On the Propagation of a Shear Band in a Steel Tube

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Marchand and Duffy tested thin-walled steel tubes in a split Hopkinson torsion bar at a nominal strain-rate of approximately 1600/s and could not determine conclusively whether a shear band initiating at a point in the tube propagated around the circumference in one direction or in both directions. They estimated the speed of propagation to be 520 m/s in the former case and 260 m/s in the latter. Here we simulate their test numerically, and find that the shear band propagates in both directions around the circumference of the tube. When the tube is twisted at a nominal strain-rate of 5000/s, the band speed varies from 180 m/s at the site of the initiation to approximately 1000 m/s at the nearly diametrically opposite point. The band speed increases with an increase in the nominal strain-rate. The material defect is modeled by assuming that a small region near the center of the tubular surface is made of a material weaker than that of the rest of the tube.

1 Introduction

Duffy and his co-workers (Marchand and Duffy, 1988; Hartley et al., 1987) have twisted thin-walled tubular steel specimens in a split Hopkinson bar at a nominal strain-rate of approximately 1600/s to study the initiation, development, and propagation of shear bands. Several investigators, e.g., Molinari and Clifton (1987), Wright and Walter (1987), Batra (1992), Wu and Freund (1984), have analyzed one-dimensional simple shearing problems to delineate factors that influence the initiation and growth of shear bands. Two-dimensional plane strain and axisymmetric problems have been studied numerically by Batra and Zhu (1991), Batra and Zhang (1990), Anand et al. (1988), and Batra and Ko (1992, 1993). Zbib and Jubran (1992) have recently studied numerically a three-dimensional problem involving the development of shear bands in a steel bar pulled in tension. Among the studies enumerated above, Batra and Zhu (1991) estimated the speed of propagation of the contours of maximum principal logarithmic strain in a steel block deformed in plane strain compression. They found that the speed depended upon the magnitude of the strain and the state of deformation of the material into which the contours propagated. Batra and Ko (1992) have developed an adaptive mesh refinement technique that generates small elements in the severely deforming region and coarse elements elsewhere.

Here we study numerically the three-dimensional dynamic thermomechanical deformations of 4340 steel thin tube twisted in a split Hopkinson bar at nominal strain-rates of 1000/s, 5000/s, and 25000/s. It is found that the increase in the nominal strain-rate delays the initiation of the shear band originating from the site of the weaker material, and the speed of propagation of the band, taken here to be the same as the speed of propagation of the contour of the effective plastic strain of two, increases with an increase in the nominal strain-rate. At a nominal strain-rate of 1000/s, the band propagates in both directions around the circumference, and its speed increases from 40 m/s at the site of initiation of the band to 260 m/s by the time it has traversed an angular distance of 150 deg.

2 Formulation of the Problem

We study dynamic, adiabatic thermomechanical deformations of a thin-walled tube governed by the following balance laws of mass, linear momentum and internal energy written in the spatial description.

$$\dot{\rho} + \rho \operatorname{div} \mathbf{v} = 0, \quad (1)$$

$$\rho \dot{\mathbf{v}} = \operatorname{div} \sigma, \quad (2)$$

$$\rho \dot{e} = tr(\sigma \mathbf{D}), \quad (3)$$

where

$$\mathbf{D} = \frac{1}{2}(\operatorname{grad} \mathbf{v} + (\operatorname{grad} \mathbf{v})^T), \quad (4)$$

is the strain-rate tensor, ρ the mass density, $\mathbf{v}$ the present velocity of a material particle, σ the Cauchy stress tensor, e the specific internal energy, and a superimposed dot indicates the material time derivative. In Eq. (3) we have neglected the effect of heat conduction thus presuming that the deformations are locally adiabatic. In view of the rather short time, of the order of a millisecond, needed for the band to form, this assumption is reasonable except possibly for copper whose thermal conductivity is greater than ten times that of steel. Also during the late stages of the shear band development when severe temperature gradients have developed, heat conduction probably plays a crucial role in determining the band width. For the simple shearing problem, Batra and Kim (1991) have shown that the band-width decreases with a decrease in the value of the thermal conductivity; however, no simple relation exists between the band-width and the thermal con-

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ductivity. In Eq. (3) we have also assumed that all of the plastic working, rather than 90 to 95 percent, as asserted by Taylor and Quinney (1925), and Sulijoadikusumo and Dillon (1979) is converted into heating.

We assume that the material of the tube is isotropic and for its constitutive relations, we take

$$\boldsymbol{\sigma} = -p\mathbf{1} + \mathbf{s}, \quad p = K\left(\frac{\rho}{\rho_0} - 1\right), \tag{5}$$

$$\dot{\boldsymbol{\sigma}} = \overset{\nabla}{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\,\mathbf{W}^T - \mathbf{W}\,\boldsymbol{\sigma}, \quad \mathbf{W} = \frac{1}{2}(\operatorname{grad}\mathbf{v} - (\operatorname{grad}\mathbf{v})^T), \tag{6}$$

$$\overset{\nabla}{\mathbf{s}} = 2\mu(\overline{\mathbf{D}} - \overline{\mathbf{D}}^p), \quad tr\,\mathbf{D}^p = 0, \quad \overline{\mathbf{D}} = \mathbf{D} - \left(\frac{1}{3}tr\,\mathbf{D}\right)\mathbf{1}, \tag{7}$$

