_{jkl,i}(\mathbf{x})], \quad (23)$$

where

$$J^S_{ikl}(\mathbf{x}) = \lim_{\omega\to 0} J_{ikl}(\mathbf{x}) = \frac{\lambda + \mu}{\lambda + 2\mu}\psi_{,ikl}(\mathbf{x}) - \frac{\lambda}{\lambda + 2\mu}\delta_{kl}\phi_{,i}(\mathbf{x}) - \delta_{ik}\phi_{,l}(\mathbf{x}) - \delta_{il}\phi_{,k}(\mathbf{x}), \quad (24)$$

$$\psi(\mathbf{x}) = \frac{1}{4\pi}\int_\Omega r d\mathbf{x}', \ \phi(\mathbf{x}) = \frac{1}{4\pi}\int_\Omega \frac{1}{r} d\mathbf{x}'. \quad (25)$$

Note that the two integrals (25) over an infinite circular cylinder diverge. However, the derivatives of the potential functions $\psi(\mathbf{x})$ and $\phi(\mathbf{x})$ appearing in (24) converge. The derivatives of $\psi(\mathbf{x})$ and $\phi(\mathbf{x})$ can be calculated in the same form as the derivatives of f in (21). Since a detailed discussion on $\psi(\mathbf{x})$ and $\phi(\mathbf{x})$ for a general ellipsoidal inclusion has been given by Mura (1982), only the relevant results for an infinite circular cylindrical inclusion are given below.

$$D^2\psi = \begin{cases} -\frac{1}{4}, & \mathbf{x} \in \Omega \\ -\dfrac{a^2}{2z^2} + \dfrac{a^4}{4z^4}, & \mathbf{x} \in R^3 - \Omega \end{cases},$$

$$D\phi = \begin{cases} -\frac{1}{2}, & \mathbf{x} \in \Omega \\ -\dfrac{a^2}{2z^2}, & \mathbf{x} \in R^3 - \Omega \end{cases} \quad (26)$$

By using (26), and recalling $S_{ijkl} = S_{jikl} = S_{ijlk}$, the nonzero components of the classical Eshelby tensor can be expressed as

$$S_{\omega\rho\alpha\beta} = \frac{4\nu - 1}{8(1-\nu)}\delta_{\alpha\beta}\delta_{\omega\rho} + \frac{3 - 4\nu}{8(1-\nu)}(\delta_{\alpha\omega}\delta_{\beta\rho} + \delta_{\alpha\rho}\delta_{\beta\omega}),$$

$$S_{3\rho 3\beta} = \frac{1}{4}\delta_{\beta\rho}, \quad S_{\omega\rho 33} = \frac{\nu}{2(1-\nu)}\delta_{\omega\rho}, \quad (27)$$

for the inside of the circular cylinder, and

$$S_{\omega\rho\alpha\beta} = \frac{a^2}{4}\left\{D_{\omega\rho\alpha\beta} - \frac{1}{1-\nu}\left[A_{\omega\rho\alpha\beta}\left(\frac{1}{z^2} - \frac{a^2}{2z^4}\right) - 2B_{\omega\rho\alpha\beta}\left(\frac{1}{z^4} - \frac{a^2}{z^6}\right) + 4C_{\omega\rho\alpha\beta}\left(\frac{2}{z^6} - \frac{3a^2}{z^8}\right)\right]\right\},$$

$$S_{3\rho 3\beta} = \frac{a^2}{4z^2}\delta_{\beta\rho} - \frac{a^2}{2z^4}x_\beta x_\rho,$$

$$S_{\omega\rho 33} = \frac{\nu a^2}{2(1-\nu)}\left(\frac{1}{z^2}\delta_{\omega\rho} - \frac{2}{z^4}x_\omega x_\rho\right), \quad (28)$$

for the outside of the circular cylinder, where ν is the Poisson ratio and

$$A_{\omega\rho\alpha\beta} = \delta_{\alpha\beta}\delta_{\omega\rho} + \delta_{\alpha\omega}\delta_{\beta\rho} + \delta_{\alpha\rho}\delta_{\beta\omega},$$

$$B_{\omega\rho\alpha\beta} = x_\alpha x_\beta \delta_{\omega\rho} + x_\alpha x_\omega \delta_{\beta\rho} + x_\alpha x_\rho \delta_{\beta\omega} + x_\beta x_\omega \delta_{\alpha\rho} + x_\beta x_\rho \delta_{\alpha\omega} + x_\omega x_\rho \delta_{\alpha\beta},$$

$$C_{\omega\rho\alpha\beta} = x_\alpha x_\beta x_\omega x_\rho,$$

$$D_{\omega\rho\alpha\beta} = \frac{2}{z^2}\left(\frac{\nu}{1-\nu}\delta_{\alpha\beta}\delta_{\omega\rho} + \delta_{\alpha\omega}\delta_{\beta\rho} + \delta_{\alpha\rho}\delta_{\beta\omega}\right) - \frac{2}{z^4}\left(\frac{2\nu}{1-\nu}x_\omega x_\rho \delta_{\alpha\beta} + x_\alpha x_\omega \delta_{\beta\rho} + x_\alpha x_\rho \delta_{\beta\omega} + x_\beta x_\omega \delta_{\alpha\rho} + x_\beta x_\rho \delta_{\alpha\omega}\right). \quad (29)$$

These expressions for the classical Eshelby tensor agree with those given in Mura (1982).

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