Effective Electroelastic Properties of a Piezocomposite with Viscoelastic and Dielectric Relaxing Matrix

B. JIANG* AND R.C. BATRA

Department of Engineering Science and Mechanics, MC 0219, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

ABSTRACT: We use the correspondence principle of linear viscoelasticity and the Mori–Tanaka averaging method to determine the effective electroviscoelastic moduli of a piezocomposite made of piezoceramic (PZT) inclusions and a viscoelastic matrix whose dielectric constants also vary with time t. For elliptic cylindrical PZT inclusions, closed form expressions for the electroviscoelastic moduli are derived. It is found that for the 1-3 piezocomposite, the relaxation of the dielectric constants of the matrix affects only the dielectric moduli and the shear components of the piezoelectric and the viscoelastic moduli of the piezocomposite. However, the viscoelasticity of the matrix influences every effective electroelastic modulus of the piezocomposite. The relaxation times of any two effective moduli are different, and the piezocomposite is an orthotropic material. For time-harmonic loading of the piezocomposite studied, only frequencies in the range of 10^{-2}–10^{2} Hz strongly influence the effective storage and the effective loss moduli. Whereas thin PZT ribbons provide the most mechanical strength to the piezocomposite, they give the least piezoelectric effect.

1 INTRODUCTION

POLYMERS usually exhibit viscoelastic (Ferry, 1970) and dielectric relaxation (Hedvig, 1977; Lovinger, 1983; Dias and Das-Gupta, 1996) phenomena. The effective properties of a composite made of elastic inclusions and a viscoelastic matrix have been derived by Li and Weng (1994), Alberola and Benzarti (1998), and Aboudi (2000). Whereas effective electroelastic properties of piezocomposites with piezoelectric (PZT) inclusions and an elastic matrix have been widely studied (e.g., see Chan and Unsworth, 1989; Dunn and Taya, 1993; Avellaneda and Swart, 1998; Agbossou et al., 1999; Jiang et al., 1999a; Hornsby and Das-Gupta, 2000), those of a piezocomposite with a viscoelastic matrix seem not to have been scrutinized. Jiang et al. (1999b, 2000) have derived effective moduli of spherical PZT inclusions embedded in a viscoelastic and dielectrically relaxing matrix without accounting for the interaction among the inclusions, and they did not give closed form expressions for the effective electroelastic moduli of the piezocomposite.

Here we use the correspondence principle of linear viscoelasticity, account for the interaction among PZT inclusions, and give closed form expressions for the effective electroviscoelastic moduli of a 1-3 piezocomposite consisting of parallel PZT cylinders of elliptic cross section embedded in a viscoelastic matrix. The relaxation of the dielectric moduli of the matrix is found to affect only the dielectric moduli and shear components of the piezoelectric and elastic moduli of the piezocomposite. However, viscous effects in the matrix influence all of the moduli of the piezocomposite. These moduli also strongly depend upon the shape of the cross section of the cylindrical PZT inclusions.

The paper is organized as follows. The problem is formulated in Section 2, and expressions for the constraint strain and the constraint electric field for a single cylindrical PZT inclusion embedded in an infinite viscoelastic matrix are deduced. The effective electroelastic moduli of a piezocomposite with several parallel cylindrical PZT inclusions in a dielectrically relaxing viscoelastic matrix are derived in Section 3. In Section 4, we compare computed values of the effective piezoelectric moduli with experimental results for an elastic matrix. We also compare the computed values of the effective moduli for elastic cylindrical inclusions in a viscoelastic matrix with both the experimental results and with the predictions from Li and Weng's (1994) model. Predictions from the present model for numerous values of material parameters of the dielectrically relaxing viscoelastic matrix and aspect ratios of the cross section of the PZT inclusions are exhibited and discussed in Section 5. Section 6 summarizes the work.

*Author to whom correspondence should be addressed.

1045-389X/01/12 0847–20 $10.00/0 DOI: 10.1106/CQCM-9WK7-4FVT-5NHP

2 FORMULATION OF THE PROBLEM

We use rectangular Cartesian coordinates to describe infinitesimal deformations of a composite made of a viscoelastic matrix and PZT inclusions; all of the PZT inclusions are cyilndrical, have the same cross section and are aligned parallel to each other. In the absence of body forces and free charges, deformations of the matrix and the inclusions are governed by

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \tag{2.1}$$

where σ is the stress tensor, D the electric displacement, a comma followed by the index j indicates partial differentiation with respect to the position x_j of a material particle, and a repeated index unless otherwise noted implies summation over the range of the index. The constitutive relations for the viscoelastic matrix are

$$\begin{aligned} \sigma_{ij}(\mathbf{x}, t) &= C^M_{ijkl}(0)\varepsilon_{kl}(\mathbf{x}, t) + \int_0^t \dot{C}^M_{ijkl}(\tau)\varepsilon_{kl}(\mathbf{x}, t-\tau)d\tau, \\ D_i(\mathbf{x}, t) &= k^M_{ij}(0)E_j(\mathbf{x}, t) + \int_0^t \dot{k}^M_{ij}(\tau)E_j(\mathbf{x}, t-\tau)d\tau, \end{aligned} \tag{2.2}$$

and those for the PZT inclusions are

$$\begin{aligned} \sigma_{ij}(\mathbf{x}, t) &= C^I_{ijkl}\varepsilon_{kl}(\mathbf{x}, t) - e^I_{lij}E_l(\mathbf{x}, t), \\ D_i(\mathbf{x}, t) &= e^I_{ijk}\varepsilon_{jk}(\mathbf{x}, t) + k^I_{ij}E_j(\mathbf{x}, t), \end{aligned} \tag{2.3}$$

where

$$E_j(\mathbf{x}, t) = -\phi_{,j}, \; 2\varepsilon_{ij}(\mathbf{x}, t) = u_i(\mathbf{x}, t)_{,j} + u_j(\mathbf{x}, t)_{,i}. \tag{2.4}$$

Here $\mathbf{C}$, $\mathbf{k}$ and $\mathbf{e}$ are respectively the elastic, the dielectric and the piezoelectric moduli, superscripts M and I denote their values for the matrix and the PZT inclusions respectively, a superimposed dot indicates partial derivative with respect to time t, $\mathbf{u}$ is the mechanical displacement, $\mathbf{E}$ the electric field and ϕ the electric potential.

It is assumed that the matrix and the inclusions are perfectly bonded together so that at their common interfaces

$$[\mathbf{u}] = \mathbf{0}, \quad [\sigma]\mathbf{n} = \mathbf{0}, \quad [\phi] = \mathbf{0}, \quad [\mathbf{D}]\cdot\mathbf{n} = 0, \tag{2.5}$$

where, $[f] = f^M - f^I$ denotes the jump in the value of f in going from the inclusion to the matrix across their common interface, and $\mathbf{n}$ is a unit outward normal to the interface.

We consider a representative volume element (RVE) of the piezocomposite that is large enough so that its effective moduli are invariant with respect to its rigid translations in the body. The spatial distributions of the similarly aligned PZT inclusions are such that the RVE can be regarded as homogeneous. Boundary conditions imposed on the surfaces of the RVE correspond to fields of uniform strain ε^0_{ij} and uniform electric field E^0_i within the RVE. Thus

$$u_i(x, t) = \varepsilon^0_{ij}(t)x_j, \quad \phi(\mathbf{x}, t) = -E^0_i(t)x_i. \tag{2.6}$$

Taking the Laplace transform of Equations (2.1)–(2.6) we obtain analogous equations in the transformed quantities except for the two equations derived from $(2.2)_1$ and $(2.2)_2$. The transformed versions of Equations $(2.2)_1$ and $(2.2)_2$ are

$$\hat{\sigma}_{ij}(\mathbf{x}, s) = s\hat{C}^M_{ijkl}(s)\hat{\varepsilon}_{kl}(\mathbf{x}, s), \quad \hat{D}_i(\mathbf{x}, s) = s\hat{k}^M_{ij}(s)\hat{E}_j(\mathbf{x}, s), \tag{2.7}$$

where a superimposed ^ indicates the Laplace transform of the quantity. The set of transformed equations is identical to the set of equations for an elastic matrix and PZT inclusions with the elastic and the dielectric moduli of the elastic matrix replaced respectively by $s\hat{\mathbf{C}}^M$ and $s\hat{\mathbf{k}}^M$. It thus follows that the Laplace transformed displacements $\hat{\mathbf{u}}$ and the electric potential $\hat{\phi}$ can be obtained from the solution of the corresponding problem for an elastic matrix.

For an ellipsoidal PZT inclusion embedded in an infinite non-electromechanically coupled elastic matrix, Jiang et al. (1999a) derived expressions for the constraint strain ε^I and the constraint electric field E^I in terms of the applied uniform fields ε^0 and $\mathbf{E}^0$. With the parameter replacing, the transformed constraint strain $\hat{\varepsilon}^I$ and the transformed constraint electric field $\hat{\mathbf{E}}^I$ for an ellipsoidal PZT inclusion embedded in an infinite viscoelastic matrix are given by

$$\hat{\varepsilon}^I_{ij} = \hat{H}^1_{ijkl}\hat{\varepsilon}^0_{kl} + \hat{H}^2_{ijkl}\hat{E}_k, \quad \hat{E}^I_i = \hat{H}^3_{ijk}\hat{\varepsilon}^0_{jk} + \hat{H}^4_{ij}\hat{E}^0_j, \tag{2.8}$$

where

$$\begin{aligned} \hat{H}^1_{ijkl} &= \hat{X}_{ijmn}\hat{A}_{mnkl}, \quad \hat{H}^2_{ijk} = \hat{X}_{ijmn}\hat{\alpha}_{mnp}\hat{B}_{pk}, \\ \hat{H}^3_{ijkl} &= \hat{Y}_{im}\hat{\beta}_{mnp}\hat{A}_{npjk}, \quad \hat{H}^4_{ij} = \hat{Y}_{im}\hat{B}_{mj}, \end{aligned} \tag{2.9}$$

are the transformed electroelastic Eshelby tensors. In Equations (2.9)

$$\begin{aligned} \hat{A}_{ijkl} &= [I_{ijkl} + \hat{S}^u_{ijmn}(s\hat{C}^M_{mnpq})^{-1}\hat{\mathbb{C}}_{pqkl}]^{-1}, \\ \hat{B}_{ij} &= [\delta_{ij} + \hat{S}^\phi_{im}(s\hat{k}^M_{mn})^{-1}\hat{K}_{nj}]^{-1}, \\ \hat{X}_{ijkl} &= (I_{ijkl} + \hat{\alpha}_{ijm}\hat{\beta}_{mkl})^{-1}, \quad \hat{Y}_{ij} = (\delta_{ij} + \hat{\beta}_{imn}\hat{\alpha}_{mnj})^{-1}, \\ \hat{\alpha}_{ijk} &= \hat{A}_{ijmn}\hat{S}^u_{mnpq}(s\hat{C}^M_{pqrs})^{-1}e^I_{krs}, \\ \hat{\beta}_{ijk} &= \hat{B}_{im}\hat{S}^\phi_{mn}(s\hat{k}^M_{np})^{-1}e^I_{pjk}, \\ I_{ijkl} &= (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2, \quad \hat{\mathbb{C}} = \mathbf{C}^I - s\hat{\mathbf{C}}^M, \; \hat{\mathbf{K}} = \mathbf{k}^I - s\hat{\mathbf{k}}^M, \end{aligned} \tag{2.10}$$

δ_{ij} is the Kronecker delta, and $\hat{S}^u$ and $\hat{S}^\phi$ are respectively the transformed Eshelby tensors for the perfect elastic and dielectric inclusion whose shape and orientation are the same as those of the PZT inclusion under consideration. The transformed Eshelby tensors can be obtained by parameter replacing.

3 EFFECTIVE CONSTITUTIVE RELATIONS FOR THE PIEZOCOMPOSITE

The transformed constitutive relations of the piezocomposite in terms of the effective moduli $\bar{\mathbf{C}}$, $\bar{\mathbf{k}}$ and $\bar{\mathbf{e}}$ can be written as

$$\hat{\sigma}^0_{ij} = \hat{\bar{C}}_{ijkl}\hat{\varepsilon}^0_{kl} - \hat{\bar{e}}_{kij}\hat{E}^0_k, \quad \hat{D}^0_i = \hat{\bar{e}}_{ijk}\hat{\varepsilon}^0_{jk} + \hat{\bar{k}}_{ij}\hat{E}^0_j. \qquad (3.1)$$

Taking the inverse Laplace transform of (3.1) we obtain the constitutive relations for the piezocomposite as

$$\begin{aligned} \sigma^0_{ij}(\mathbf{x}, t) &= \int_{-\infty}^{t} \bar{C}_{ijkl}(t-\tau)\frac{\partial \varepsilon^0_{kl}(\mathbf{x},\tau)}{\partial \tau}d\tau \\ &\quad - \int_{-\infty}^{t} \bar{e}_{kij}(t-\tau)\frac{\partial E^0_k}{\partial \tau}(\mathbf{x},\tau)d\tau, \\ D^0_i(\mathbf{x}, t) &= \int_{-\infty}^{t} \bar{e}_{ijkl}(t-\tau)\frac{\partial \varepsilon^0_{jk}}{\partial \tau}(\mathbf{x},\tau)d\tau \\ &\quad + \int_{-\infty}^{t} \bar{k}_{ij}(t-\tau)\frac{\partial E^0_j}{\partial \tau}(x,\tau)d\tau. \end{aligned} \qquad (3.2)$$

The transformed effective moduli, obtained from the solution of the corresponding problem of the elastic matrix by the parameter replacement, are

$$\begin{aligned} \hat{\bar{C}}_{ijkl} &= s\hat{C}_{ijkl} + f\,\mathrm{sym}[\hat{\mathbb{C}}_{ijmn}\hat{M}_{mnkl} - \hat{e}_{mij}\hat{P}_{mkl}], \\ \hat{\bar{k}}_{ij} &= s\hat{k}_{ij} + f\,\mathrm{sym}[\hat{e}_{imn}\hat{N}_{mnj} + \hat{K}_{im}\hat{Q}_{mj}], \\ \hat{\bar{e}}_{ijk} &= \tfrac{1}{2}f[\hat{K}_{im}\hat{P}_{mjk} - \hat{\mathbb{C}}_{jkmn}\hat{N}_{mni} + \hat{Q}_{im}\hat{e}_{mjk} + \hat{e}_{imn}\hat{M}_{mnjk}], \end{aligned} \qquad (3.3)$$

where $\mathrm{sym}[F_{ijkl}] = \frac{1}{2}(F_{ijkl} + F_{klij})$, $\mathrm{sym}(J_{ij}) = \frac{1}{2}(J_{ij} + J_{ji})$, and f is the volume fraction of the PZT inclusions. Moreover, tensors $\hat{\mathbf{M}}$, $\hat{\mathbf{N}}$, $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$ reflect the relations between the transformed constraint fields, $\hat{\varepsilon}^I$, $\hat{\mathbf{E}}^I$, of the piezoelectric inclusion, and the transformed applied fields, $\hat{\varepsilon}^0$ and $\hat{\mathbf{E}}^0$, i.e.,

$$\hat{\varepsilon}^I_{ij} = \hat{M}_{ijkl}\hat{\varepsilon}^0_{kl} + \hat{N}_{ijk}\hat{E}^0_k, \quad \hat{E}^I_i = \hat{P}_{ijk}\hat{\varepsilon}^0_{jk} + \hat{Q}_{ij}\hat{E}^0_j. \qquad (3.4)$$

Note that Equation (2.8) are for a single ellipsoidal PZT inclusion embedded in an infinite viscoelastic matrix, but Equations (3.4) account for the interaction among neighboring inclusions. Two approximate techniques of finding $\hat{\mathbf{M}}$, $\hat{\mathbf{N}}$, $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$ are discussed below.

3.1 Dilute Solutions

If the volume fractions of the PZT inclusions are small so that the interaction among the inclusions can be neglected, then the constraint strain and the constraint electric fields inside the inclusions are approximated by the strain and the electric fields that would occur in an isolated PZT inclusion embedded in an infinite non-piezoelectromechanically coupled matrix. Therefore,

$$\hat{\mathbf{M}} = \hat{\mathbf{H}}^1, \quad \hat{\mathbf{N}} = \hat{\mathbf{H}}^2, \quad \hat{\mathbf{P}} = \hat{\mathbf{H}}^3, \quad \hat{\mathbf{Q}} = \hat{\mathbf{H}}^4, \qquad (3.5)$$

where tensors $\hat{H}^1$, $\hat{H}^2$ etc. are given by (2.9).