$$\dot{e} = c\dot{\theta} - p\left(\frac{\dot{\rho}}{\rho^2}\right). \tag{8}$$

Here K is the bulk modulus of the material of the tube, ρ_0 its mass density in the stress-free reference configuration, μ the shear modulus, and c the specific heat. Furthermore, $\overset{\nabla}{\boldsymbol{\sigma}}$ denotes the Jaumann derivative of $\boldsymbol{\sigma}$, $\mathbf{W}$ the spin tensor, $\mathbf{s}$ the deviatoric Cauchy stress tensor, and $\overline{\mathbf{D}}$ the deviatoric strain-rate tensor. The deviatoric part $\overline{\mathbf{D}}^p$ of the plastic strain-rate tensor is assumed to be given by

$$\overline{\mathbf{D}}^p = \frac{\dot{\gamma}_p}{\tau}\mathbf{s}, \quad \dot{\gamma}_p = \left(\frac{2}{3}tr(\overline{\mathbf{D}}^p\overline{\mathbf{D}}^p)\right)^{1/2}, \tag{9}$$

$$\tau = (A + B\gamma_p^n)(1 + D\ln\dot{\gamma}_p)(1 - T^m), \quad \gamma_p = \int \dot{\gamma}_p dt, \tag{10}$$

$$T = \frac{\theta - \theta_0}{\theta_m - \theta_0}. \tag{11}$$

Here $\dot{\gamma}_p$ equals the effective plastic strain rate, γ_p the effective plastic strain, τ the equivalent stress, θ_0 the room temperature, θ_m the melting temperature of the material, and A, B, n, D, and m are material parameters. Equation (9) was proposed by Johnson and Cook (1983) based upon the torsion tests of thin tubular specimens conducted at different strain-rates and temperatures. However, the range of temperatures and strain-rates studied is not close to that likely to occur in a shear band problem. For the initial conditions we take

$$\boldsymbol{\sigma}(\mathbf{x}, 0) = \mathbf{0}, \quad \rho(\mathbf{x}, 0) = \rho_0, \quad \mathbf{v}(\mathbf{x}, 0) = \mathbf{0}, \quad \theta(\mathbf{x}, 0) = \theta_0. \tag{12}$$

That is, the body is initially at rest, is stress free at a uniform temperature θ_0, and the initial mass density of every material point is ρ_0. For the boundary conditions, we take

$$\boldsymbol{\sigma}\,\mathbf{n} = \mathbf{0} \quad \text{on the inner and outer surfaces of the tube,} \tag{13}$$

$$\mathbf{v}(x_1, x_2, 0, t) = \mathbf{0}, \tag{14}$$

$$\mathbf{v}(x_1, x_2, L, t) = \omega(t)\,(x_1^2 + x_2^2)^{1/2}\mathbf{e}, \tag{15}$$

$$\operatorname{grad}\theta \cdot \mathbf{n} = 0 \text{ on all bounding surfaces,} \tag{16}$$

where $\mathbf{n}$ is a unit outward normal to the surface, ω is the angular speed of the end surface $x_3 = L$ of the tube, and $\mathbf{e}$ is a unit vector in the surface $x_3 = L$ and is perpendicular to a radial line. The boundary condition (16) is consistent with the assumption that the deformations of the tube are locally adiabatic. The end $x_3 = 0$ of the tube is kept fixed while the other end is twisted at an angular speed $\omega(t)$. We assume that

$$\omega(t) = \begin{cases} \omega_0\, t/20, & 0 \le t \le 20\ \mu\text{s}, \\ \omega_0, & t > 20\ \mu\text{s}, \end{cases} \tag{17}$$

where ω_0 is the steady value of the angular speed on the end surface $x_3 = L$. The rise time of 20 μs is typical for torsional tests done in a split Hopkinson bar.

3 Computation and Discussion of Results

In order to compute numerical results we assigned following values to various material and geometric parameters.

$$\rho = 7860\ \text{kg/m}^3,\ \mu = 76\ \text{GPa},\ \theta_m = 1520^\circ\text{C},\ c = 473\ \text{J/kg}^\circ\text{C},$$
$$\theta_0 = 25^\circ\text{C},\ A = 792.19\ \text{MPa},\ B = 509.51\ \text{MPa},\ D = 0.014,$$
$$n = 0.26,\ m = 1.03,\ \text{Tube thickness} = 0.38\ \text{mm},$$
$$\text{Inner radius of the tube} = 4.75\ \text{mm},$$
$$\text{axial length of the tube} = 2.5\ \text{mm}. \tag{18}$$

The values of material parameters for the 4340 steel in the Johnson-Cook model are taken from Rajendran (1992). The thickness of the tube, its inner radius and its gage length correspond to that used by Marchand and Duffy (1988) in their tests. The value of the angular speed ω_0 was varied so as to produce the desired nominal strain-rate.

The aforestated problem was solved by using the explicit large scale finite element code DYNA3D (Whirley and Hallquist, 1991). The code computes the size of the time step based on the Courant condition thereby ensuring the stability of the computed solution. For the parameters listed above, the computed shear stress versus nominal shear strain curves for the 4340 steel tube twisted at nominal strain-rates of 1000/s, 5000/s, and 25000/s are depicted in Fig. 1. It is clear that softening of the material because of its being heated up overcomes its hardening due to strain and strain-rate effects at nominal strains of 0.64, 0.67, and 0.78, respectively.

The finite element mesh consisted of 8-noded brick elements with 30 elements along the gage length of the tube, 4 elements across the thickness, and 80 elements along the circumference. A material defect was modeled by presuming that the yield stress in a quasistatic simple compression test for the material in the region consisting of two rows of four elements through the thickness and located near the center of the gage length

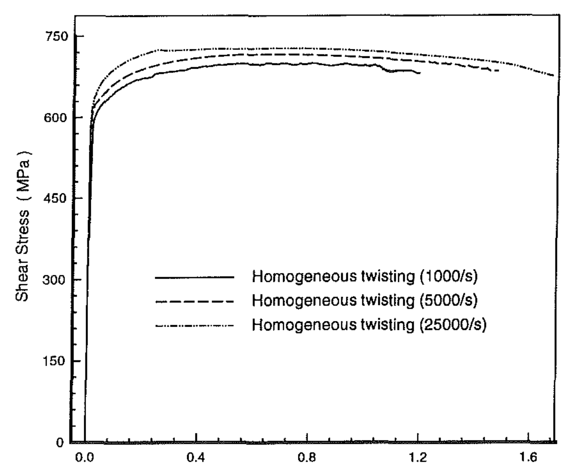


Fig. 1 Shear stress-shear strain curves for homogeneous deformations of the tube at nominal strain-rates of 1000/s, 5000/s, and 25000/s

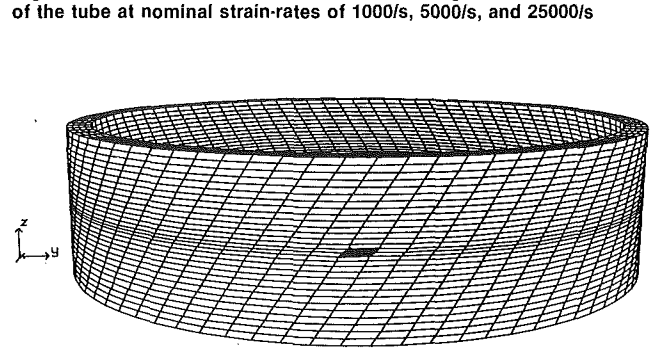


Fig. 2(*a*)

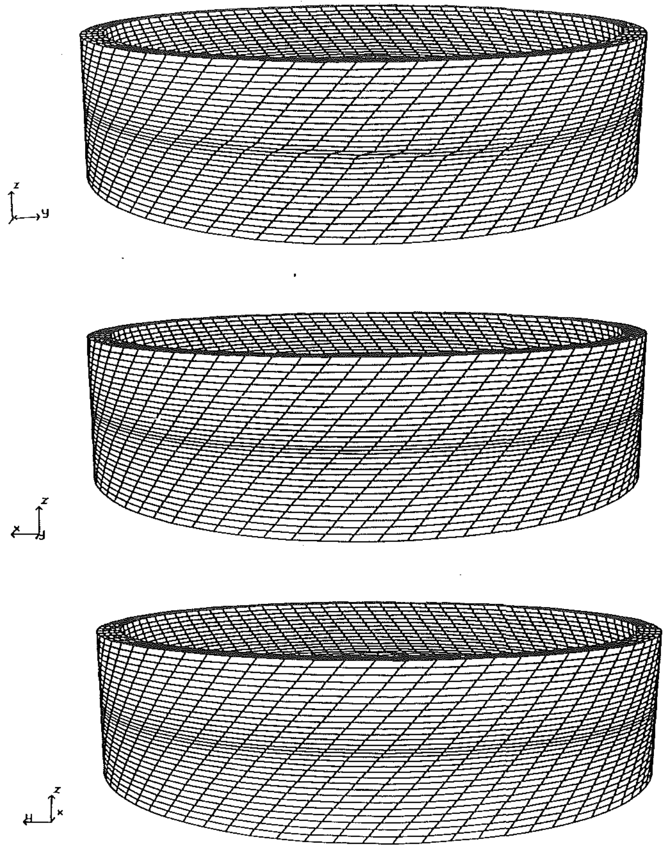


Fig. 2(*b*)

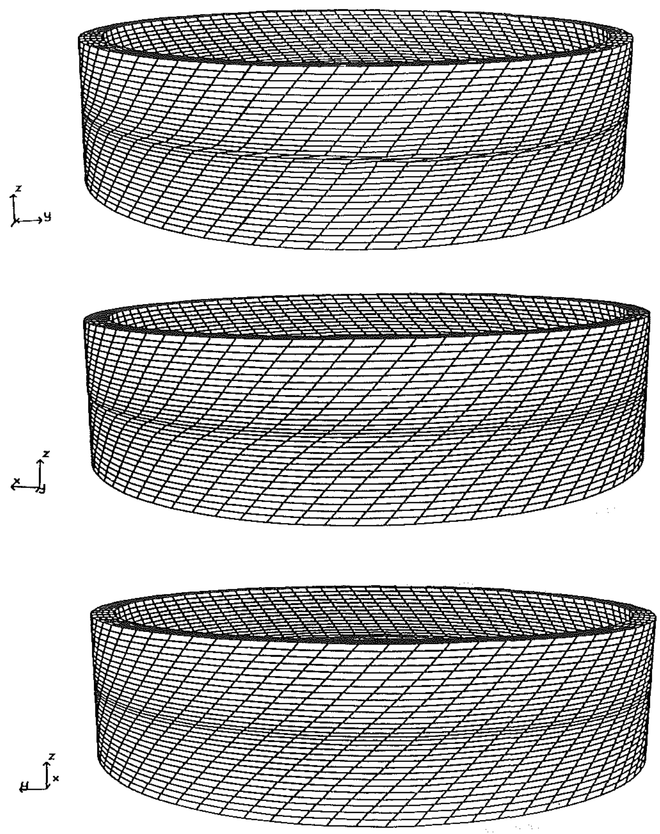


Fig. 2(*c*)