3.2 Dense Solutions

We use the Mori–Tanaka (1973) method to account for the interaction among inclusions; they are assumed to be embedded in an infinite non-electromechanically coupled matrix and the RVE is subjected to the average matrix strain and the average matrix electric field in the composite rather than to the applied strain and the applied electric field. Hence $\hat{\mathbf{M}}$, $\hat{\mathbf{N}}$, $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$ are given by

$$\begin{aligned} \hat{M}_{ijkl} &= \hat{H}^1_{ijpq}\hat{\mathcal{M}}_{pqkl} + \hat{H}^2_{ijp}\hat{\mathcal{P}}_{pkl}, \\ \hat{N}_{ijk} &= \hat{H}^1_{ijmn}\hat{\mathcal{N}}_{mnk} + \hat{H}^2_{ijm}\hat{\mathcal{Q}}_{mk}, \\ \hat{P}_{ijk} &= \hat{H}^3_{imn}\hat{\mathcal{M}}_{mnjk} + \hat{H}^4_{im}\hat{\mathcal{P}}_{mjk}, \\ Q_{ij} &= \hat{H}^3_{imn}\hat{\mathcal{N}}_{mnj} + \hat{H}^4_{im}\hat{\mathcal{Q}}_{mj}, \end{aligned} \qquad (3.6)$$

where

$$\begin{aligned} \hat{\mathcal{M}}_{ijkl} &= [\hat{\mathcal{H}}^1_{ijkl} - \hat{\mathcal{H}}^2_{ijm}(\hat{\mathcal{H}}^4_{mn})^{-1}\hat{\mathcal{H}}^3_{nkl}]^{-1}, \\ \hat{\mathcal{N}}_{ijk} &= -\hat{\mathcal{M}}_{ijmn}\hat{\mathcal{H}}^2_{mnp}(\hat{\mathcal{H}}^4_{pk})^{-1}, \\ \hat{\mathcal{P}}_{ijk} &= -(\hat{\mathcal{H}}^4_{im})^{-1}\hat{\mathcal{H}}^3_{mpq}\hat{\mathcal{M}}_{pqjk}, \\ \hat{\mathcal{Q}}_{ij} &= (\hat{\mathcal{H}}^4_{im})^{-1}[\delta_{mj} - \hat{\mathcal{H}}^3_{mpq}\hat{\mathcal{N}}_{pqj}], \\ \hat{\mathcal{H}}^1_{ijkl} &= I_{ijkl} - f(I_{ijkl} - \hat{H}^1_{ijkl}), \quad \hat{\mathcal{H}}^2_{ijk} = f\hat{H}^2_{ijk}, \\ \hat{\mathcal{H}}^3_{ijk} &= f\hat{H}^3_{ijk}, \quad \hat{\mathcal{H}}^4_{ij} = \delta_{ij} - f(\delta_{ij} - \hat{H}^4_{ij}). \end{aligned} \qquad (3.7)$$

3.3 Effective Electroelastic Properties

We now give closed-form expressions for the effective electroelastic properties of a piezocomposite made of elliptic cylindrical PZT inclusions and an isotropic homogeneous viscoelastic matrix. The PZT inclusions are assumed to be transversely isotropic about the x_3-axis which is also the axis of polarization. The nonzero electroelastic moduli of the PZT inclusions are

$$\begin{aligned} &C^I_{1111} = C^I_{2222},\ C^I_{3333},\ C^I_{1133} = C^I_{2233},\ C^I_{1313} = C^I_{2323},\ C^I_{1122}, \\ &C^I_{1212} = (C^I_{1111} - C^I_{1122})/2,\ e^I_{131} = e^I_{232},\ e^I_{311} = e^I_{322},\ e^I_{333}, \\ &k^I_{11} = k^I_{22},\ k^I_{33}. \end{aligned} \qquad (3.8)$$

In order to simplify the problem, we assume that the Poisson's ratio, ν, for the matrix is a real constant. Thus the ratio of the shear and the bulk relaxation functions must be a constant. In terms of the nondimensional parameter

$$\hat{p}_0 = p_0 = \frac{C_{1122}(t)}{C_{1111}(t)} = \frac{\nu}{(1-\nu)}, \tag{3.9}$$

the elastic and the dielectric moduli of the matrix are given by

$$\begin{aligned} C^M_{ijkl}(t) &= c(1+\rho_c e^{-\tau_e t})[p_0\delta_{ij}\delta_{kl} + (1-p_0)I_{ijkl}], \\ k^M_{ij}(t) &= \kappa(1+\rho_\kappa e^{-\tau_d t})\delta_{ij}, \end{aligned} \tag{3.10}$$

where c, κ, ρ_c and ρ_k are time-independent material constants of the matrix, and $\tau_e > 0$ and $\tau_d > 0$ are respectively, reciprocals of the elastic and the dielectric relaxation times of the matrix. The Laplace transforms of Equations (3.10) give

$$\begin{aligned} s\hat{C}^M_{ijkl} &= \frac{c[\tau_e + (1+\rho_c)s]}{(s+\tau_e)}[p_0\delta_{ij}\delta_{kl} + (1-p_0)I_{ijkl}], \\ s\hat{k}^M_{ij} &= \frac{\kappa[\tau_d + (1+\rho_\kappa)s]}{s+\tau_d}\delta_{ij}. \end{aligned} \tag{3.11}$$

Jiang and Batra (2001) have given closed form expressions for the effective electroelastic moduli of a piezocomposite with $\rho_c = \rho_\kappa = 0$. By using the correspondence principle, we obtain closed form expressions for the transformed electroelastic moduli of the piezocomposite being studied here. By taking their inverse transform, the effective electroelastic moduli of the piezocomposite are computed. The nonzero effective moduli of the piezocomposite are

$$\bar{C}_{1111},\ \bar{C}_{2222},\ \bar{C}_{3333},\ \bar{C}_{1122},\ \bar{C}_{1133},\ \bar{C}_{2233},\ \bar{C}_{1313},\ \bar{C}_{2323},\ \bar{C}_{1212},\ \bar{e}_{311},\ \bar{e}_{322},\ \bar{e}_{333},\ \bar{e}_{131},\ \bar{e}_{232},\ \bar{k}_{11},\ \bar{k}_{22} \text{ and } \bar{k}_{33} \tag{3.12}$$

with the following symmetry relations among them

$$\bar{C}_{ijkl} = \bar{C}_{ijlk} = \bar{C}_{klij}, \quad \bar{e}_{ijk} = \bar{e}_{ikj}. \tag{3.13}$$

Thus the piezocomposite is orthotropic.

The relaxation times of the piezocomposite are related to a_L and b_L; expressions for a_L and b_L are given in the Appendix. The subscript L in a_L, b_L and w_L equals A for $\bar{C}_{1313}$, $\bar{e}_{131}$ and $\bar{k}_{11}$; it equals B for $\bar{C}_{2323}$, $\bar{e}_{232}$ and $\bar{k}_{22}$; it equals C for the remaining moduli except for $\bar{C}_{1212}$, and it equals D for $\bar{C}_{1212}$. Note that a_L and b_L are not components of a vector and in the expressions given below for the effective piezoelectric moduli there is no summation implied on the repeated indices i, j, k, l and L.

If $b_L > 0$,

$$\frac{\bar{C}_{ijkl}}{c} = \bar{C}^0_{ijkl} + \bar{C}^1_{ijkl}e^{-\tau_e t} + \frac{\bar{C}^2_{ijkl}}{2\sqrt{b_L}}\left\{\left[\frac{\Psi_{ijkl}-\Phi_{ijkl}}{(a_L-\sqrt{b_L})}\right]e^{-(a_L-\sqrt{b_L})t} - \left[\frac{\Psi_{ijkl}-\Phi_{ijkl}}{(a_L+\sqrt{b_L})}\right]e^{-(a_L+\sqrt{b_L})t}\right\}, \tag{3.14}$$

$$\bar{e}_{ijk} = \bar{e}^0_{ijk} + \frac{\bar{e}^1_{ijk}}{2\sqrt{b_L}}\left\{\left[\frac{\Psi_{ijk}-\Phi_{ijk}}{(a_L-\sqrt{b_L})}\right]e^{-(a_L-\sqrt{b_L})t} - \left[\frac{\Psi_{ijk}-\Phi_{ijk}}{(a_L+\sqrt{b_L})}\right]e^{-(a_L+\sqrt{b_L})t}\right\}, \tag{3.15}$$

$$\bar{k}_{ij} = \bar{k}^0_{ij} + \bar{k}^1_{ij}e^{-\tau_d t} + \frac{\bar{k}^2_{ij}}{2\sqrt{b_L}}\left\{\left[\frac{\Psi_{ij}-\Phi_{ij}}{(a_L-\sqrt{b_L})}\right]e^{-(a_L-\sqrt{b_L})t} - \left[\frac{\Psi_{ij}-\Phi_{ij}}{(a_L+\sqrt{b_L})}\right]e^{-(a_L+\sqrt{b_L})t}\right\}. \tag{3.16}$$

If $b_L = 0$,

$$\frac{\bar{C}_{ijkl}}{c} = \bar{C}^0_{ijkl}e^{-\tau_e t} + \bar{C}^2_{ijkl}\left\{t\Psi_{ijkl} - \frac{(1+a_L t)\Phi_{ijkl}}{w_L}\right\}e^{-a_L t}, \tag{3.17}$$

$$\bar{e}_{ijk} = \bar{e}^0_{ijk} + \bar{e}^1_{ijk}\left\{t\Psi_{ijk} - \frac{(1+a_L t)\Phi_{ijk}}{w_L}\right\}e^{-a_L t}, \tag{3.18}$$

$$\frac{\bar{k}_{ij}}{\kappa} = \bar{k}^0_{ij} + \bar{k}^1_{ij}e^{-\tau_d t} + \bar{k}^2_{ij}\left\{\frac{t\Psi_{ij} - (1+a_L t)\Phi_{ij}}{w_L}\right\}e^{-a_L t}. \tag{3.19}$$

If $b_L < 0$,

$$\frac{\bar{C}_{ijkl}}{c} = \bar{C}^0_{ijkl} + \bar{C}^1_{ijkl}e^{-\tau_e t} + \frac{\bar{C}^2_{ijkl}}{\sqrt{-b_L}}\left\{\Psi_{ijkl}\sin\left(t\sqrt{-b_L}\right) - \frac{\Phi_{ijkl}}{\sqrt{w_L}}\sin\left(t\sqrt{-b_L} + \tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right\}e^{-a_L t}, \tag{3.20}$$

$$\bar{e}_{ijk} = \bar{e}^0_{ijk} + \frac{\bar{e}^1_{ijk}}{\sqrt{-b_L}}\left\{\Psi_{ijk}\sin\left(t\sqrt{-b_L}\right) - \frac{\Phi_{ijk}}{\sqrt{w_L}}\sin\left(t\sqrt{-b_L} + \tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right\}e^{-a_L t}, \tag{3.21}$$

$$\frac{\bar{k}_{ij}}{\kappa} = \bar{k}^0_{ij} + \bar{k}^1_{ij}e^{-\tau_d t} + \frac{\bar{k}^2_{ij}}{\sqrt{-b_L}}\left\{\Psi_{ij}\sin\left(t\sqrt{-b_L}\right) - \frac{\Phi_{ij}}{\sqrt{w_L}}\sin\left(t\sqrt{-b_L} + \tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right\}e^{-a_L t}. \tag{3.22}$$

For $\bar{C}_{1212}, b_D \neq \tau_e$. If $b_D = \tau_e, \varphi_D^2$ defined by Equation $(A6)_2$ in the Appendix will be zero which is impossible because p_0, m_1 and m_2 are positive and $0 \leq f \leq 1$. Therefore,

$$\frac{\bar{C}_{1212}}{c} = \frac{1}{2}(1-p_0)\left\{1+2f\frac{p_5}{\varpi_D}\frac{\tau_e}{b_D}+\rho_c\left[1+f\frac{\rho_c}{\varpi_D}\frac{\tau_e}{(b_D-\tau_e)}\right]e^{-\tau_e t} - \frac{f}{\varpi_D}\left[2p_5\frac{\tau_e-b_D}{b_D}+\rho_c\right]\left[1+\frac{\rho_c b_D}{b_D-\tau_e}\right]e^{-b_D t}\right\}. \tag{3.23}$$

Parameters appearing in Equation (3.14)–(3.23) are defined in the Appendix.

For the 1-3 piezocomposite being studied, the relaxation of the dielectric moduli of the matrix influences values of $\bar{k}_{ij}$, $\bar{C}_{1313}$, $\bar{C}_{2323}$, $\bar{e}_{131}$ and $\bar{e}_{232}$. However, the relaxation of the elastic moduli of the matrix affects values of all nonzero components of $\bar{\mathbf{C}}$, $\bar{\mathbf{e}}$ and $\bar{\mathbf{k}}$.

When $\rho_c = \rho_\kappa = 0$, it is seen from Equations (3.14)–(3.23) and those given in the Appendix that the effective electroelastic moduli of the piezocomposite reduce to those obtained earlier by Jiang and Batra (2001) for a composite made of an elastic matrix and PZT inclusions. Also, if $f = 0$ or 1, the effective electroelastic moduli of the piezocomposite equal those of the viscoelastic matrix or the PZT respectively. As $f \to 1, a_A + \sqrt{b_A}$ and $a_B + \sqrt{b_B} \to \tau_d/(1+\rho_\kappa)$, and $a_A - \sqrt{b_A}$ and $a_B - \sqrt{b_B} \to \tau_e/(1+\rho_c)$ and $a_C \to \tau_e/(1+\rho_c)$. Thus with an increase in the volume fraction of the PZT inclusions, the effect of the material properties and the shapes of the PZT inclusions on the effective relaxation times of the piezocomposite decreases. When $a_2 \to \infty$, we have $m_1 \to 1$ and $m_2 \to \infty$. In this case, $a_B + \sqrt{b_B} \to \tau_d/(1+\rho_\kappa)$ and $a_B - \sqrt{b_B} \to \tau_e/(1+\rho_c)$. Although Poisson's ratio of the viscoelastic matrix has been taken to be a constant, that of the piezocomposite is a function of time, t.

It is clear from expressions (3.14)–(3.23) and Equation (A1) of the Appendix that the relaxation time of the piezocomposite is determined by a_L and b_L. Values of a_L and b_L are related to the material properties of the matrix and the PZT inclusions, and also to the volume fraction and the shapes of the inclusions. Different components of the effective moduli of the piezocomposite have different relaxation times. Even for the case of $\rho_\kappa = \rho_c$ and $\tau_d = \tau_e$, different effective electroelastic moduli do not relax at the same rate.

4 COMPARISON OF RESULTS WITH EXPERIMENTAL FINDINGS AND PREDICTIONS FROM OTHER MODELS

We are not aware of any experimental data for the 1-3 piezocomposite with a viscoelastic matrix and cylindrical PZT inclusions with their axes aligned along the x_3-axis. Therefore, we first compare computed values of some of the effective moduli of an 1-3 piezocomposite whose matrix is elastic with test values of Chan and Unsworth (1989). Subsequently, we compare results for a composite comprised of a viscoelastic matrix and elastic elliptic cylindrical inclusions with those of Li and Weng (1994).

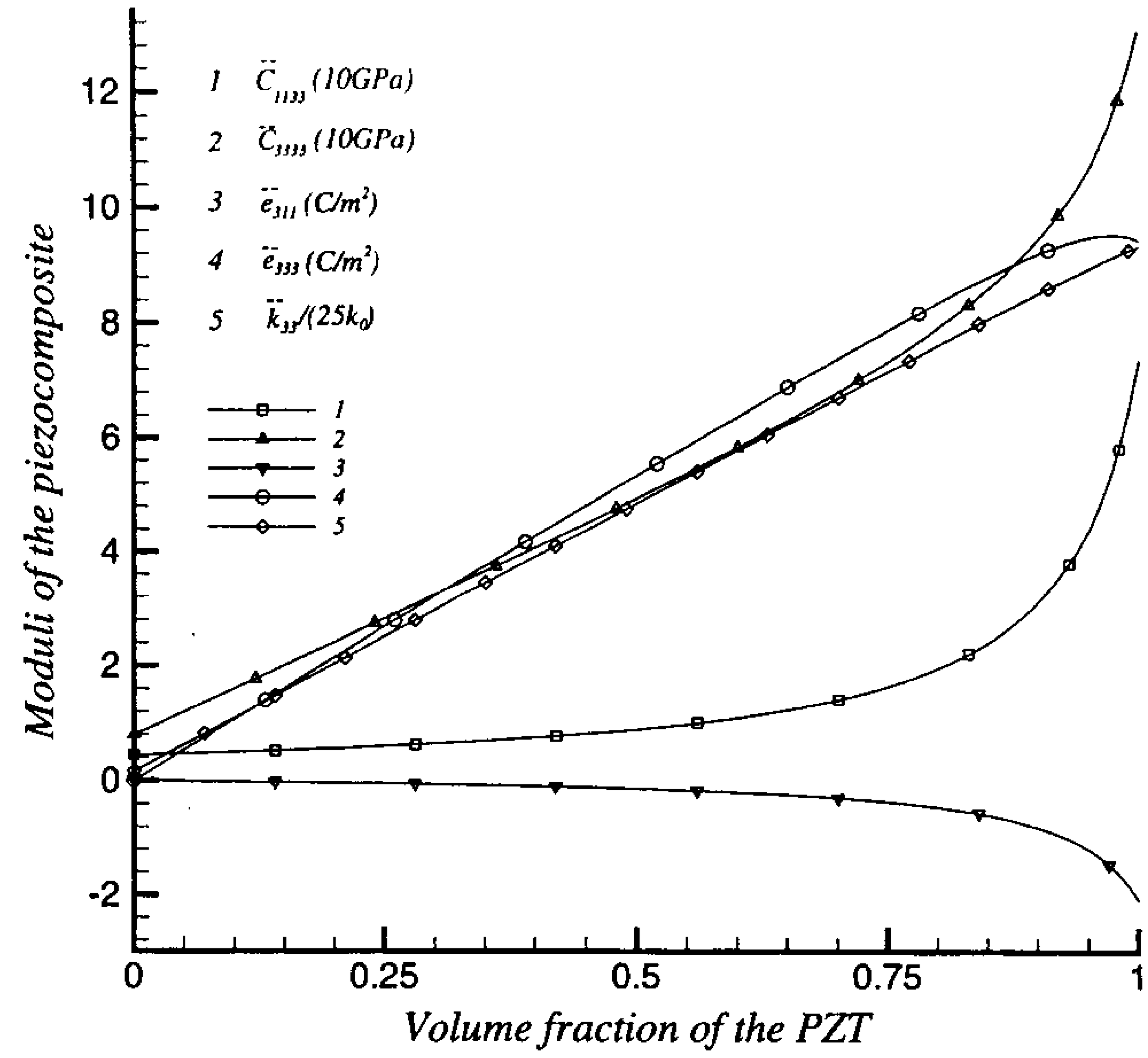

Figure 1. *Comparison of computed electroelastic moduli with the experimental values of Chan and Unsworth. Solid curves are the computed values and symbols denote those observed in experiments.*

Figure 1 depicts the presently computed values of the effective electroelastic moduli $\bar{C}_{1133}$, $\bar{C}_{3333}$, $\bar{e}_{311}$, $\bar{e}_{333}$ and $\bar{k}_{33}$ for the 1-3 piezocomposite made of PZT7A cylindrical wires embedded along the x_3-axis in an isotropic elastic Araldite *D* matrix; the material properties are given in Chan and Unsworth's (1989) paper. The PZT7A is modeled as transversely isotropic with the axis of polarization along the x_3-axis, and viscous effects in the matrix are eliminated by setting $\rho_\kappa = \rho_c = 0$. The solid curves denote the computed values and the symbols test values; it is clear that the two sets of results agree well with each other.