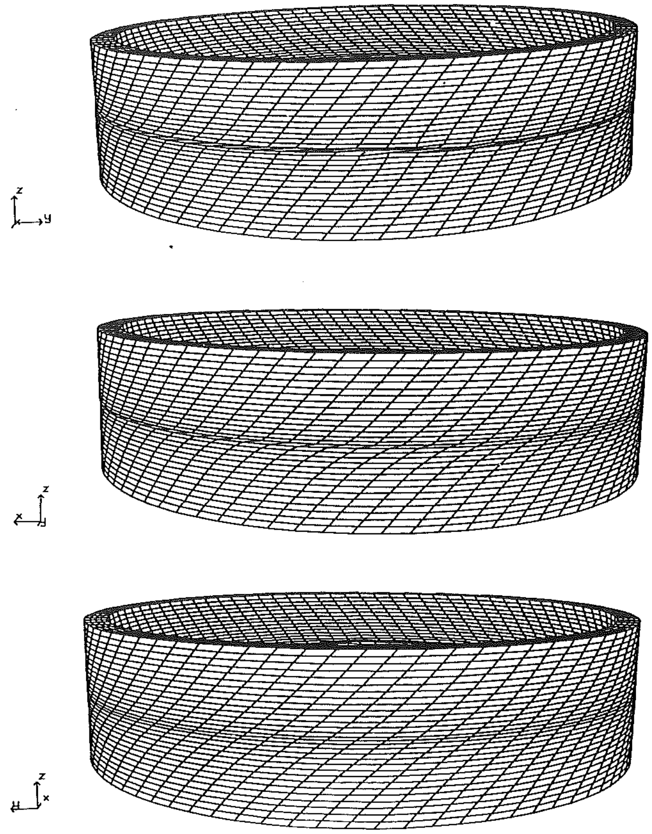


Fig. 2(*d*)

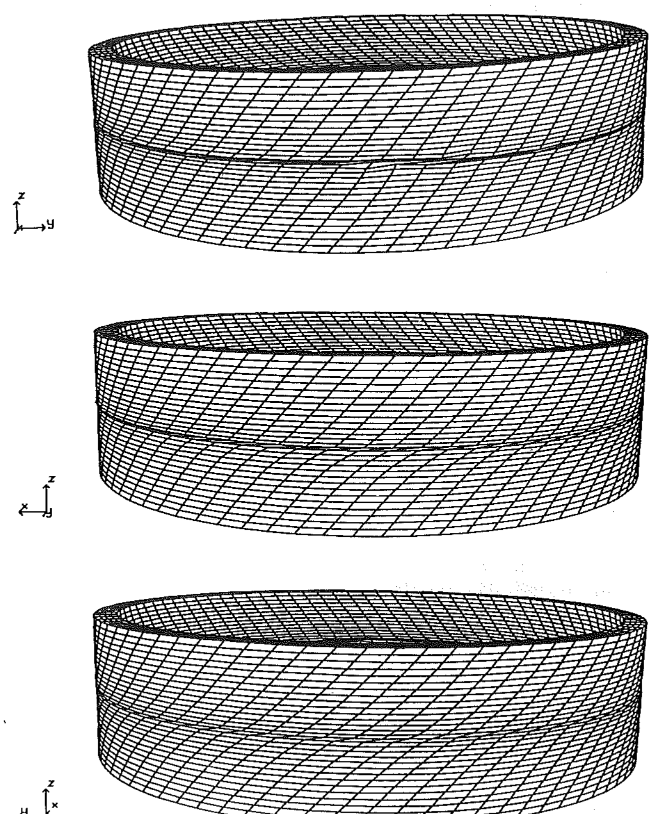


Fig. 2(*e*)

Fig. 2 Deformed meshes at (*a*) $t = 140\ \mu s$, (*b*) 180 μs, (*c*) 200 μs, (*d*) 220 μs, and (*e*) 240 μs for the tube twisted at an average strain-rate of 5000/s. For Figs. 2(*b*) through (*e*), the top, middle, and bottom pictures depict views of the deformed mesh from the front, right side, and the back, respectively.

was 10 percent lower than that of the rest of the material. It was accomplished by reducing the aforelisted values of A and B by 10 percent. The two weak elements as seen from the front are shown as shaded in Fig. 2(*a*).