Li and Weng (1994) have found the effective moduli of a composite made of glass fibers or ribbons embedded in an ED-6 resin. The glass fiber is modeled as an isotropic elastic matrix with $C_{1111} = 77.23$ GPa and $C_{1122} = 20.53$ GPa. The creep properties of an ED-6 resin tested at 20° and axial stresses of 63.97 and 14.80 MPa are given by Skudra and Auzukalns (1970). Li and Weng (1994) used a 4-parameter rheological model comprised of Maxwell and Voigt elements connected in series to simulate the test data. Here, we evaluate the three parameters c, ρ_c and τ_e appearing in

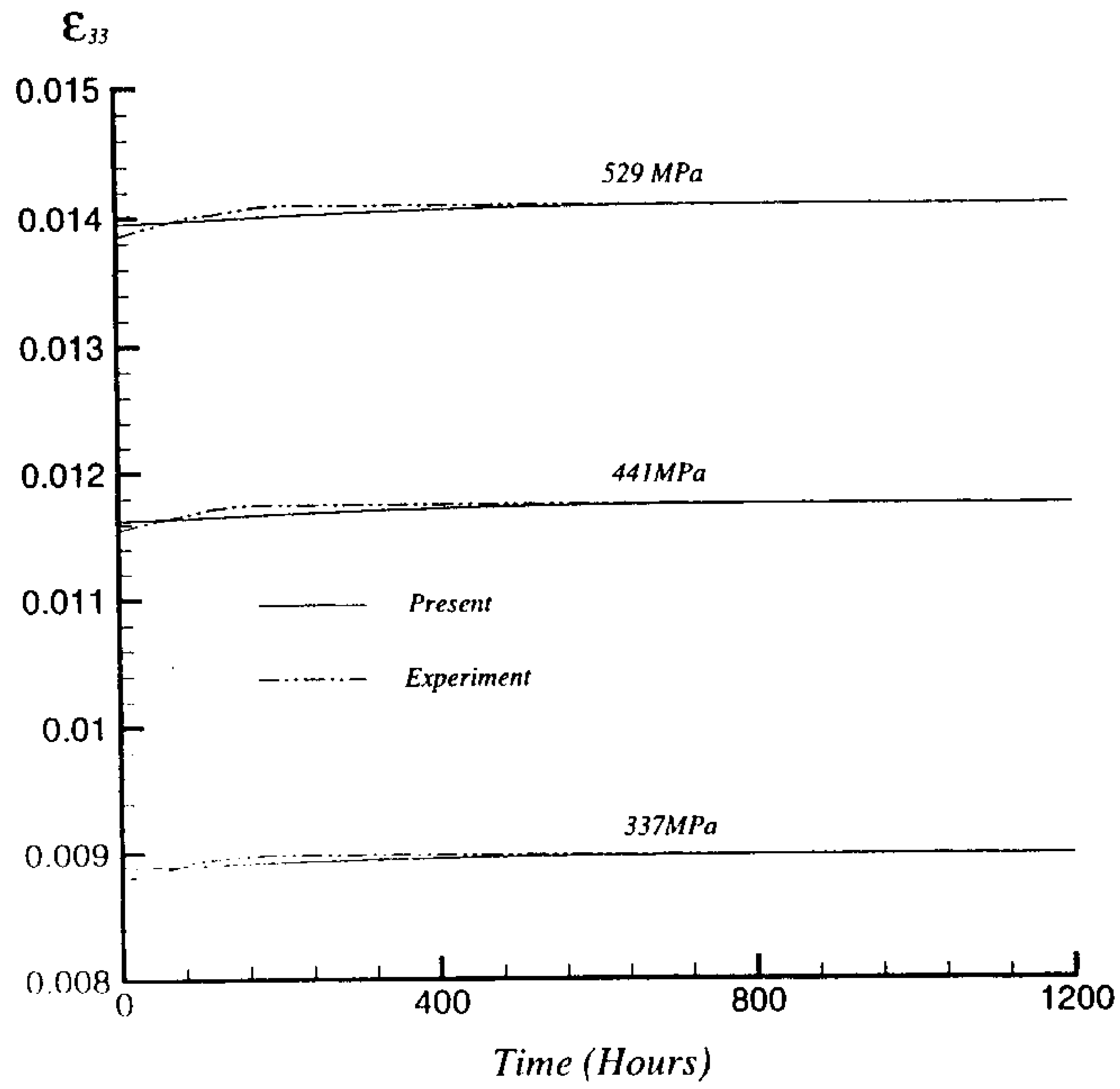

Figure 2. Comparison between the experimental and the computed creep curves for three uniaxial creep tests performed on a viscoelastic matrix containing glass fibers aligned in the loading direction.

Equation $(3.10)_1$ from the test values at time $t=0$, 500 and 2000 h with the following result:

$$c = 1.71\,\text{GPa}, \quad \rho_c = 2.5, \quad \tau_e = 5.04 \times 10^{-3}/\text{h}.$$

Poisson's ratio of the ED-6 resin is taken to be 0.38. Figure 2 compares the computed time history of ε_{33} with the experimental one of Skudra and Auzukalns (1970) for 54% fiber content and three creep tests conducted at axial stress levels of 337, 441 and 529 MPa. Whereas computed curves match well with the experimental ones for large values of time, they differ for short times. The viscoelastic relation $(3.10)_1$ with one relaxation time does not capture well the creep of the material at short times. However, the viscoelastic material model with two relaxation times used by Li and Weng (1994) captured well this effect. We could not use more terms in Equation $(3.10)_1$ because of the increased complexity of the resulting algebraic work.

In Figures 3(a) and 3(b), we compare the presently computed time histories of the compliance coefficients $\tilde{M}_{3333}$ and $\tilde{M}_{2222}$ for the composite with 20% volume fraction of the elastic fibers and the composite loaded axially in the x_3-direction with those of Li and Weng (1994). Here $\tilde{\mathbf{M}} = \tilde{\mathbf{C}}^{-1}$, and $R = a_2/a_1$ equals the aspect ratio of the fibers; $R=1$ corresponds to circular cylindrical fibers and $R=0$ to long thin fibers. Li and Weng noted that the compliance coefficient $\tilde{M}_{3333}$ is independent of the aspect ratio of the inclusions. Our results for $\tilde{M}_{3333}$ show a very slight dependence on the aspect ratio and are close to those of Li and Weng. The

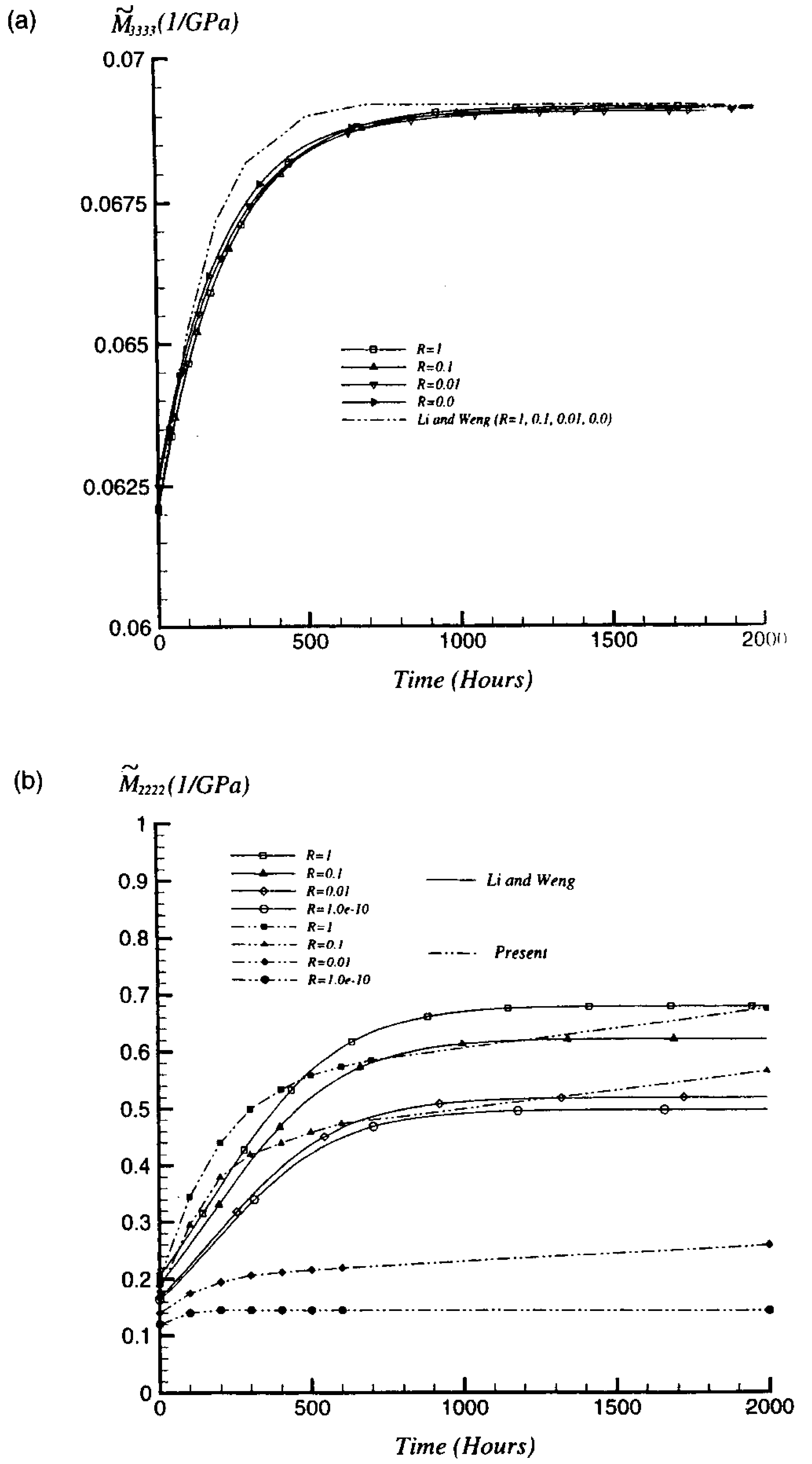

***Figure 3.** For different values of the aspect ratio, R, of the 20% elastic fibers embedded along the x_3-axis in a viscoelastic matrix, comparison between the presently computed time histories of the compliance coefficients (a) $\tilde{M}_{3333}$ and (b) $\tilde{M}_{2222}$ and those computed by Li and Weng.*

time histories of $\tilde{M}_{2222}$ for four different values of R, depicted in Figure 3(b), agree qualitatively with but differ quantitatively from those of Li and Weng. The difference between the two sets of results is least for $R=1$ and most for $R \approx 0$. For $R \neq 1$, these differences are mainly due to the assumption of randomly distributed inclusions in Li and Weng's work and likewise oriented inclusions in the present work, and, to a lesser extent, the slightly different viscoelastic relations for the polymeric matrix. For $R=0$ and 1, the effective moduli of the composite differ because of the different constitutive relations employed for the matrix.

5 RELAXATION BEHAVIOR OF THE PIEZOCOMPOSITE WITH CYLINDRICAL PZT INCLUSIONS

We study the effect of varying values of ρ_c, ρ_κ, τ_e and the volume fraction, f, of the PZT inclusions on the relaxation behavior of $\bar{C}_{3333}$, $\bar{e}_{333}$, $\bar{e}_h = \bar{e}_{3ii}$ and $\bar{k}_{33}$ for the piezocomposite made of the viscoelastic matrix and circular cylindrical (i.e., $R = 1$) PZT inclusions aligned along the x_3-axis. Results presented in Section 3 show that the relaxation of the dielectric constants of the matrix affects only values of $\bar{k}_{ij}$ and the shear components of $\bar{C}_{ijkl}$ (i.e., components for which $i \neq j$ and $k \neq j$) and $\bar{e}_{ijk}$ (i.e., components for which $j \neq k$). Therefore we need to analyze the effect of ρ_κ and τ_e on $\bar{C}_{3333}$, $\bar{e}_{333}$ and $\bar{e}_h$, and of ρ_c, ρ_κ, τ_d and τ_e on $\bar{k}_{33}$.

For a piezocomposite with nondielectrically relaxing viscoelastic matrix, Figures 4(a)–(d) exhibit time histories of the evolution of $\bar{C}_{3333}$, $\bar{e}_{333}$, $\bar{e}_h$ and $\bar{k}_{33}$ for seven combinations of values of ρ_c, τ_e and f which are listed in Figure 4(a). Also given therein are the values of the resulting relaxation time of the piezocomposite obtained by fitting expressions of the type $\bar{C}_{3333} = \bar{c}(1 + \bar{\rho}_c e^{\bar{\tau}_e t})$ to the values of $\bar{C}_{3333}$ at times $t = 0, 1$, and 200 h. The relaxation times of the matrix used in these calculations are considerably smaller than that of the ED-6 resin. It is clear that the addition of the hard PZT inclusions increases the value of $\bar{C}_{3333}$ but

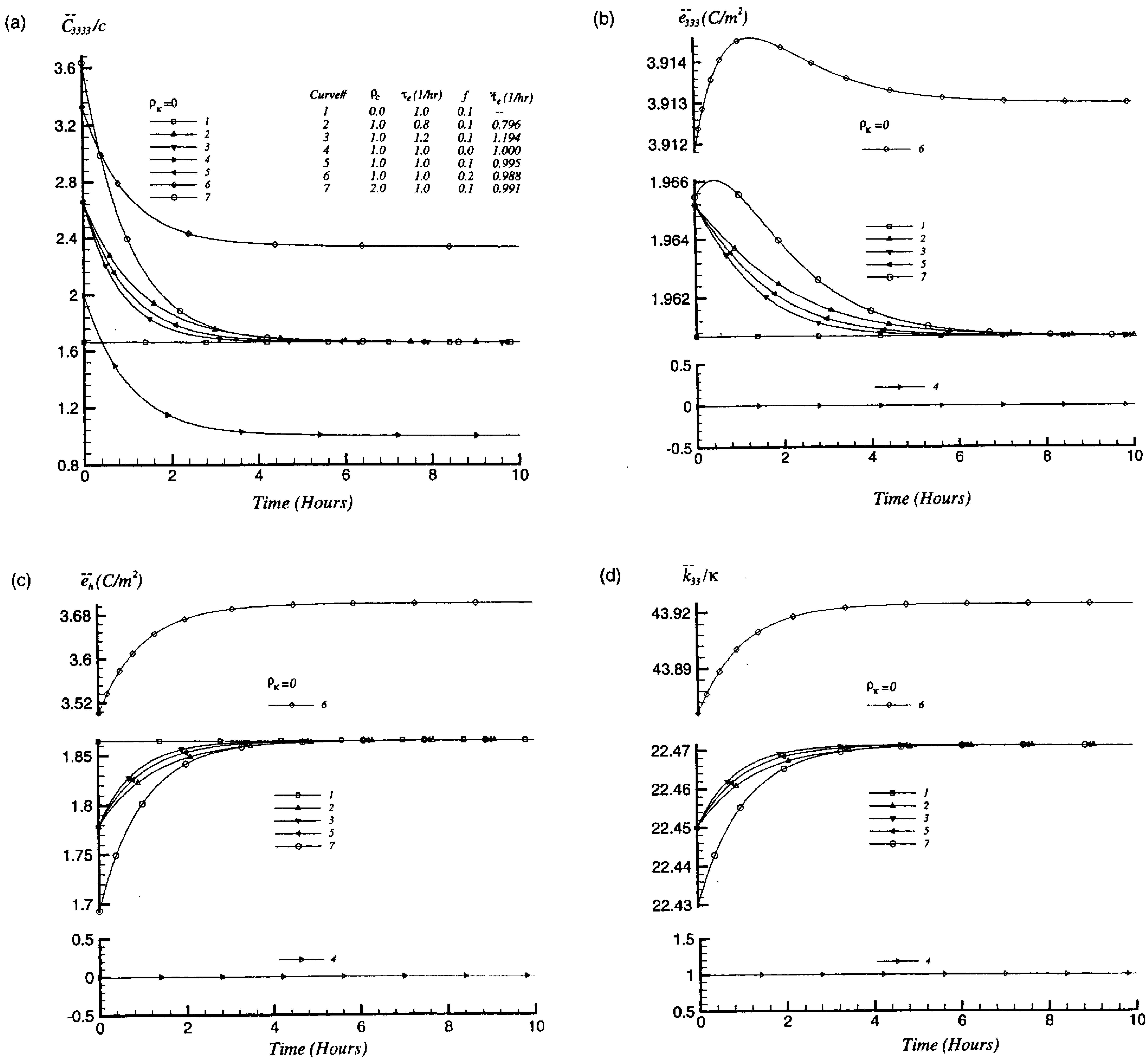

Figure 4. *For seven different combinations of the values of ρ_c, τ_e and f, time histories of the effective moduli (a) $\bar{C}_{3333}$, (b) $\bar{e}_{333}$, (c) $\bar{e}_h$, and (d) $\bar{k}_{33}$ for a piezocomposite comprised of cylindrical PZT inclusions in a nondielectrically relaxing viscoelastic matrix.*

does not change much the effective relaxation time of the piezocomposite. For each one of the six cases studied in which the matrix is viscoelastic, the effective relaxation time of the piezocomposite is essentially the same as that of the matrix. When either $\rho_c = 2$ or $f = 0.2$, the value of $\bar{e}_{333}$ increases initially, albeit by a very small amount, and then relaxes gradually to its equilibrium value. The difference between the maximum and the minimum values of $\bar{e}_{333}$ is less than 0.3% so that for all practical purposes, $\bar{e}_{333}$ can be regarded as a constant. Whereas values of $\bar{C}_{3333}$ approach their equilibrium values from above as the time t approaches infinity, those of $\bar{e}_h$ approach their equilibrium values from below. The largest difference in the maximum and the minimum values of $\bar{e}_h$ is about 10% signifying that the difference in the initial and the equilibrium values of $(\bar{e}_{311} + \bar{e}_{322})$ is more than that in the corresponding values of $\bar{e}_{333}$. Like $\bar{e}_{333}$, values of $\bar{k}_{33}$ approach their equilibrium values from below, but the total change in the initial and the equilibrium values of $\bar{k}_{33}$ is only about 0.2%. We note that the effective piezoelectric moduli and the dielectric constants of the piezocomposite are strongly influenced by the viscoelastic moduli of the matrix. Also, viscoelasticity of the matrix increases the initial values (i.e., at time $t = 0$) of $\bar{e}_{333}$, but decreases the initial values of $\bar{k}_{33}$.

Figure 5 exhibits the time history of the evolution of the dielectric constant $\bar{k}_{33}$ when the matrix is elastic but dielectrically relaxing. In this case, the creep behavior of $\bar{k}_{33}$ is similar to that of $\bar{C}_{3333}$ discussed above for a viscoelastic matrix with nonrelaxing dielectric constants. An increase in the volume fraction of the PZT inclusions enhances the value of the effective $\bar{k}_{33}$. For each one of the six cases studied, the relaxation time of the dielectric constant of the piezocomposite is the same as that of the dielectric constant of the elastic matrix.

We have plotted in Figure 6 the time history of the evolution of $\bar{k}_{33}$for seven different combinations of the values of ρ_c, ρ_κ, τ_e and τ_d. Except for the case of a viscoelastic matrix with nonrelaxing dielectric constant, the values of $\bar{k}_{33}$ monotonically decrease from their initial values and approach their equilibrium values in about 6 h. The relaxation time of the effective value of $\bar{k}_{33}$ is essentially unchanged for the other six cases studied herein.

5.1 Effect of the Shape of the PZT Inclusions on the Relaxation Behavior of the Piezocomposite

Figures 7(a)–(d) evince the effect of the aspect ratio, R, of the cross section of the elliptic cylindrical PZT inclusions on the relaxation behavior of $\bar{C}_{3333}$, $\bar{e}_{333}$, $\bar{e}_h$ and $\bar{k}_{33}$ respectively for $f = 0.1$, $\rho_c = \rho_\kappa$ and $\tau_e = \tau_d = 1/\text{h}$.The PZT cylinders are circular for $R = 1$ and thin ribbons for $R = 0$; values for $R = 0$ are calculated by taking $R = 1.0 \times 10^{-10}$. For $R = 1$, 0.1, 0.01 and 0, the reciprocals of the effective relaxation times of the piezocomposite computed from the relaxation behavior of $\bar{C}_{3333}$ equal 0.995, 0.988, 0.989 and 0.997 (1/h) respectively, and those obtained from the

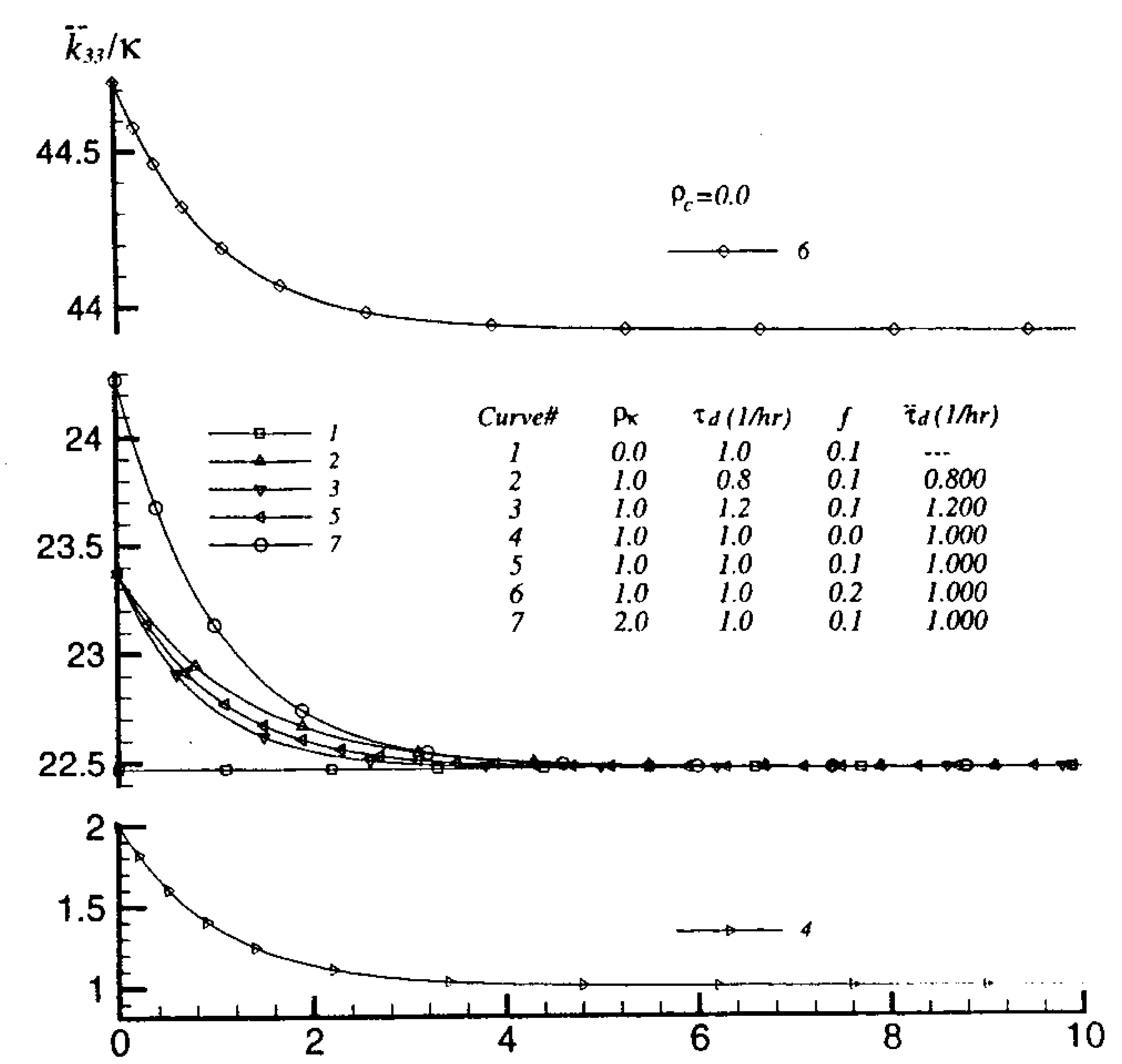

Figure 5. For seven different combinations of the values of ρ_κ, τ_d and f, time history of the effective dielectric relaxation modulus $\bar{k}_{33}$ for a piezocomposite made of cylindrical PZT inclusions and a dielectrically relaxing elastic matrix.

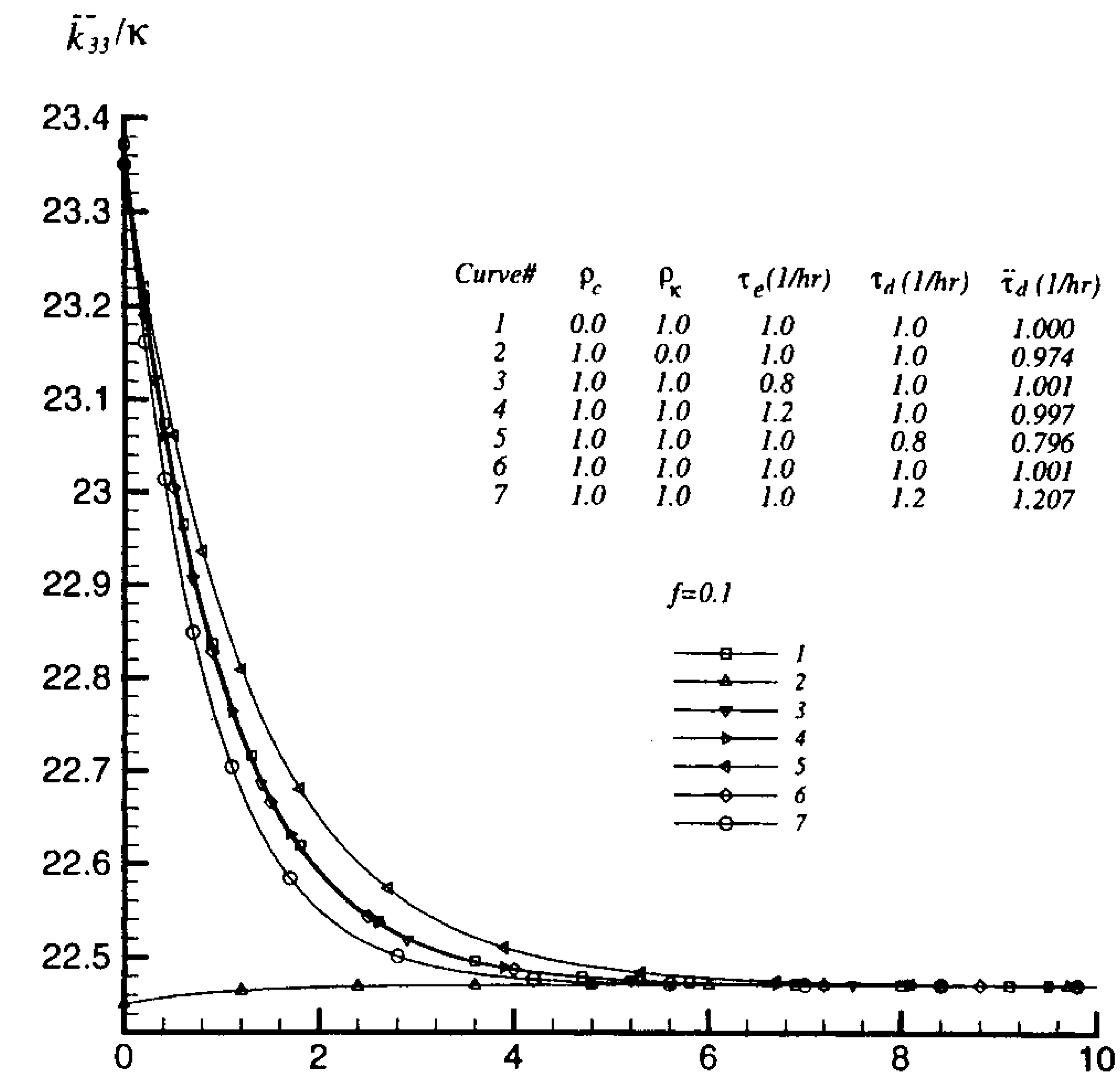

Figure 6. For seven different combinations of the values of ρ_c, ρ_κ, τ_e and τ_d, time history of the effective dielectric relaxation modulus $\bar{k}_{33}$ for a piezocomposite made of cylindrical PZT inclusions and a viscoelastic matrix.

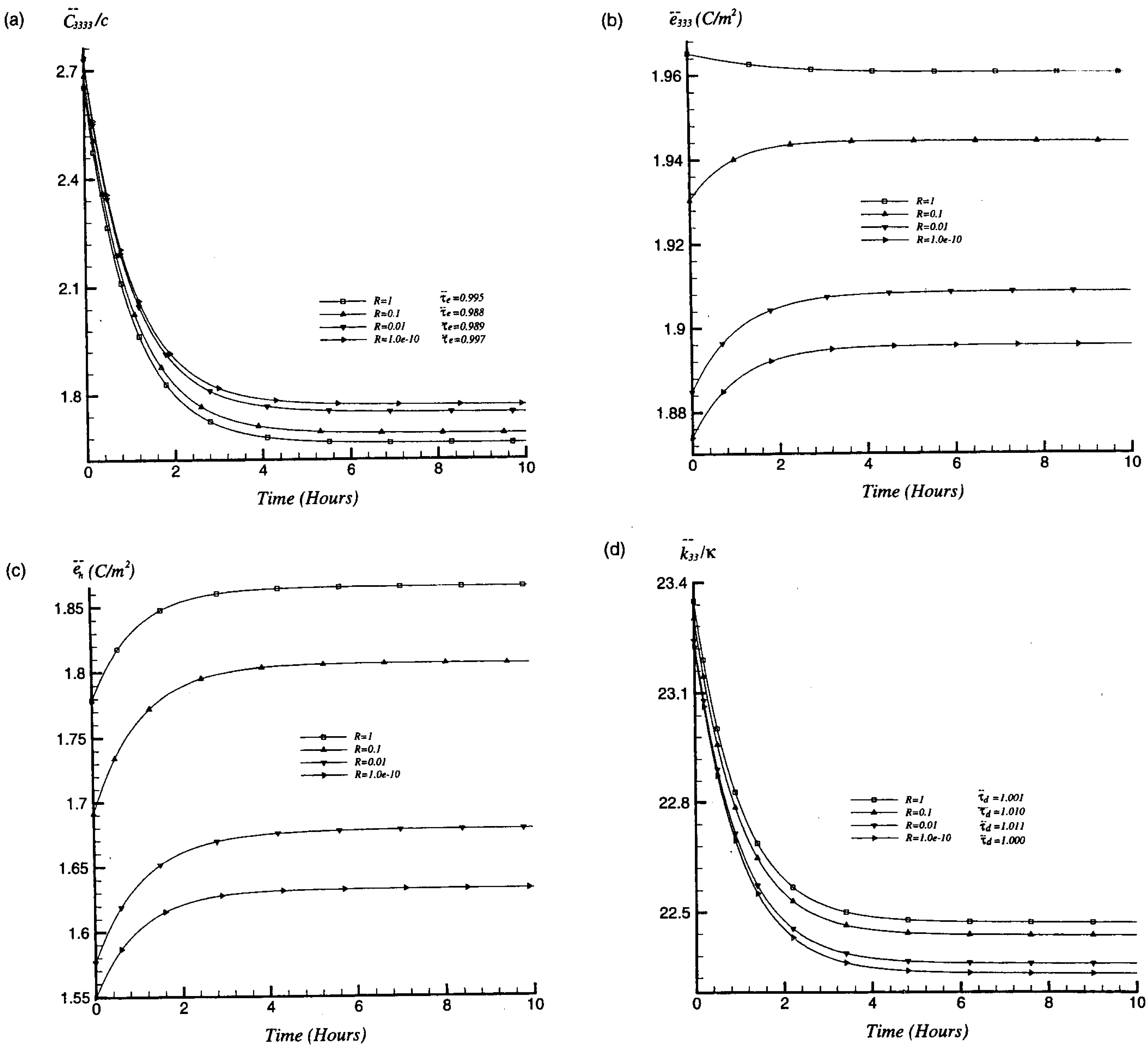

Figure 7. *For four different values of the aspect ratio, R, of the elliptic cylindrical PZT inclusions embedded in a viscoelastic matrix, time histories of (a) $\bar{C}_{3333}$, (b) $\bar{e}_{333}$, (c) $\bar{e}_h$ and (d) $\bar{k}_{33}$.*

relaxation curves of $\bar{k}_{33}$ are 1.001, 1.010, 1.011 and 1.000 (1/h) respectively. For each one of the four shapes of the PZT inclusions, $\bar{C}_{3333}$ and $\bar{k}_{33}$ approach their equilibrium values from above but $\bar{e}_h$ from below. For $R=1$, the relaxation behavior of $\bar{e}_{333}$ is similar to that of $\bar{C}_{3333}$ in the sense that its values gradually decrease to its equilibrium value. However, for $R=0$, 0.01 and 0.1, values of $\bar{e}_{333}$ gradually rise to the corresponding equilibrium values. Note that thin ribbons provide the most reinforcing effect in the sense that, at any time t, the effective value of $\bar{C}_{3333}$ is the greatest out of the four values computed with $R=0$, 0.01, 0.1 and 1.0. Also, the initial and the final values of $\bar{e}_h$ and $\bar{e}_{333}$ are more noticeably influenced by the aspect ratio, R, of the PZT inclusions than the initial and the equilibrium values of $\bar{C}_{3333}$ and $\bar{k}_{33}$. For any fixed value of time t, values of $\bar{e}_{333}$, $\bar{e}_h$ and $\bar{k}_{33}$ increase with an increase in the value of R but those of $\bar{C}_{3333}$ decrease. Thus thin PZT ribbons provide the most mechanical strength but the least piezoelectric effect in the piezocomposite. In Figures 8(a) and (b) we have plotted the variation of the relaxation times of $\bar{C}_{3333}$ and $\bar{k}_{33}$ with the volume fraction of the PZT inclusions. For results included in Figures 8(a) and (b), we have set $\rho_c = 1$, $\rho_\kappa = 0$, $\tau_e = 1(1/h)$. It is evident that the aspect ratio of the cross section of the inclusions significantly affects the relaxation time of $\bar{k}_{33}$ but has less noticeable effect on the relaxation time of $\bar{C}_{3333}$. Whereas the reciprocal of the relaxation time of $\bar{C}_{3333}$ monotonically decreases with an increase in the volume fraction of the PZT