Figure 2 depicts the deformed meshes at time $t = 140\ \mu s$, 180 μs, 200 μs, 220 μs, and 240 μs for the tube twisted at an average strain-rate of 5000/s. The initially vertical lines on the outer surface of the untwisted tube correspond to the lines scribed on the outer surface of the tube and parallel to its axis in Marchand and Duffy's (1988) experiments. The tangent of the angle through which one of these lines rotates clockwise at a point is a measure of the shear strain induced there. At $t = 140$ μs, from the front view of the deformed mesh shown in Fig. 2(*a*), one can conclude that the weaker region has been deformed more than the rest of the body and deformations everywhere except in the small weaker region are essentially homogeneous. In each of Figs. 2(*b*) through Fig. 2(*e*), the top, middle, and bottom pictures depict the deformed meshes as seen from the front, right side, and the back. At $t = 180\ \mu s$ (cf. Fig. 2(*b*)) the severe deformations initiating from the weaker region have propagated along the circumference of the tube. However, views from the side and the back reveal that these intense deformations are confined to a small arc length around the weak region. At $t = 200\ \mu s$ (Fig. 2(*c*)) and 220 μs (Fig. 2(*d*)), the severe deformations have propagated further along the circumference, but the views from the back indicate that they have not progressed all around the circumference. At $t = 240\ \mu s$ (Fig. 2(*e*)), a thin narrow region near the middle of the tube and all around its circumference has undergone significant deformations. The deformed shapes of initial vertical lines match closely with those reported by Marchand and Duffy (1988). Notice that only one element near the center of the tube has been distorted severely. The deformed shapes of initially vertical lines outside of this element are virtually straight and parallel to each other implying thereby that the material there is deforming homogeneously. Once a shear band has formed across the circumference of the tube, the material for subsequent deformations is divided into three regions; the lower one adjoining the fixed end remains essentially stationary, the middle one consisting of one element which undergoes nearly all of the deformations, and the upper one adjoining the moving end which rotates as a rigid body. These results are similar to those obtained by Batra and Kim (1992) who analyzed simple shearing deformations of a viscoplastic block and studied the problem for twelve different materials. In the present work, the deformations were found to be uniform through the thickness of the tube.

In order to delineate further whether a shear band propagates in one direction or in both directions along the circumference of the tube and to determine its speed of propagation, we have plotted in Figs. 3(*a*), 3(*b*), and 3(*c*) the evolution of the effective plastic strain at several points along the circumference of the tube being twisted at nominal strain-rates of 1000 s^{-1}, 5000 s^{-1}, and 25000 s^{-1}, respectively. The angular location of these points is also shown in the figures. These points are on the circumference passing through point A which is at the center of the band in the weak region. We note that the curves corresponding to points B and H, C and G, and D and F coincide with each other implying thereby that the material instability initiating from point A propagates in both directions along the circumference at the same speed. Since the lengths of circular segments AB, BC, CD, and DE are equal to each other, unequal horizontal distance between the curves for points B and C, C and D, and D and E suggests that the speed of propagation is a function of the angular position θ and hence of the state of deformation at a material point. At a nominal strain-rate of 1000/s, the curve for point E is parallel to that for points B, C, and D but such is not the case at nominal strain-rates of 5000/s and 25000/s. An explanation for this could be that there is a strong interaction between the circumferentially propagating disturbances meeting at point E at the higher strain-rates. We note that Batra (1988) pointed out that inertia forces begin playing a noticeable role in the simple shearing problem at a nominal strain-rate of 5000/s.

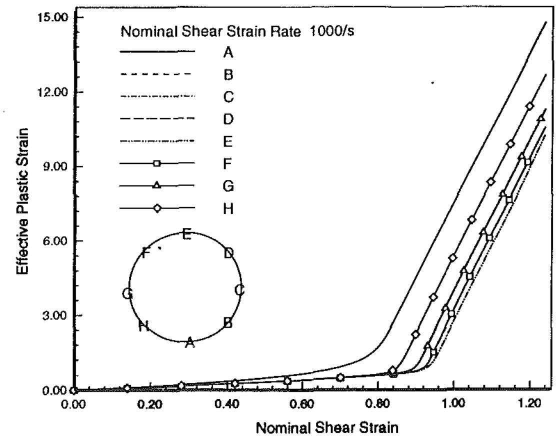


Fig. 3(*a*)

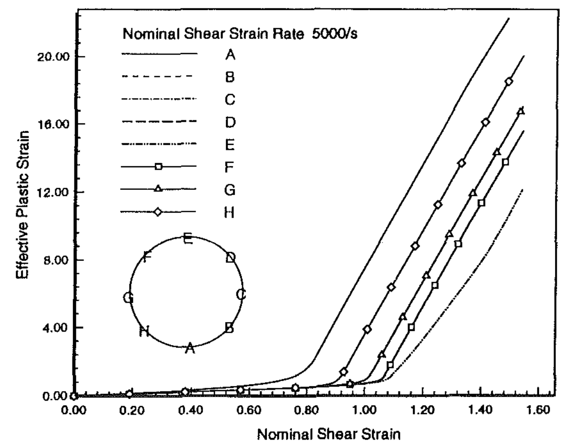


Fig. 3(*b*)

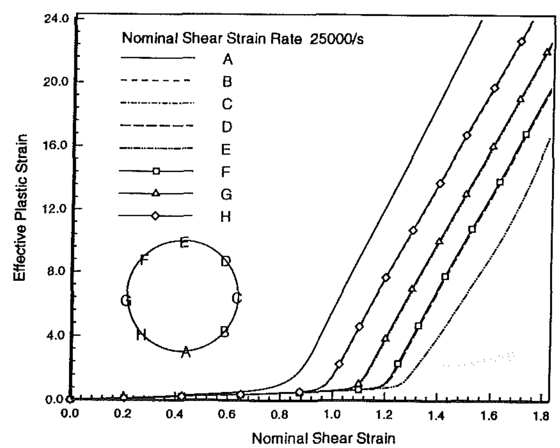


Fig. 3(*c*)

Fig. 3 Evolution of the effective plastic strain at several points along the circumference of the tube. (*a*) $\dot{\gamma}_0 = 1000/s$, (*b*) $\dot{\gamma}_0 = 5000/s$, (*c*) $\dot{\gamma}_0 = 25000/s$.