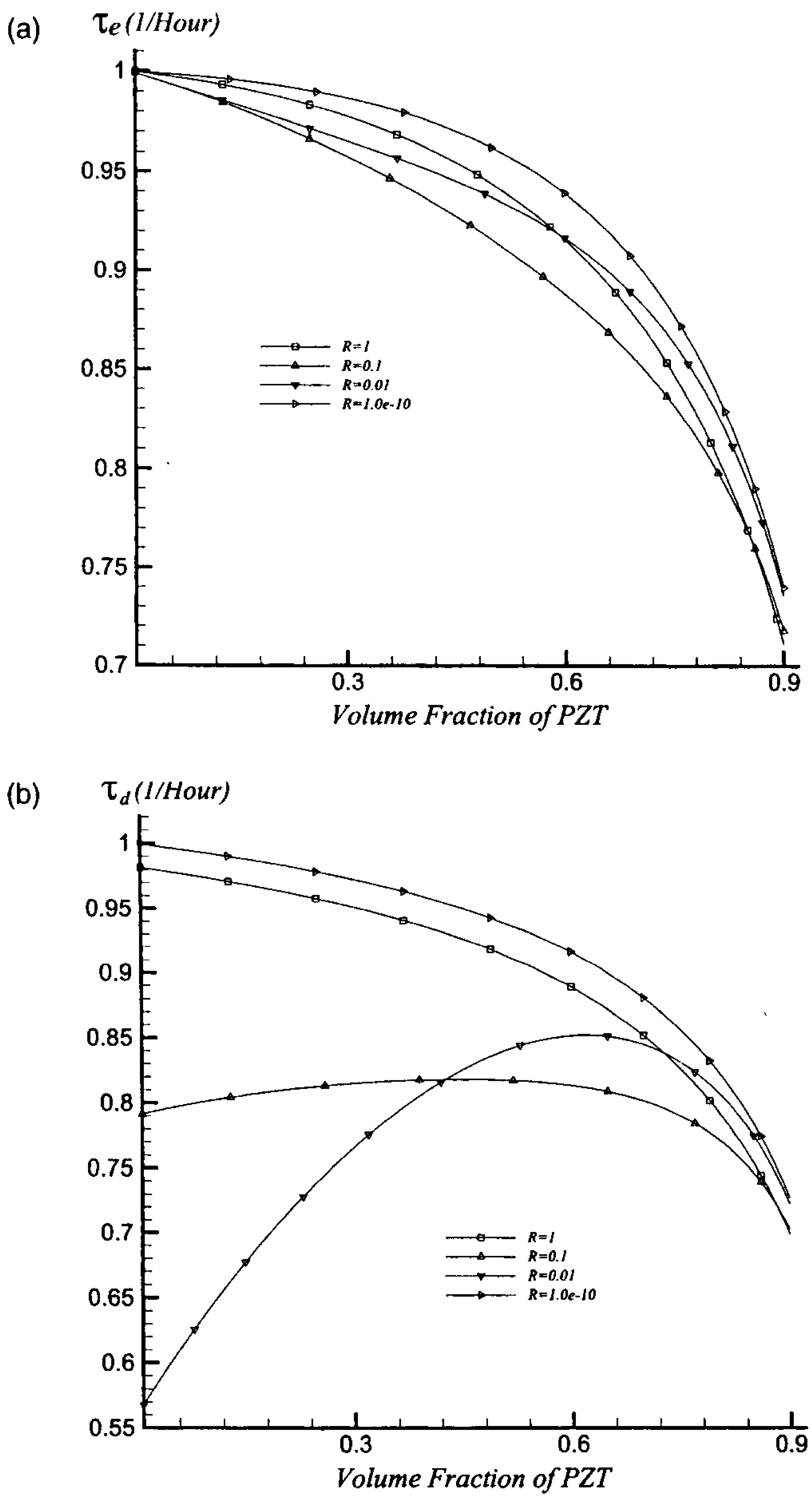

Figure 8. For four different values of the aspect ratio, R, of the elliptic cylindrical PZT inclusions embedded in a viscoelastic matrix, the dependence of the relaxation times of the effective moduli upon the volume fraction of the PZT inclusions (a) $\bar{C}_{3333}$ and (b) $\bar{k}_{33}$.

inclusions, such is the case for $\bar{k}_{33}$ only for $R = 1.0$ and 0.1. For thin PZT ribbons, $\bar{\tau}_d$ has a maximum value at $f \simeq 0.66$. For $\rho_c = 0, \rho_\kappa = 1, \ \tau_d = 1$, the relaxation time of the effective $\bar{k}_{33}$ is independent of the volume fraction and the aspect ratio of the cylindrical PZT inclusions.

5.2 Complex Effective Moduli of the Piezocomposite

We assume that boundary conditions prescribed on the boundary of the representative volume element are such as to produce the following homogeneous time harmonic strain and electric fields within the RVE:

$$\varepsilon^0_{ij}(t) = \varepsilon^{\max}_{ij} e^{J\omega_\varepsilon t}, \quad E^0_i(t) = E^{\max}_i e^{J\omega_E t}, \tag{5.1}$$

where $J = \sqrt{-1}$, $\varepsilon^{\max}_{ij}$ and $E^{\max}_i$ are the amplitudes of the strain and the electric field respectively, and ω_ε and ω_E are the corresponding frequencies. We replace the effective moduli in Equation (3.1) by their complex values signified by a superscript $*$ and denote their real and imaginary parts by a prime and a double prime respectively. For example,

$$C^*_{ijkl} = C'_{ijkl} + JC''_{ijkl}. \tag{5.2}$$

Note that C^*, C' and C'' may depend upon the frequency of the applied fields; C' and C'' are often referred to as the storage and the loss moduli respectively.

For the 1-3 piezocomposite being studied here, expressions for the storage and the loss moduli are given below wherein indices i, j, k etc. and L are not summed.

$$\begin{aligned}
\frac{\bar{C}'_{1212}}{c} &= \frac{1}{2}(1-p_0)\left\{1+\frac{2fp_5\tau_e}{\varpi_D b_D}\right\} + \frac{1}{2}(1-p_0)\omega_\varepsilon^2 \\
&\times \left\{\left[1+\frac{f\rho_c\tau_e}{(b_D-\tau_e)\varpi_D}\right]\frac{\rho_c}{\tau_e^2+\omega_\varepsilon^2} - \frac{f}{\varpi_D(b_D^2+\omega_\varepsilon^2)}\right. \\
&\times \left.\left[2p_5\frac{\tau_e-b_D}{b_D}+\rho_c\right]\left[1+\frac{\rho_c b_D}{b_D-\tau_e}\right]\right\}, \\
\frac{\bar{C}''_{1212}}{c} &= \frac{1}{2}(1-p_0)\omega_\varepsilon\left\{\frac{\rho_c\tau_e}{\tau_e^2+\omega_\varepsilon^2}\left[1+\frac{f\rho_c\tau_e}{(b_D-\tau_e)\varpi_D}\right]\right. \\
&\left. - \frac{fb_D}{\varpi_D(b_D^2+\omega_\varepsilon^2)}\left[2p_5\frac{\tau_e-b_D}{b_D}+\rho_c\right]\left[1+\frac{\rho_c b_D}{b_D-\tau_e}\right]\right\},
\end{aligned} \tag{5.3}$$

If $b_L > 0$

$$\begin{aligned}
\frac{\bar{C}'_{ijkl}}{c} &= \bar{C}^0_{ijkl} + \frac{\omega_\varepsilon^2\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2} + \frac{\omega_\varepsilon^2\bar{C}^2_{ijkl}}{(w_L+\omega_\varepsilon^2)^2+4b_L\omega_\varepsilon^2} \\
&\times \left\{2a_L\Psi_{ijkl} - (3a_L^2+b_L+\omega_\varepsilon^2)\frac{\Phi_{ijkl}}{w_L}\right\}, \\
\frac{\bar{C}''_{ijkl}}{c} &= \frac{\tau_e\omega_\varepsilon\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2} + \frac{\omega_\varepsilon\bar{C}^2_{ijkl}}{(w_L+\omega_\varepsilon^2)^2+4b_L\omega_\varepsilon^2} \\
&\times \left\{(w_L-\omega_\varepsilon^2)\Psi_{ijkl} - 2b_L\Phi_{ijkl}\right\}, \\
\bar{e}'_{ijk} &= \bar{e}^0_{ijk} + \frac{\omega_r^2\bar{e}^1_{ijk}}{(w_L+\omega_r^2)^2+4b_L\omega_r^2} \\
&\times \left\{2a_L\Psi_{ijk} - (3a_L^2+b_L+\omega_\varepsilon^2)\frac{\Phi_{ijk}}{w_L}\right\}, \\
\bar{e}''_{ijk} &= \frac{\omega_r\bar{e}^1_{ijk}}{(w_L+\omega_r^2)^2+4b_L\omega_r^2}\left\{(w_L-\omega_r^2)\Psi_{ijk} - 2a_L\Phi_{ijk}\right\}, \\
\frac{\bar{k}'_{ij}}{\kappa} &= \bar{k}^0_{ij} + \frac{\omega_E^2\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2} + \frac{\omega_E^2\bar{k}^2_{ij}}{(w_L+\omega_E^2)^2+4b_L\omega_E^2} \\
&\times \left\{2a_L\Psi_{ij} - (3a_L^2+b_L+\omega_E^2)\frac{\Phi_{ij}}{w_L}\right\},
\end{aligned}$$

$$\frac{\bar{k}''}{\kappa}=\frac{\tau_d\omega_E\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2}+\frac{\omega_E\bar{k}^2_{ij}}{(w_L+\omega_E^2)^2+4b_L\omega_E^2}\times\left\{(w_L-\omega_E^2)\Psi_{ij}-2a_L\Phi_{ij}\right\}. \tag{5.4}$$

If $b_L=0$

$$\begin{aligned}
\frac{\bar{C}'_{ijkl}}{c}&=\bar{C}^0_{ijkl}+\frac{\omega_\varepsilon^2\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2}+\frac{\omega_\varepsilon^2\bar{C}^2_{ijkl}}{(a_L^2+\omega_\varepsilon^2)^2}\times\left[2a_L\Psi_{ijkl}-(3a_L^2+\omega_\varepsilon^2)\frac{\Phi_{ijkl}}{w_L}\right],\\
\frac{\bar{C}''_{ijkl}}{c}&=\frac{\tau_e\omega_\varepsilon\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2}+\frac{\omega_\varepsilon\bar{C}^2_{ijkl}}{(a_L^2+\omega_\varepsilon^2)^2}\times\left[(a_L^2-\omega_\varepsilon^2)\Psi_{ijkl}-2a_L\Phi_{ijkl}\right],\\
\bar{e}'_{ijk}&=\bar{e}^0_{ijk}+\frac{\omega_r^2\bar{e}^1_{ijk}}{(a_L^2+\omega_r^2)^2}\left[2a_L\Psi_{ijkl}-(3a_L^2+\omega_r^2)\frac{\Phi_{ijk}}{w_L}\right],\\
\bar{e}''_{ijk}&=\frac{\omega_r\bar{e}^1_{ijk}}{(a_L^2+\omega_r^2)^2}\left[(a_L^2-\omega_r^2)\Psi_{ijk}-2a_L\Phi_{ijk}\right],\\
\frac{\bar{k}'_{ij}}{\kappa}&=\bar{k}^0_{ij}+\frac{\omega_E^2\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2}+\frac{\omega_E^2\bar{k}^2_{ij}}{(a_L^2+\omega_E^2)^2}\times\left[2a_L\Psi_{ij}-(3a_L^2+\omega_E^2)\frac{\Phi_{ij}}{w_L}\right],\\
\frac{\bar{k}''}{\kappa}&=\frac{\tau_d\omega_E\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2}+\frac{\omega_E\bar{k}^2_{ij}}{(a_L^2+\omega_E^2)^2}\left[(a_L^2-\omega_E^2)\Psi_{ij}-2a_L\Phi_{ij}\right].
\end{aligned} \tag{5.5}$$

If $b_L<0$

$$\begin{aligned}
\frac{\bar{C}'_{ijkl}}{c}&=\bar{C}^0_{ijkl}+\frac{\omega_\varepsilon^2\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2}+\frac{\omega_\varepsilon^2\bar{C}^2_{ijkl}}{(w_L+\omega_\varepsilon^2)^2+4b_L\omega_\varepsilon^2}\times\left\{2a_L\Psi_{ijkl}-\frac{\Phi_{ijkl}}{\sqrt{w_L}}\left[2a_L\cos\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+(a_L^2+b_L+\omega_\varepsilon^2)\frac{1}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right]\right\},\\
\frac{\bar{C}''_{ijkl}}{c}&=\frac{\tau_e\omega_\varepsilon\bar{C}^1_{ijkl}}{\tau_e^2+\omega_\varepsilon^2}+\frac{\omega_\varepsilon\bar{C}^2_{ijkl}}{(w_L+\omega_\varepsilon^2)^2+4b_L\omega_\varepsilon^2}\left\{(w_L-\omega_\varepsilon^2)\Psi_{ijkl}-\frac{\Phi_{ijkl}}{\sqrt{w_L}}\left[(w_L-\omega_\varepsilon^2)\cos\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+(w_L+\omega_\varepsilon^2)\frac{a_L}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right]\right\},\\
\bar{e}'_{ijk}&=\bar{e}^0_{ijk}+\frac{\omega_r^2\bar{e}^1_{ijk}}{(w_L+\omega_r^2)^2+4b_L\omega_r^2}\left\{2a_L\Phi_{ijk}-\frac{\Phi_{ijk}}{\sqrt{w_L}}\left[2a_L\cos\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+\frac{a_L^2+b_L+\omega_r^2}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right]\right\},\\
\bar{e}''_{ijk}&=\frac{\omega_r\bar{e}^1_{ijk}}{(w_L+\omega_r^2)^2+4b_L\omega_r^2}\left\{(w_L-\omega_r^2)\Psi_{ijk}-\frac{\Phi_{ijk}}{\sqrt{w_L}}\left[(w_L-\omega_r^2)\cos\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+(w_L+\omega_r^2)\frac{a_L}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right]\right\},\\
\frac{\bar{k}'_{ij}}{\kappa}&=\bar{k}^0_{ij}+\frac{\omega_E^2\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2}+\frac{\omega_E^2\bar{k}^2_{ij}}{(w_L+\omega_E^2)^2+4b_L\omega_E^2}\times\left\{2a_L\Psi_{ij}-\frac{\Phi_{ij}}{\sqrt{w_L}}\left[2a_L\cos\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+\frac{a_L^2+b_L+\omega_E^2}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-L}}{a_L}\right)\right]\right\},\\
\frac{\bar{k}''_{ij}}{\kappa}&=\frac{\tau_d\omega_E\bar{k}^1_{ij}}{\tau_d^2+\omega_E^2}+\frac{\omega_E\bar{k}^2_{ij}}{(w_L+\omega_E^2)^2+4b_L\omega_E^2}\left\{(w_L-\omega_E^2)\Psi_{ij}-\frac{\Phi_{ij}}{\sqrt{w_L}}\left[(w_L-\omega_E^2)\cos\left(\tan^{-1}\frac{\sqrt{-b_l}}{a_L}\right)\right.\right.\\
&\quad\left.\left.+(w_L+\omega_E^2)\frac{a_L}{\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-L}}{a_L}\right)\right]\right\}.
\end{aligned} \tag{5.6}$$

From Equations (5.3)–(5.6), one can see that if the frequency $\omega_r=0$ ($r=\varepsilon$ or E), no matter $b_L>0,=0$ or <0, we obtain

$$\begin{aligned}
\bar{C}'_{ijkl}/c&=\bar{C}^0_{ijkl},\quad \bar{C}''_{ijkl}/c=0,\\
\bar{e}'_{ijk}&=\bar{e}^0_{ijk},\quad \bar{e}''_{ijk}=0,\\
\bar{k}'_{ij}/\kappa&=\bar{k}^0_{ij},\quad \bar{k}''_{ij}/\kappa=0.
\end{aligned} \tag{5.7}$$

That is, in the absence of time harmonic strain and electricfields, the imaginary electroelastic moduli vanish. However, if $\omega_r\to\infty$ ($r=\varepsilon$, or E) when $b_L>0$ or $=0$, we have

$$\begin{aligned}
\frac{\bar{C}'_{ijkl}}{c}&=\bar{C}^0_{ijkl}+\bar{C}^1_{ijkl}-\frac{\Phi_{ijkl}}{w_L}\bar{C}^2_{ijkl},\quad \frac{\bar{C}''_{ijkl}}{c}=0,\\
\bar{e}'_{ijk}&=\bar{e}^0_{ijk}-\frac{\Phi_{ijk}}{w_L}\bar{e}^1_{ijk},\quad \bar{e}''_{ijk}=0,\\
\frac{\bar{k}'_{ij}}{\kappa}&=\bar{k}^0_{ij}+\bar{k}^1_{ij}-\frac{\Phi_{ij}}{w_L}\bar{k}^2_{ij},\quad \frac{\bar{k}''_{ij}}{\kappa}=0,
\end{aligned} \tag{5.8}$$

and when $b_L<0$

$$\begin{aligned}
\frac{\bar{C}'_{ijkl}}{c}&=\bar{C}^0_{ijkl}+\bar{C}^1_{ijkl}-\frac{\Phi_{ijkl}\bar{C}^2_{ijkl}}{\sqrt{w_L}\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right),\\
\frac{\bar{C}''_{ijkl}}{c}&=0,\\
\bar{e}'_{ijk}&=\bar{e}^0_{ijk}-\frac{\Phi_{ijk}\bar{e}^1_{ijk}}{\sqrt{w_L}\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-b_L}}{a_L}\right),\quad \bar{e}''_{ijk}=0,\\
\frac{\bar{k}'_{ij}}{\kappa}&=\bar{k}^0_{ij}+\bar{k}^1_{ij}-\frac{\Phi_{ij}\bar{k}^2_{ij}}{\sqrt{w_L}\sqrt{-b_L}}\sin\left(\tan^{-1}\frac{\sqrt{-a_L}}{b_L}\right),\\
\frac{\bar{k}''_{ij}}{\kappa}&=0.
\end{aligned} \tag{5.9}$$

For $\bar{C}^{*}_{1212}$, when $\omega_\epsilon = 0$

$$\frac{\bar{C}''_{1212}}{c} = \frac{1}{2}(1 - p_0)\left\{1 + \frac{2fp_5\tau_e}{\varpi_D V_D}\right\}, \quad \frac{C''_{1212}}{c} = 0. \tag{5.10}$$

If $\omega_\varepsilon \to \infty$, then

$$\frac{\bar{C}'_{1212}}{c} = \frac{1}{2}(1 - p_0)\left\{1 + \rho_c + \frac{f\tau_e}{\varpi_D}\left[\frac{2p_5}{b_D} + \frac{\rho_c}{b_D - \tau_e}\right] - \frac{f}{\varpi_D}\left[\rho_c + \frac{2p_5(\tau_e - b_D)}{b_D}\right]\left[1 + \frac{\rho_c b_D}{b_D - \tau_e}\right]\right\},$$

$$\frac{\bar{C}''_{1212}}{c} = 0. \tag{5.11}$$

Thus the loss electroelastic moduli of the piezocomposite tend to zero and the storage electroelastic moduli approach constants as the frequency of the applied loads goes to either zero or infinity. However, the two limiting values of the storage electroelastic moduli are different.

The dependence upon the frequency ω of the effective storage moduli $\bar{C}'_{3333}$, $\bar{e}'_{333}$, $\bar{e}'_h$ and $\bar{k}'_{33}$ for the piezocomposite with a dielectrically nonrelaxing viscoelastic matrix is displayed in Figures 9(a)–(d) respectively; values of ρ_c, τ_e and f for different curves are given in Figure 9(a). Whereas $\bar{e}'_h$ is nearly independent of the frequency, and $\bar{e}'_{333}$ and $\bar{k}'_{33}$ vary slightly with the frequency in the range $10^{-2} \le \omega \le 10^2$, $\bar{C}'_{3333}$ exhibits

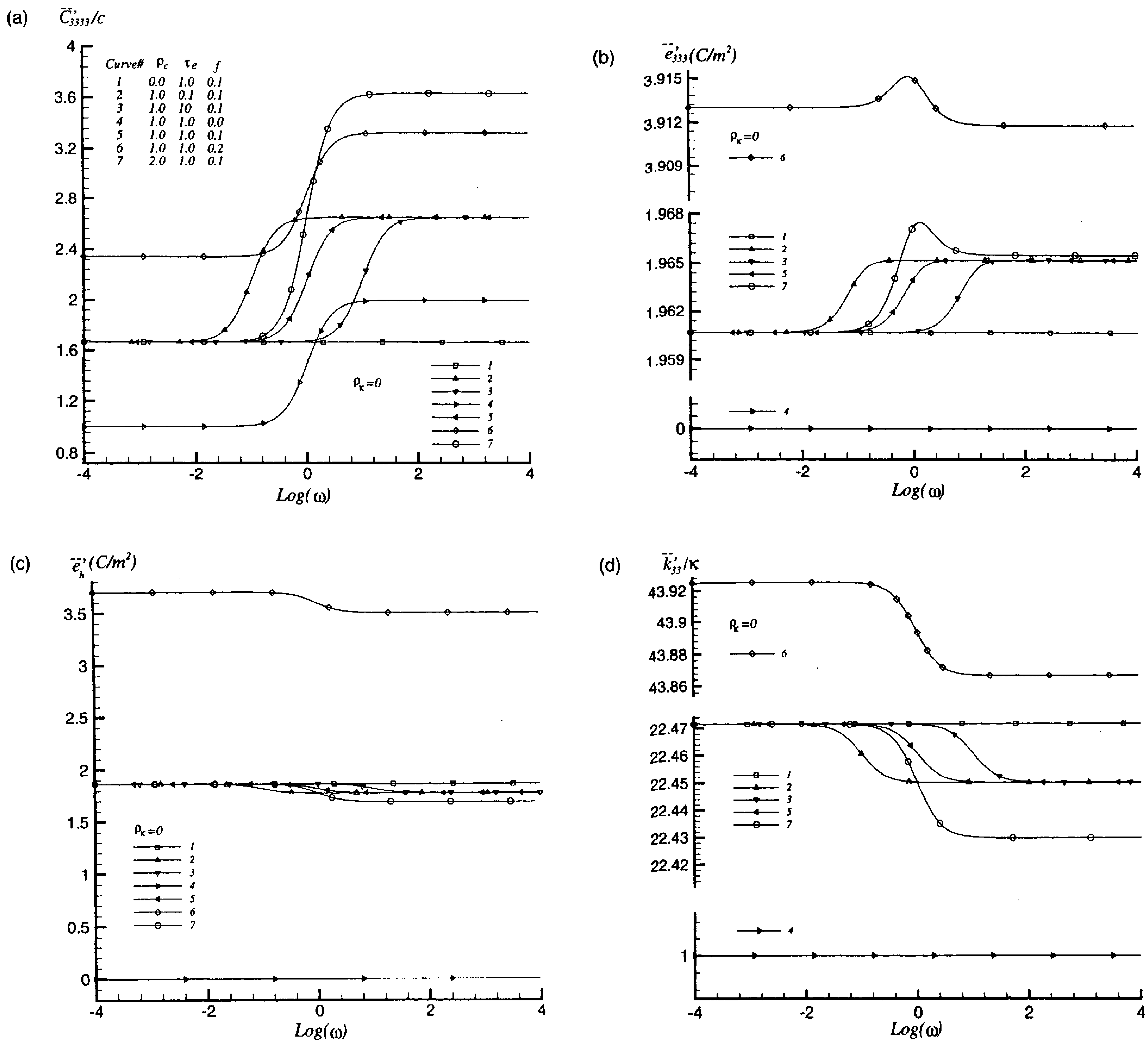

Figure 9. Dependence upon the frequency ω of the effective storage moduli (a) $\bar{C}'_{3333}$, (b) $\bar{e}'_{333}$, (c) $\bar{e}'_h$, and (d) $\bar{k}'_{33}$ for the piezocomposite with a dielectrically nonrelaxing viscoelastic matrix.

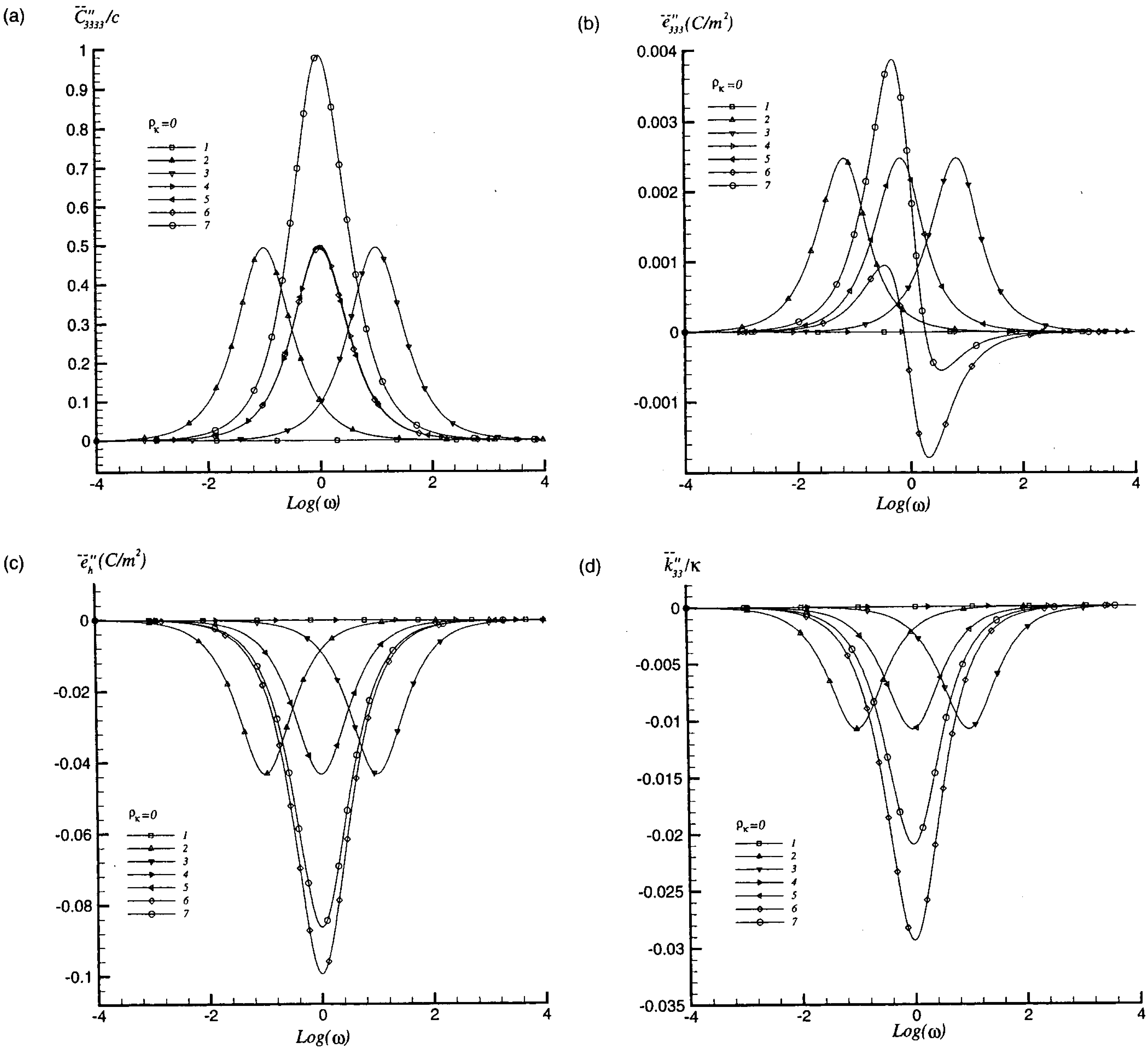

Figure 10. *Variation with the frequency ω of the effective loss moduli (a) $\bar{C}''_{3333}$, (b) $\bar{e}''_{333}$, (c) $\bar{e}''_h$ and (d) $\bar{k}''_{33}$ for the piezocomposite with a dielectrically nonrelaxing viscoelastic matrix. See Figure 9(a) for the identifications of different curves.*

noticeable dependence upon the frequency in the range of 10^{-2}–10^{2} Hz. In general, with an increase in the frequency of the applied loading, values of $\bar{k}'_{33}$ decrease but those of $\bar{C}'_{3333}$ increase. The difference in the values of $\bar{C}'_{3333}$ at high and low frequencies can be as high as 50%, that in the values of $\bar{e}'_{333}$ and $\bar{k}'_{33}$ is < 0.5%. Figures 10(a)–(d) show the variation of the loss moduli $\bar{C}''_{3333}$, $\bar{e}''_{333}$, $\bar{e}''_h$ and $\bar{k}''_{33}$ with the frequency ω of the applied load. A change in the values of τ_e shifts the frequency at which the maximum value of $\bar{C}''_{3333}$ occurs but does not alter its maximum value which is proportional to ρ_c. Comparing curves 5 and 6, we see that its maximum value and the range of frequencies for which the loss modulus is positive do not depend upon the volume fraction of the PZT inclusions; however, values of f strongly influence the storage moduli $\bar{C}'_{3333}$, and $\bar{e}'_{333}$. The maximum values of $\bar{e}''_{333}$, $\bar{e}''_h$ and $\bar{k}''_{33}$ are negligible as compared to those of $\bar{e}'_{333}$, $\bar{e}'_h$ and $\bar{k}'_{33}$ respectively.

Figures 11(a) and (b) exhibit the dependence of $\bar{k}'_{33}$ and $\bar{k}''_{33}$ upon the frequency ω for the piezocomposite with a dielectrically relaxing elastic matrix; values of ρ_κ, τ_d and f for different curves are given in Figure 11(a). Whereas the storage modulus $\bar{k}'_{33}$ is essentially independent of ω, the loss modulus $\bar{k}''_{33}$ strongly depends upon ω and this dependence is similar to that of $\bar{C}''_{3333}$. The doubling of the volume fraction of the PZT inclusions doubles the storage modulus $\bar{k}'_{33}$ but decreases slightly the loss modulus $\bar{k}''_{33}$. In Figures 12(a) and (b), we have plotted the dependence of the storage, $\bar{k}'_{33}$, and the loss, $\bar{k}''_{33}$, moduli upon the frequency, ω, for different values of ρ_c, ρ_κ, τ_e and τ_d listed in Figure 11(a). The maximum increase in the value of $\bar{k}'_{33}$ is about 5% and its values increase monotonically from those at low frequencies to the ones at high frequencies. Values of the loss modulus vanish at very low and at very high frequencies and the frequency at which its peak value occurs depends upon the values of τ_d. An increase (decrease) in the value of τ_d increases (decreases) this frequency.

Figures 13(a)–(d) and 14(a)–(d) exhibit respectively how the dependence of the effective storage and the effective loss moduli upon the frequency vary with the aspect ratio $R = a_2/a_1$ of the cross section of the elliptic cylindrical PZT inclusions. The aspect ratio of the cross section of the PZT inclusions does not influence much the variation with the frequency of the storage and the loss moduli $\bar{C}'_{3333}$, $\bar{e}'_{333}$, $\bar{k}'_{33}$, $\bar{C}''_{3333}$, and $\bar{k}''_{33}$. However, values of $\bar{e}'_h$, $\bar{e}''_h$ and $\bar{e}''_{333}$ do depend noticeably upon the aspect ratio R.

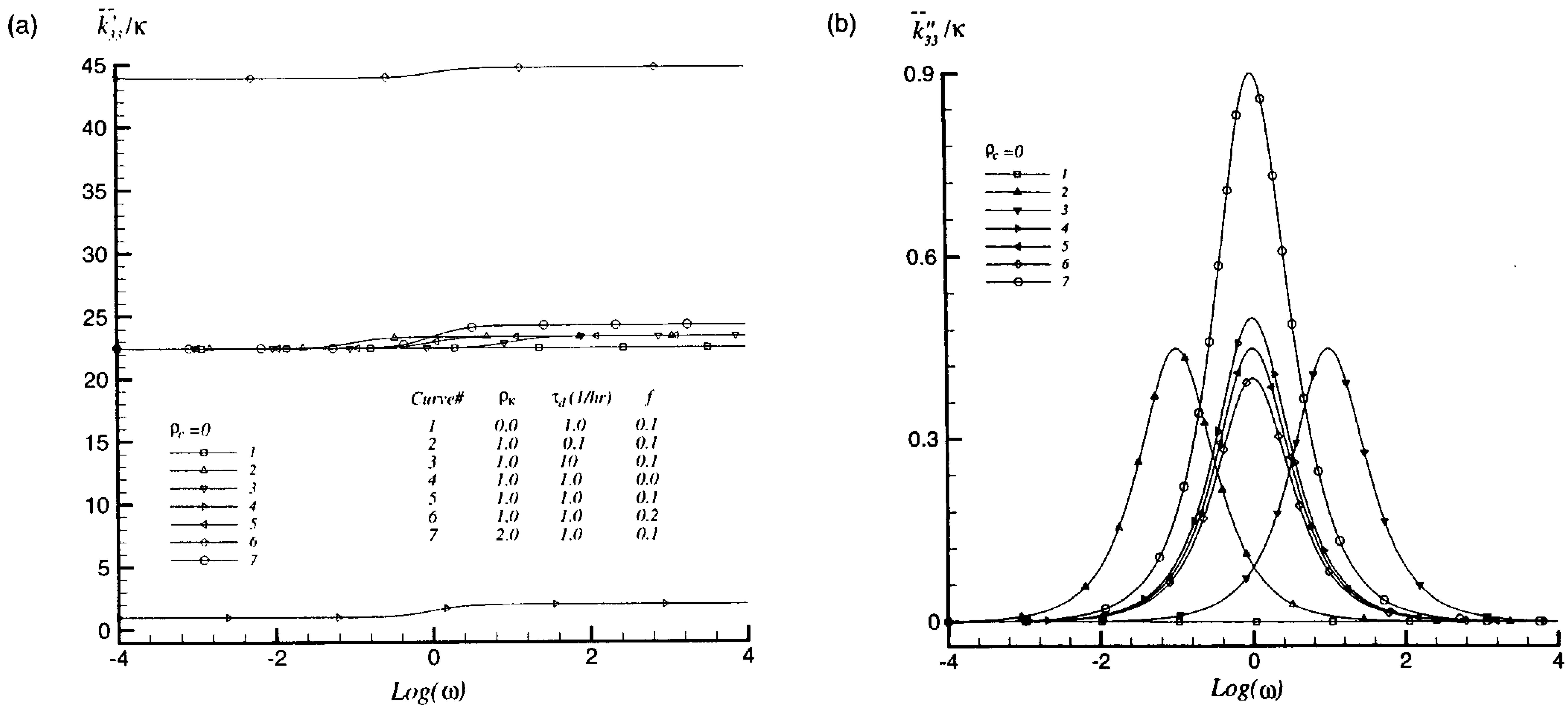

Figure 11. Dependence upon the frequency ω of (a) the effective storage modulus $\bar{k}'_{33}$, and (b) the loss modulus $\bar{k}''_{33}$ for the piezocomposite with a dielectrically relaxing elastic matrix.

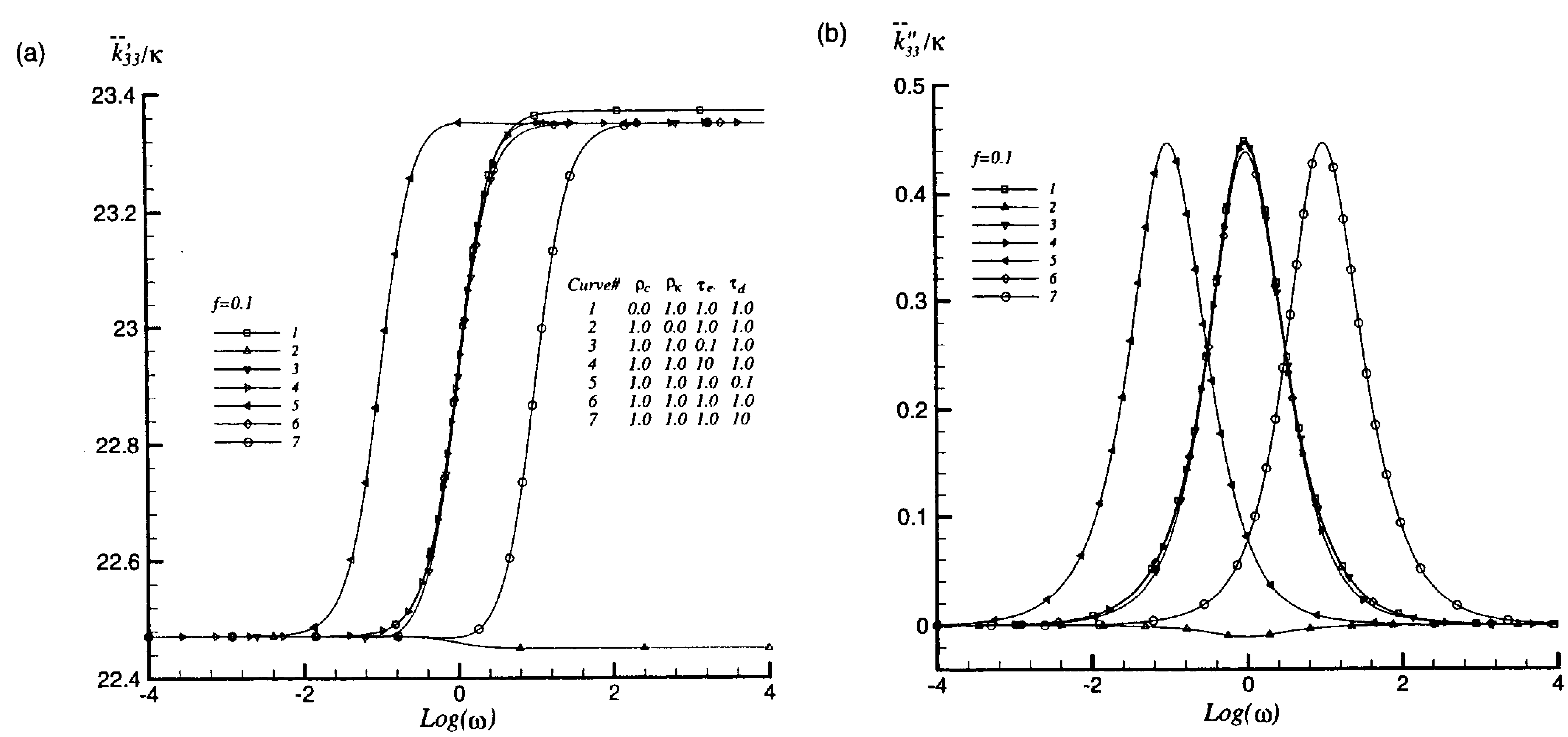

Figure 12. For different values of ρ_c, ρ_κ, τ_e and τ_d, the dependence upon the frequency ω of (a) the effective storage modulus $\bar{k}'_{33}$, and (b) the loss modulus $\bar{k}''_{33}$ for the piezocomposite with a viscoelastic matrix.