In Fig. 3 the curves depicting the evolution of the effective plastic strain at points A, B, C, and D after the shear band has initiated stay essentially parallel to each other suggesting that the speed of propagation of different strain levels is nearly the same. In order to illustrate the spatial variation of the speed of propagation of the shear band, we concentrate on how the strain level $\gamma_p = 2$ propagates along the circumference.

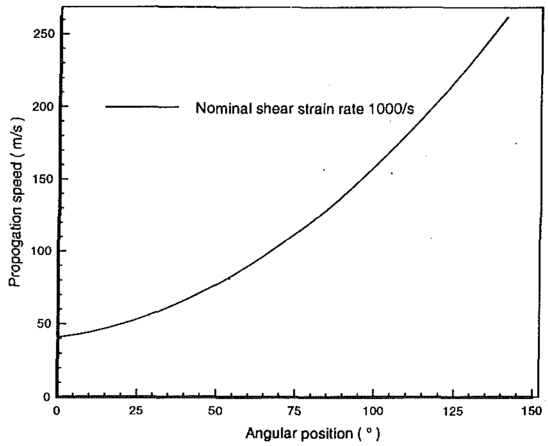


Fig. 4(*a*)

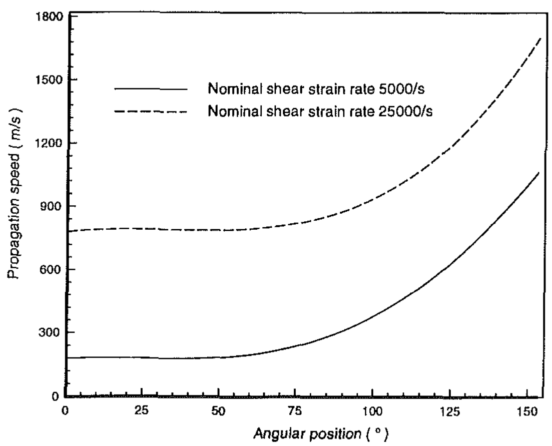
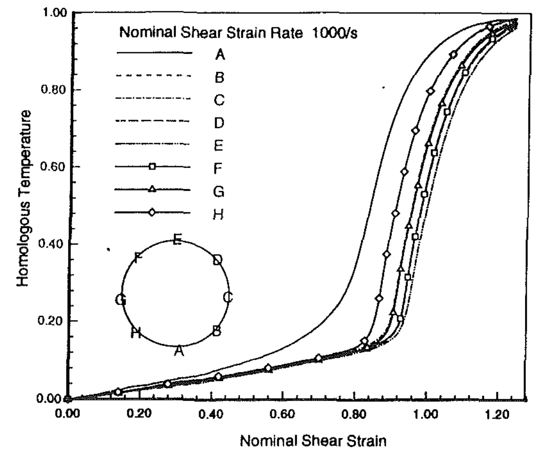
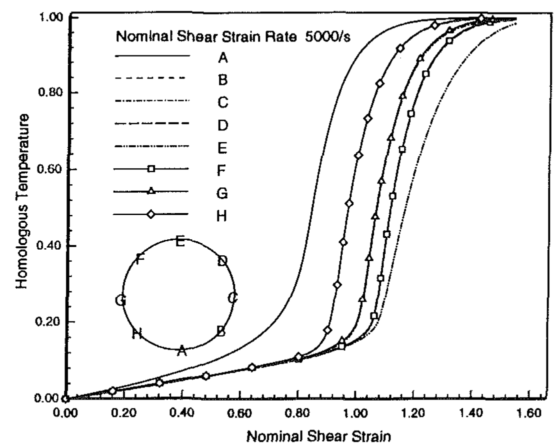


Fig. 4(*b*)

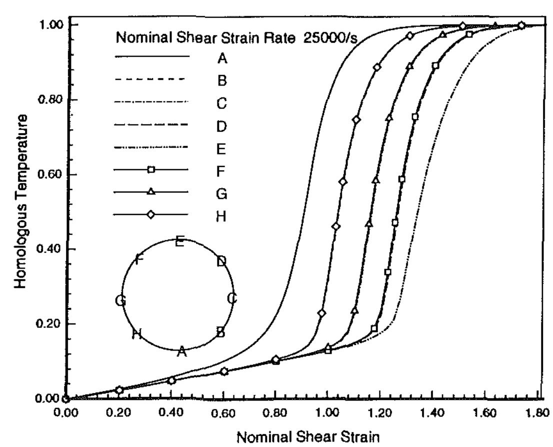
Fig. 4 Variation of the band speed with the angular position. (*a*) $\dot{\gamma}_0 =$ 1000/s, (*b*) $\dot{\gamma}_0 =$ 5000/s, and 25000/s.



(*a*)



(*b*)



(*c*)

Fig. 5 Evolution of the homologous temperature at several points along the circumference of the tube. (*a*) $\dot{\gamma}_0 =$ 1000/s, (*b*) $\dot{\gamma}_0 =$ 5000/s, (*c*) $\dot{\gamma}_0 =$ 25000/s.