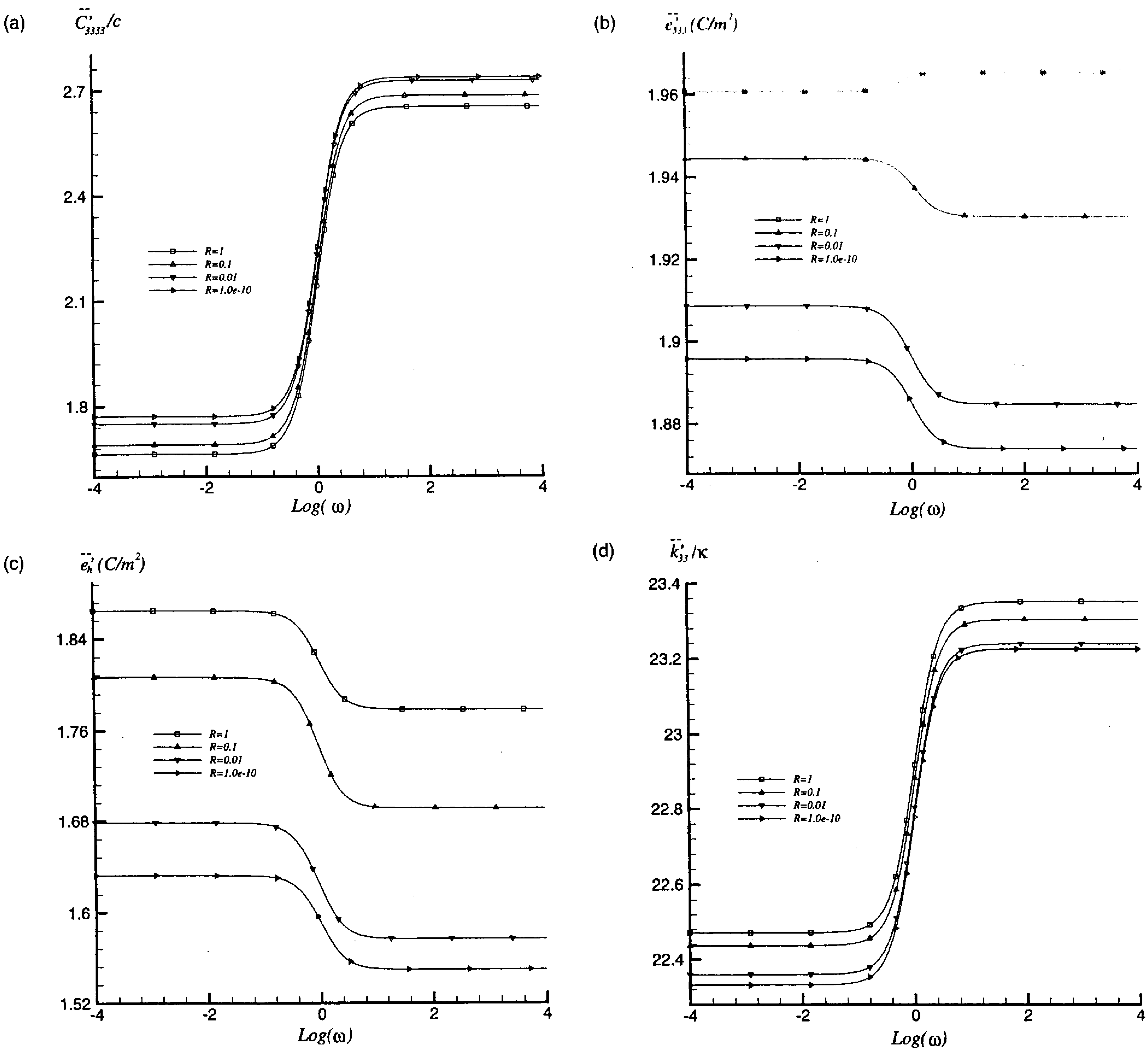

Figure 13. *For four different values of the aspect ratio of the cross section of the cylindrical PZT inclusions, the dependence upon the frequency* ω *of the effective storage moduli (a)* $\bar{C}'_{3333}$*, (b)* $\bar{e}'_{333}$*, (c)* $\bar{e}'_h$ *and (d)* $\bar{k}'_{33}$ *of the piezocomposite with a viscoelastic matrix.*

6 CONCLUSIONS

We have derived closed form expressions for the effective moduli of a piezocomposite comprised of elliptic cylindrical PZT inclusions aligned parallel to each other and embedded in a dielectrically relaxing viscoelastic matrix. The interaction among inclusions is accounted for by using the Mori–Tanaka method. The correspondence principle of linear viscoelasticity is employed to deduce expressions for the constraint strain tensor and the constraint electric field in the PZT inclusions from those for the geometrically identical piezocomposite but the viscoelastic matrix replaced by an elastic matrix. All of the viscoelastic moduli are assumed to have the same relaxation time which may be different from that of the dielectric constants.

It is found that relaxation times of different effective moduli of the piezocomposite are not the same. The relaxation of the dielectric constants of the matrix affects only the effective dielectric constants and the shear components of the effective piezoelectric and the visco-elastic components of the piezocomposite. The relaxation of the viscoelastic moduli of the matrix influences all of the effective moduli of the piezocomposite. The effective loss moduli of the piezocomposite vanish for

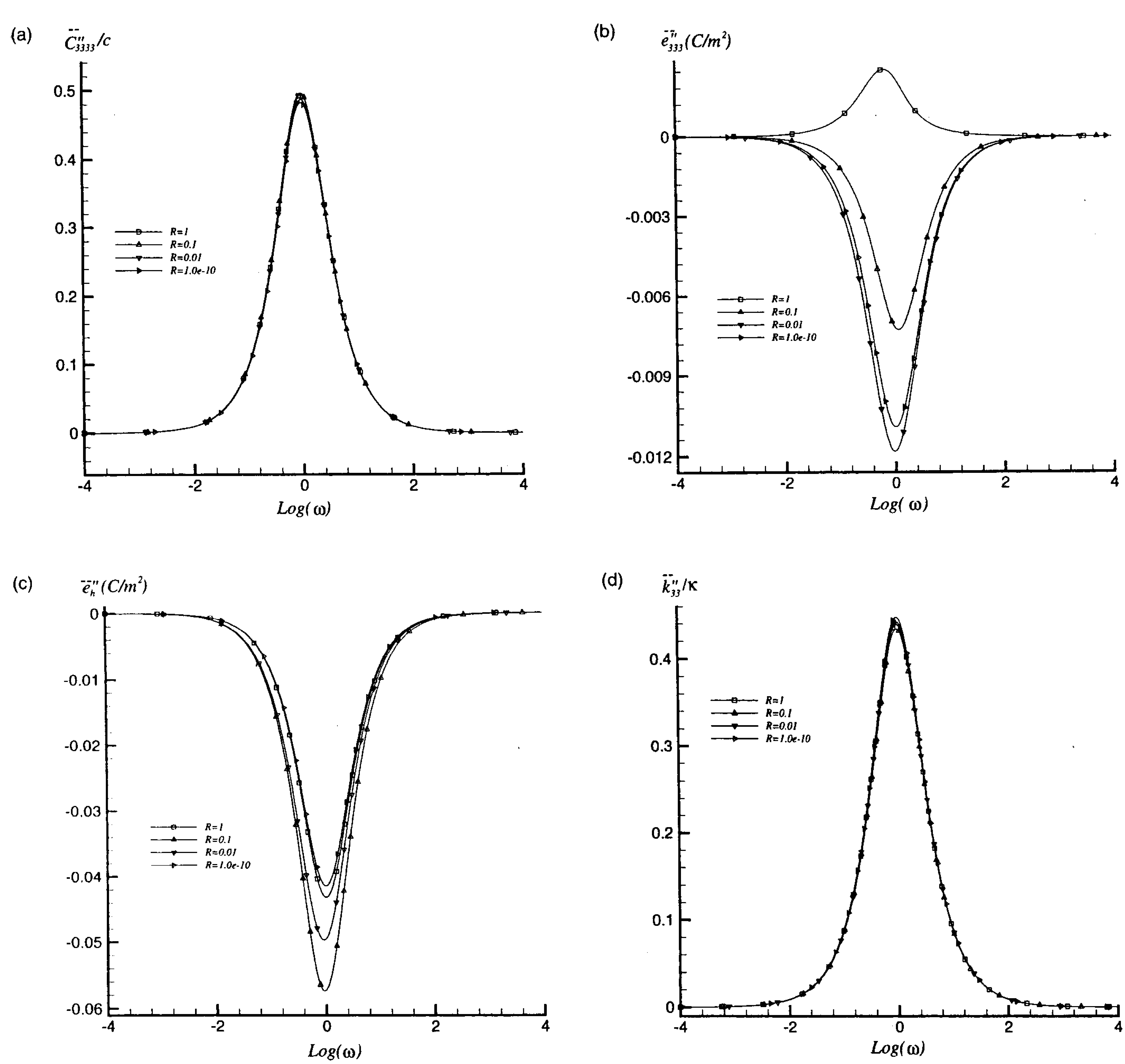

Figure 14. *For four different values of the aspect ratio of the cross section of the cylindrical PZT inclusions, the dependence upon the frequency* ω *of the effective loss moduli (a)* $\bar{C}''_{3333}$, *(b)* $\bar{e}''_{333}$, *(c)* $\bar{e}''_h$ *and (d)* $\bar{k}''_{33}$ *of the piezocomposite with a viscoelastic matrix.*

very low and very high frequencies, and the frequency at which they have peak values varies with the relaxation times of the moduli of the dielectrically relaxing viscoelastic matrix, and the ratio of the instantaneous to the equilibrium values of the moduli of the matrix.

For steady loading of the piezocomposite, the effective viscoelastic moduli of the piezocomposite have the largest value for ribbon like PZT inclusions and the smallest value for circular cylindrical PZT inclusions. The aspect ratio of the elliptic cross section of the PZT inclusions affects significantly the effective value of the $\bar{e}_h = \bar{e}_{3ii}$ but does not alter much the value of $\bar{e}_{333}$ where $\bar{e}_{333}$ is the effective piezoelectric moduli of the piezocomposite along the axis of the PZT inclusions. Whereas the volume fraction of the PZT inclusions significantly affects the relaxation times of the effective moduli $\bar{C}_{3333}$ and $\bar{k}_{33}$, the aspect ratio of the cross section of the PZT inclusions has a noticeable effect on the relaxation time of $\bar{k}_{33}$ but not that much effect on the relaxation time of $\bar{C}_{3333}$.

ACKNOWLEDGMENT

This work was partially supported by the NSF grant CMS9713453 and the ARO grant DAAG55-98-1-0030 to Virginia Polytechnic Institute and State University.

APPENDIX

A Determination of Parameters in Equations (32)–(41)

In order to simplify equations, we introduce following dimensionless parameters:

$$p_1 = \frac{C^I_{1111} - c}{(1-p_0)c}, \quad p_2 = \frac{C^I_{1122} - p_0 c}{(1-p_0)c}, \quad p_3 = \frac{C^I_{1133} - p_0 c}{(1-p_0)c},$$

$$p_4 = \frac{C^I_{1313} - \frac{1}{2}(1-p_0)c}{(1-p_0)c}, \quad p_5 = \frac{C^I_{1212} - \frac{1}{2}(1-p_0)c}{(1-p_0)c},$$

$$p_6 = \frac{C^I_{3333} - c}{(1-p_0)c},$$

$$q_1 = q_2 = \frac{k^I_{11} - \kappa}{\kappa}, \quad q_3 = \frac{k^I_{33} - \kappa}{\kappa},$$

$$r_1 = \frac{(e^I_{131})^2}{(1-p_0)\kappa c}, \quad r_2 = \frac{(e^I_{311})^2}{(1-p_0)\kappa c},$$

$$r_3 = \frac{(e^I_{333})^2}{(1-p_0)\kappa c}, \quad r_4 = \frac{e^I_{311} e^I_{333}}{(1-p_0)\kappa c},$$

$$p^+_{12} = \tfrac{1}{2}(p_1 + p_2), \quad p^-_{12} = \tfrac{1}{2}(p_1 - p_2),$$

$$m_1 = \frac{a_2}{a_1 + a_2}, \quad m_2 = \frac{a_1}{a_1 + a_2},$$

where $2a_1$ and $2a_2$ are principal axes along x_1 and x_2 directions, repectively, of the elliptic cross section of the cylinder.

A.1 RELAXATION TIMES OF THE PIEZOCOMPOSITE

The relaxation times of the piezocomposite are related to a_L and b_L ($L = A, B, C, D$) whose expressions are given below.

$$a_A = \tfrac{1}{2}[(1-\chi^d_A)\tau_d + (1-\chi^e_A)\tau_e], \quad b_A = (a_A)^2 - w_A,$$
$$a_B = \tfrac{1}{2}[(1-\chi^d_B)\tau_d + (1-\chi^e_B)\tau_e], \quad b_B = (a_B)^2 - w_B,$$
$$a_C = \tfrac{1}{2}(1+\chi^e_C)\tau_e, \quad b_C = (a_C)^2 - w_C, \quad b_D = \frac{\varphi^1_D}{\varpi_D}\tau_e, \tag{A1}$$

where

$$w_A = \chi^{de}_A \tau_d \tau_e, \quad w_B = \chi^{de}_B \tau_d \tau_e, \quad w_C = \chi^{ee}_C (\tau_e)^2,$$
$$\chi^d_A = \frac{\varphi^2_A + \varphi^4_A}{\varpi_A}, \quad \chi^e_A = \frac{\varphi^3_A + \varphi^4_A}{\varpi_A}, \quad \chi^{de}_A = \frac{\varphi^1_A}{\varpi_A},$$
$$\chi^d_B = \frac{\varphi^2_B + \varphi^4_B}{\varpi_B}, \quad \chi^e_B = \frac{\varphi^3_B + \varphi^4_B}{\varpi_B}, \quad \chi^{de}_B = \frac{\varphi^1_B}{\varpi_B},$$
$$\chi^e_C = \frac{\varphi^1_C - \varphi^3_C}{\varpi_C}, \quad \chi^{ee}_C = \frac{\varphi^1_C}{\varpi_C}, \tag{A2}$$

$$\varphi^1_A = 1 + (1-f)m_1\{2p_4 + q_1 + 2(1-f)(r_1 + p_4 q_1)m_1\},$$
$$\varphi^2_A = \rho_\kappa\{1 - (1-f)m_1[1 - 2p_4 + 2(1-f)p_4 m_1]\},$$
$$\varphi^3_A = \rho_c\{1 - (1-f)m_1[1 - q_1 + (1-f)q_1 m_1]\},$$
$$\varphi^4_A = \rho_\kappa\rho_c\{1 - (1-f)m_1[2 - (1-f)m_1]\},$$
$$\varpi_A = \sum_{i=1}^{4} \varphi^i_A, \tag{A3}$$

$$\varphi^1_B = 1 + (1-f)m_2\{2p_4 + q_1 + 2(1-f)(r_1 + p_4 q_1)m_2\},$$
$$\varphi^2_B = \rho_\kappa\{1 - (1-f)m_2[1 - 2p_4 + 2(1-f)p_4 m_2]\},$$
$$\varphi^3_B = \rho_c\{1 - (1-f)m_2[1 - q_1 + (1-f)q_1 m_2]\},$$
$$\varphi^4_B = \rho_\kappa\rho_c\{1 - (1-f)m_2[2 - (1-f)m_2]\},$$
$$\varpi_B = \sum_{i=1}^{4} \varphi^i_2, \tag{A4}$$

$$\varphi^1_C = 1 + (1-f)\{(1-p_0)p_1$$
$$+ 2m_1 m_2 p^-_{12}[1 + p_0 + (1-f)(1-p_0)(3-p_0)p^+_{12}]\},$$
$$\varphi^2_C = \rho_c\{2 + (1-f)[(1-p_0)p_1 - 1]$$
$$+ (1-f)m_1 m_2[(1+p_0)(p_1 - p_2 - 1),$$
$$- (1-f)(3-p_0)(p_1 - p_0 p_2)]\},$$
$$\varphi^3_C = (\rho_c)^2\{f - (1-f)(1+p_0)m_1 m_2$$
$$\times \left(1 - \tfrac{1}{2}(1-f)(3-p_0)\right)\},$$
$$\varpi_C = \sum_{i=1}^{3} \varphi^i_C, \tag{A5}$$

$$\varphi^1_D = 1 + 2(1-f)[1 - (1+p_0)m_1 m_2]p_5,$$
$$\varphi^2_D = \rho_c\{1 - (1-f)[1 - (1+p_0)m_1 m_2]\},$$
$$\varpi_D = \sum_{m=1}^{2} \varphi^m_D. \tag{A6}$$