The speed of propagation of $\gamma_p = 2$ was determined by dividing the arc length joining the centroids of two adjoining elements by the time taken for γ_p to equal 2 at these points. The value of band speed so computed was assigned to the midpoint of the arc length joining the centroids of the abutting elements. As evidenced in Figs. 4(*a*) and 4(*b*), the band speed varies significantly along the circumference of the tube and at a point increases with the nominal strain-rate. At a nominal strain-rate of 25000/s, the band speed increases from 750 m/s at point A to 1700 m/s at a point 150 deg along the circumference through A. However, at a nominal strain-rate of 1000/s, the band speed varies from 40 m/s to 260 m/s between the same two points. Thus, the band speed is a function of the angular position, and hence of the state of deformation at a point.

The evolution of the homologous temperature at points A through H at the three nominal strain-rates studied is depicted in Fig. 5. The homologous temperature of a material point is defined as its absolute temperature divided by the melting temperature of the material. Since deformations have been assumed to be locally adiabatic, the temperature rise at a point is proportional to the energy dissipated there. As for the evolution of the effective plastic strain-rate, the curves for the evolution of the temperature at points B, C, and D coincide with those for points H, G, and F, respectively. At the nominal strain-rate of 1000/s the curves for points B, C, D, and E are parallel to each other, but at the higher strain-rates of 5000/s and 25000/s curves for points B, C, and D only are parallel to each other.

The torque required to twist the tube as a function of the nominal strain is plotted in Fig. 6. The value of the torque was determined from the shear stresses computed at numerous points on the end face of the tube. Since the radius of a material point and hence the tube thickness remains essentially unchanged during the deformations of the tube, the ordinate can also be interpreted as the nominal shear stress. The torque first increases linearly because of the elastic deformations of the tube, and subsequently increases slowly when plastic deformations of the tube are predominant. The peak value of the torque increases with the increase in the nominal strain-rate, however the increase in the peak value of the torque is less when the nominal strain-rate is increased from 5000/s to 25000/s than when it is increased from 1000/s to 5000/s. For each one of

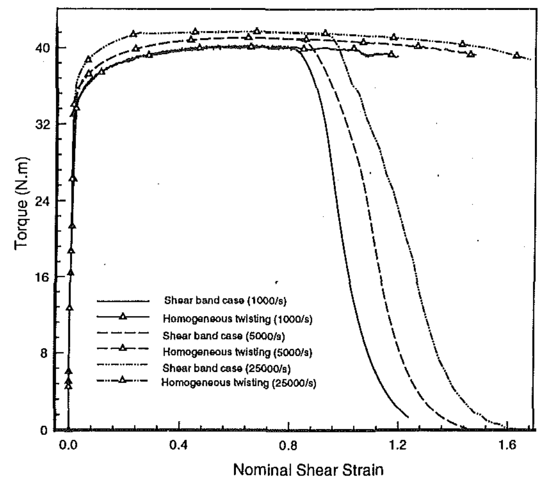


Fig. 6 Torque required to twist the tube versus the nominal strain

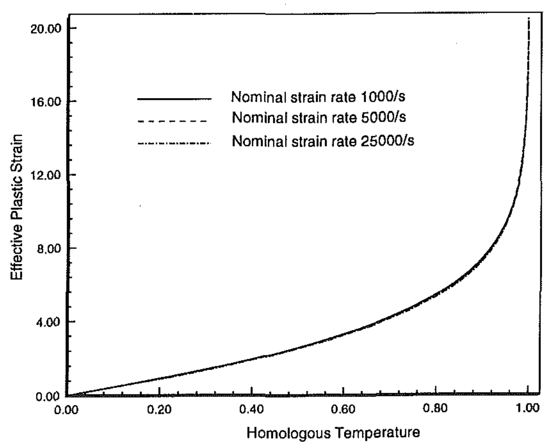


Fig. 7 Effective plastic strain versus the homologous temperature at the band center for the three values of the nominal strain-rate studied

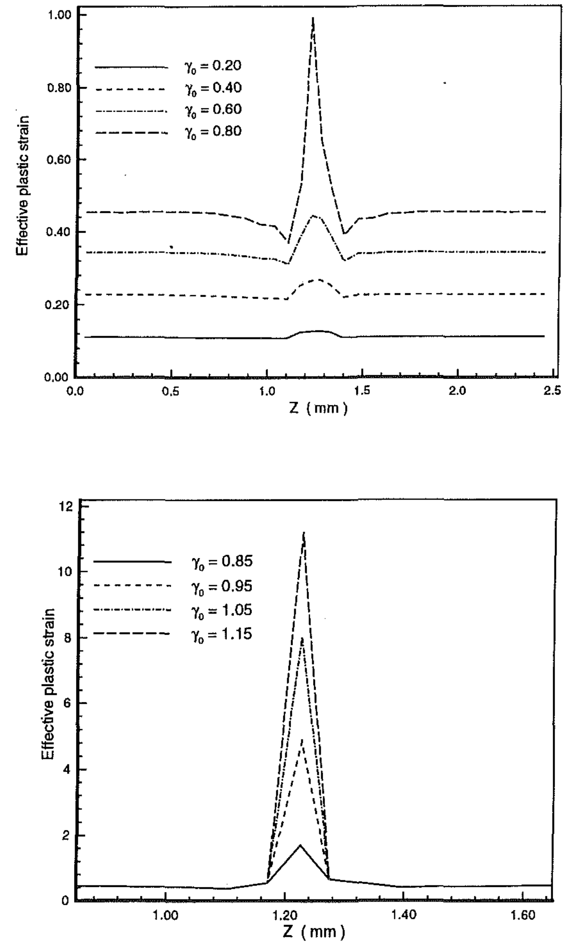


Fig. 8 Distribution, at different times, of the effective plastic strain along an axial line

these three values of the nominal strain-rates, the torque versus nominal shear strain curve follows that for the homogeneous tube well beyond the peak value of the torque. Similar observation was first made by Wright and Batra (1985) who studied a simple shearing problem. The precipitous drop in the torque signifies the initiation of a shear band, and suggests that a shear band initiates at nominal strains of 0.80,0.84, and 0.94 for $\dot{\gamma}_0 = 1000/s$, 5000/s, and 25000/s, respectively. These values of the nominal strains equal essentially those in Fig. 3 at which the effective plastic strain at point A begins to increase sharply. This sudden drop in the torque, and thus the load carrying capacity of the tube, was also observed by Marchand and Duffy (1988) in their tests. Thus, the nominal strain at which a shear band initiates increases with an increase in the value of the nominal strain-rate. The incremental increase in the value of the nominal strain at the initiation of the shear band is more when the nominal strain-rate is increased from 5000/s to 25000/s than that when it is increased from 1000/s to 5000/s. However, in every case the rate of drop of the torque appears to be nearly the same. From Fig. 5 we see that the homologous temperature at point A at the instant of the precipitous drop in the torque equals 0.35, 0.48, and 0.65 for $\dot{\gamma}_0 = 1000/s$, 5000/s, and 25000/s, respectively. For each one of the three values of $\dot{\gamma}_0$, the homologous temperature at point A begins to rise sharply prior to the initiation of the shear band.

Batra and Ko (1993) studied the development of a shear band in a cylindrical block undergoing either axisymmetric or plane strain deformations. They found that the nominal axial strain at the initiation of the shear band was significantly more for axisymmetric deformations as compared to that for plane strain deformations. However, in both cases, the temperature at the band center was nearly the same when the band initiated. They thus stated that the temperature at the band center rather than the nominal strain was a better indicator of when a shear band initiates. The observation made above that the homologous temperature at the band center equalled 0.35, 0.48, and 0.65 for $\dot{\gamma}_0 = 1000/s$, 5000/s, and 25000/s respectively implies that the homologous temperature is not necessarily a good indicator of when a shear band initiates. Nevertheless, we have plotted in Fig. 7 the effective plastic strain versus the homologous temperature at the band center at point A for the three nominal strain-rates studied. It is clear that the three curves essentially coincide with each other. However, it is hard to decipher from them the value of the homologous temperature at the instant of the initiation of the shear band. Why the three curves overlap should be apparent from the following equation (Cao, 1993)

$$\rho c \dot{\theta} = (A + B\gamma_p^n)(1 + D \ln \dot{\gamma}_p)(1 - T^m)\dot{\gamma}_p \qquad (19)$$

obtained from Eqs. (3), (9), and (10). Since $D \ll 1$, Eq. (19) is a functional relationship between the temperature and the effective plastic strain which is virtually independent of the nominal strain-rate, and the three curves in Fig. 7 depict this functional relationship. If the effect of heat conduction were included in Eq. (3), then Eq. (19) will still hold to a good approximation since the thermal conductivity of steel is very small. It should explain the observations of Batra and Ko (1993) who incorporated the effect of heat conduction in the balance of internal energy.

In Figs. 8(*a*) and 8(*b*) we have plotted, at different times, the variation of the effective plastic strain along an axial line for the case when the tube is twisted at a nominal strain-rate of 5000/s. The time is measured in terms of the nominal strain. Note that the horizontal scale in Fig. 8(*b*) is enlarged to demonstrate vividly the strain distribution. These plots show that once the deformation has localized into a shear band at a nominal strain of 0.85, the material outside of it undergoes essentially no deformations during the subsequent twisting of the tube and all of the deformations are concentrated within the band. When the nominal strain increases from 0.85 to 1.15, the peak value of the effective plastic strain within the band increases from 1.6 to 11.5. Because of the rather coarse mesh used, it is not feasible to ascertain the band-width. A finer mesh could not be used because of the limited computational resources available.

Conclusions

We have studied the initiation, growth, and the propagation of a shear band in a thin steel tube twisted at nominal strain-rates of 1000/s, 5000/s, and 25000/s. One end of the tube is kept fixed while the other end is twisted at a prescribed rate. The inner surface and the mantle of the tube are taken to be traction free, and its deformations are presumed to be locally adiabatic. A material defect has been modeled by presuming that a small region consisting of two elements along the axis of the tube situated midway between the tube ends is made of a material weaker than that of the rest of the tube. This weak region extends through the thickness of the tube and 6 deg across its circumference. It is found that a shear band initiates from the site of the weak region and propagates in both directions along the circumference. Its speed of propagation varies with the nominal strain-rate and also depends upon the state of deformation at a point. At a nominal strain-rate of 1000/s, the band speed varies from 40 m/s to 260 m/s. The nominal strain at which a shear band initiates increases with the nominal strain-rate.

Acknowledgments

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