A.2 PARAMETERS RELATED TO EFFECTIVE ELECTROELASTIC MODULI OF THE PIEZOCOMPOSITE

The parameters related to effective electroelastic moduli of the piezocomposite are

$$
\begin{aligned}
&\bar{C}^0_{1111} = 1 + \bar{C}^2_{1111}[1 + \rho_c(1-\theta_{1111}) + \Phi_{1111}/w_C],\\
&\bar{C}^1_{1111} = \rho_c(1 + \bar{C}^2_{1111}\vartheta_{1111}),\\
&\bar{C}^2_{1111} = (1-p_0)\frac{f}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{1111},\\
&\bar{C}^0_{2222} = 1 + \bar{C}^2_{2222}[1 + \rho_c(1-\theta_{2222}) + \Phi_{2222}/w_C],\\
&\bar{C}^1_{2222} = \rho_c(1 + \bar{C}^2_{2222}\vartheta_{2222}),\\
&\bar{C}^2_{2222} = (1-p_0)\frac{f}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{2222},\\
&\bar{C}^0_{3333} = 1 + f(1-p_0)p_6 + \bar{C}^2_{3333}[(1-p_0)p_3 - p_0\rho_c(1-\theta_{3333})\\
&\qquad + \Phi_{3333}/w_C],\\
&\bar{C}^1_{3333} = \rho_c[1 - f - p_0\bar{C}^2_{3333}\vartheta_{3333}],\\
&\bar{C}^2_{3333} = -f(1-f)(1-p_0)\frac{1}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{3333},\\
&\bar{C}^0_{1122} = p_0 + \bar{C}^2_{1122}[1 + \rho_c(1-\theta_{1122}) + \Phi_{1122}/w_C],\\
&\bar{C}^1_{1122} = \rho_c(p_0 + \bar{C}^2_{1122}\vartheta_{1122}),\\
&\bar{C}^2_{1122} = (1-p_0)\frac{f}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{1122},\\
&\bar{C}^0_{1133} = p_0 + \bar{C}^2_{1133}[1 + \rho_c(1-\theta_{1133}) + \Phi_{1133}/w_C],\\
&\bar{C}^1_{1133} = \rho_c(p_0 + \bar{C}^2_{1133}\vartheta_{1133}),\\
&\bar{C}^2_{1133} = (1-p_0)\frac{f}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{1133},\\
&\bar{C}^0_{2233} = p_0 + \bar{C}^2_{2233}[1 + \rho_c(1-\theta_{2233}) + \Phi_{2233}/w_C],\\
&\bar{C}^1_{2233} = \rho_c(p_0 + \bar{C}^2_{2233}\vartheta_{2233}), \quad \bar{C}^2_{2233} = (1-p_0)\frac{f}{\varpi_C}\sum_{m=1}^{3}\breve{C}^m_{2233},\\
&\bar{C}^0_{1313} = \tfrac{1}{2}(1-p_0) + \bar{C}^2_{1313}[1 + \rho_c(1-\theta_{1313}) + \Phi_{1313}/w_A],\\
&\bar{C}^1_{1313} = \rho_c\left[\tfrac{1}{2}(1-p_0) + \bar{C}^2_{1313}\vartheta_{1313}\right],\\
&\bar{C}^2_{1313} = (1-p_0)\frac{f}{\varpi_A}\sum_{m=1}^{4}\breve{C}^m_{1313},\\
&\bar{C}^0_{2323} = \tfrac{1}{2}(1-p_0) + \bar{C}^2_{2323}[1 + \rho_c(1-\theta_{2323}) + \Phi_{2323}/w_B],\\
&\bar{C}^1_{2323} = \rho_c\left[\tfrac{1}{2}(1-p_0) + \bar{C}^2_{2323}\vartheta_{2323}\right],\\
&\bar{C}^2_{2323} = (1-p_0)\frac{f}{\varpi_B}\sum_{m=1}^{4}\breve{C}^m_{2323},
\end{aligned}
\tag{A7}
$$

$$
\begin{aligned}
&\bar{e}^0_{131} = \bar{e}^1_{131}(1 + \Phi_{131}/w_A), \quad \bar{e}^1_{131} = f(1+\rho_\kappa)(1+\rho_c)\frac{e^I_{131}}{\varpi_A},\\
&\bar{e}^0_{232} = \bar{e}^1_{232}(1 + \Phi_{232}/w_B), \quad \bar{e}^1_{232} = f(1+\rho_\kappa)(1+\rho_c)\frac{e^I_{131}}{\varpi_B},\\
&\bar{e}^0_{311} = \bar{e}^1_{311}(1 + \Phi_{311}/w_C), \quad \bar{e}^1_{311} = \frac{f e^I_{311}}{\varpi_C}\sum_{m=1}^{3}\breve{e}^m_{311},\\
&\bar{e}^0_{322} = \bar{e}^1_{322}(1 + \Phi_{322}/w_C), \quad \bar{e}^1_{322} = \frac{f e^I_{311}}{\varpi_C}\sum_{m=1}^{3}\breve{e}^m_{322},\\
&\bar{e}^0_{333} = f e^I_{333} + \bar{e}^1_{333}(1 + \Phi_{333}/w_C),\\
&\bar{e}^1_{333} = -f(1-f)(1-p_0)\frac{e^I_{311}}{\varpi_C}\sum_{m=1}^{3}\breve{e}^m_{333},
\end{aligned}
\tag{A8}
$$

$$
\begin{aligned}
&\bar{k}^0_{11} = 1 + \bar{k}^2_{11}[1 + \rho_\kappa(1-\theta_{11}) + \Phi_{11}/w_A],\\
&\bar{k}^1_{11} = \rho_\kappa(1 + \bar{k}^2_{11}\vartheta_{11}), \quad \bar{k}^2_{11} = \frac{f}{\varpi_A}\sum_{m=1}^{4}\breve{k}^m_{11},\\
&\bar{k}^0_{22} = 1 + \bar{k}^2_{22}[1 + \rho_\kappa(1-\theta_{22}) + \Phi_{22}/w_B],\\
&\bar{k}^1_{22}\rho_\kappa(1 + \bar{k}^2_{22}\vartheta_{22}), \quad \bar{k}^2_{22} = \frac{f}{\varpi_B}\sum_{m=1}^{4}\breve{k}^m_{22},\\
&\bar{k}^0_{33} 1 + f q_3 + \bar{k}^2_{33}(1 + \Phi_{33}/w_C),\\
&\bar{k}^1_{33}(1-f)\rho_\kappa, \quad \bar{k}^2_{33} = f(1-f)(1-p_0)\frac{r_2}{\varpi_C}\sum_{m=1}^{2}\breve{k}^m_{33}.
\end{aligned}
\tag{A9}
$$

For $ijkl = 1111$, 2222, 1122, 1133, 2233, we have

$$
\begin{aligned}
&\Psi_{ijkl} = [(1+\rho_c)(\Lambda^e_{ijkl} - \chi^e_C) + \rho_c(\vartheta_{ijkl} - 1)]\tau_e,\\
&\Phi_{ijkl} = [\Lambda^{ee}_{ijkl} - \chi^{ee}_C + \rho_c\chi^{ee}_C(\vartheta_{ijkl} - 1)](\tau_e)^2,\\
&\vartheta_{ijkl} = \frac{\Lambda^e_{ijkl} - \Lambda^{ee}_{ijkl}}{\chi^e_C - \chi^{ee}_C}.
\end{aligned}
\tag{A10}
$$

Moreover

$$
\begin{aligned}
&\Psi_{3333} = \{[(1-p_0)p_3 - p_0\rho_c](\Lambda^e_{3333} - \chi^e_C) - p_0\rho_c(\vartheta_{3333} - 1)\}\tau_e\\
&\Phi_{3333} = \{(1-p_0)p_3(\Lambda^{ee}_{3333} - \chi^{ee}_C) - p_0\rho_c\chi^{ee}_C(\vartheta_{3333} - 1)\}(\tau_e)^2\\
&\vartheta_{3333} = \frac{\Lambda^e_{3333} - \Lambda^{ee}_{3333}}{\chi^e_C - \chi^{ee}_C}.
\end{aligned}
\tag{A11}
$$

For $ijkl = 1313$ and 2323.

$$
\begin{aligned}
&\Psi_{ijkl} = (1+\rho_c)\left[(\chi^e_L - \Lambda^e_{ijkl})\tau_e + (\chi^d_L - \Lambda^d_{ijkl})\tau_d\right]\\
&\qquad + \rho_c\tau_e(\vartheta_{ijkl} - 1),\\
&\Phi_{ijkl} = \left[\Lambda^{de}_{ijkl} - \chi^{de}_L + \rho_c\chi^{de}_L(\vartheta_{ijkl} - 1)\right]\tau_d\tau_e,\\
&\vartheta_{ijkl} = \frac{(1 - \Lambda^d_{ijkl} - \Lambda^{de}_{ijkl})\tau_d - \Lambda^e_{ijkl}\tau_e}{(1 - \chi^d_L - \chi^{de}_L)\tau_d - \chi^e_L\tau_e},
\end{aligned}
\tag{A12}
$$

where $L=A$ for $ijkl=1313$ and B for $ijkl=2323$. For $ijk=311$, 322 and 333, we have

$$\Psi_{ijk}=(\Lambda^{e}_{ijk}-\chi^{e}_{C})\tau_e,\quad \Phi_{ijk}=(\Lambda^{ee}_{ijk}-\chi^{ee}_{C})(\tau_e)^2, \quad (A13)$$

and for $ijk=131$ and 232

$$\Psi_{ijk}=\left[\chi^{d}_{L}-\frac{\rho_\kappa}{1+\rho_\kappa}\right]\tau_d+\left[\chi^{e}_{L}-\frac{\rho_c}{1+\rho_c}\right]\tau_e,$$
$$\Phi_{ijk}=\left[\frac{1}{(1+\rho_c)(1+\rho\kappa)}-\chi^{de}_{L}\right]\tau_e\tau_d, \quad (A14)$$

where $L=A$ for $ijk=131$ and B for $ijk=232$. For $ij=11$ and 22

$$\Psi_{ij}=(1+\rho_\kappa)\Big[(\chi^{e}_{L}-\Lambda^{e}_{ij})\tau_e+(\chi^{d}_{L}-\Lambda^{d}_{ij})\tau_d\Big]+\rho_\kappa\tau_d(\vartheta_{ij}-1),$$
$$\Phi_{ij}=\Big[\Lambda^{de}_{ij}-\chi^{de}_{L}+\rho_\kappa\chi^{de}_{L}(\vartheta_{ij}-1)\Big]\tau_e\tau_d,$$
$$\vartheta_{ij}=\frac{[1-(\Lambda^{e}_{ij}+\Lambda^{de}_{ij})]\tau_e-\Lambda^{d}_{ij}\tau_d}{[1-(\chi^{e}_{L}+\chi^{de}_{L})]\tau_e-\chi^{d}_{L}\tau_d}, \quad (A15)$$

where $L=A$ for $ij=11$ and B for $ij=22$. Expressions for Ψ_{33} and Φ_{33} are

$$\Psi_{33}=(\Lambda_{33}-\chi^{e}_{C})\tau_e,\quad \Phi_{33}=(\Lambda_{33}-\chi^{ee}_{C})(\tau_e)^2. \quad (A16)$$

In Equations (A10)–(A16), non-zero components of Λ^{e}_{ijkl} and Λ^{ee}_{ijkl}, for $ijkl=1111$, 2222, 3333, 1122, 1133, and 2233 are

$$\Lambda^{e}_{ijkl}=\frac{\breve{C}^{1}_{ijkl}-\breve{C}^{3}_{ijkl}}{\sum_{m=1}^{3}\breve{C}^{m}_{ijkl}},\quad \Lambda^{ee}_{ijkl}=\frac{\breve{C}^{1}_{ijkl}}{\sum_{m=1}^{3}\breve{C}^{m}_{ijkl}} \quad (A17)$$

The components of Λ^{d}_{ijkl}, Λ^{e}_{ijkl} and Λ^{de}_{ijkl} for $ijkl=1313$ and 2323 have the forms

$$\Lambda^{d}_{ijkl}=\frac{\breve{C}^{2}_{ijkl}+\breve{C}^{4}_{ijkl}}{\sum_{m=1}^{4}\breve{C}^{m}_{ijkl}},\quad \Lambda^{e}_{ijkl}=\frac{\breve{C}^{3}_{ijkl}+\breve{C}^{4}_{ijkl}}{\sum_{m=1}^{4}\breve{C}^{m}_{ijkl}},$$
$$\Lambda^{de}_{ijkl}=\frac{\breve{C}^{1}_{ijkl}}{\sum_{m=1}^{4}\breve{C}^{m}_{ijkl}}. \quad (A18)$$

The non-zero components of Λ^{e}_{ijk} and Λ^{ee}_{ijk} for $ijk=311$, 322, 333, are given by

$$\Lambda^{e}_{ijk}=\frac{\breve{e}^{1}_{ijk}-\breve{e}^{3}_{ijk}}{\sum_{m=1}^{3}\breve{e}^{m}_{ijk}},\quad \Lambda^{ee}_{ijk}=\frac{\breve{e}^{1}_{ijk}}{\sum_{m=1}^{3}\breve{e}^{m}_{ijk}}, \quad (A19)$$

and

$$\Lambda^{d}_{11}=\frac{\breve{k}^{2}_{11}+\breve{k}^{4}_{11}}{\sum_{m=1}^{4}\breve{k}^{m}_{11}},\quad \Lambda^{e}_{11}=\frac{\breve{k}^{3}_{11}+\breve{k}^{4}_{11}}{\sum_{m=1}^{4}\breve{k}^{m}_{11}},\quad \Lambda^{de}_{11}=\frac{\breve{k}^{1}_{11}}{\sum_{m=1}^{4}\breve{k}^{m}_{11}},$$
$$\Lambda^{d}_{22}=\frac{\breve{k}^{2}_{22}+\breve{k}^{4}_{22}}{\sum_{m=1}^{4}\breve{k}^{m}_{22}},\quad \Lambda^{e}_{22}=\frac{\breve{k}^{3}_{22}+\breve{k}^{4}_{22}}{\sum_{m=1}^{4}\breve{k}^{m}_{22}},\quad \Lambda^{de}_{22}=\frac{\breve{k}^{1}_{22}}{\sum_{m=1}^{4}\breve{k}^{m}_{22}},$$
$$\Lambda_{33}=\frac{\breve{k}^{1}_{33}}{\sum_{m=1}^{2}\breve{k}^{m}_{33}}. \quad (A20)$$

Furthermore

$$\breve{C}^{1}_{1111}=p_1+2(1-f)m_2[2(1-p_0)+(1+p_0)m_1]p^{+}_{12}p^{-}_{12}$$
$$\breve{C}^{2}_{1111}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_1-1-(1-f)m_2[2(1-p_0)+(1+p_0)m_1](p_1-p_0p_2)\}$$
$$\breve{C}^{3}_{1111}=-\frac{(\rho_c)^2}{1-p_0}\{1-\tfrac{1}{2}(1-f)(1+p_0)m_2[2(1-p_0)+(1+p_0)m_1]\}$$
$$\breve{C}^{1}_{2222}=p_1+2(1-f)m_1[2(1-p_0)+(1+p_0)m_2]p^{+}_{12}p^{-}_{12}$$
$$\breve{C}^{2}_{2222}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_1-1-(1-f)m_1[2(1-p_0)+(1+p_0)m_2](p_1-p_0p_2)\}$$
$$\breve{C}^{3}_{2222}=-\frac{(\rho_c)^2}{1-p_0}\{1-\tfrac{1}{2}(1-f)(1+p_0)m_1[2(1-p_0)+(1+p_0)m_2]\}$$
$$\breve{C}^{1}_{3333}=p_3[1+2(1-f)(3-p_0)m_1m_2p^{-}_{12}]$$
$$\breve{C}^{2}_{3333}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_3-p_0-(1-f)(3-p_0)m_1m_2[(1-p_0)p_3+2p_0p^{-}_{12}]\}$$
$$\breve{C}^{3}_{3333}=-\frac{p_0(\rho_c)^2}{1-p_0}[1-(1-f)(3-p_0)m_1m_2]$$
$$\breve{C}^{1}_{1122}=p_2+2(1-f)(1+p_0)m_1m_2p^{+}_{12}p^{-}_{12}$$
$$\breve{C}^{2}_{1122}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_2-p_0-(1-f)m_1m_2(p_1-p_0p_2)$$
$$\breve{C}^{3}_{1122}=-\frac{(\rho_c)^2}{1-p_0}\{p_0-\tfrac{1}{2}(1-f)(1+p_0)^2m_1m_2\}$$
$$\breve{C}^{1}_{1133}=p_3\{1+2(1-f)m_2[1-p_0+(1+p_0)m_1]p^{-}_{12}\}$$
$$\breve{C}^{2}_{1133}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_3-p_0-(1-f)m_2[1-p_0+(1+p_0)m_1][(1-p_0)p_3-2p_0p^{-}_{12}]\}$$
$$\breve{C}^{3}_{1133}=-\frac{p_0(\rho_c)^2}{1-p_0}\{1-(1-f)m_2[1-p_0+(1+p_0)m_1]\}$$
$$\breve{C}^{1}_{2233}=p_3\{1+2(1-f)m_1[1-p_0+(1+p_0)m_2]p^{-}_{12}\}$$
$$\breve{C}^{2}_{2233}=\frac{\rho_c}{1-p_0}\{(1-p_0)p_3-p_0-(1-f)m_1[1-p_0+(1+p_0)m_2][(1-p_0)p_3-2p_0p^{-}_{12}]\}$$

$$\check{C}^3_{2233} = -\frac{p_0(\rho_c)^2}{1-p_0}\{1-(1-f)m_1[1-p_0+(1+p_0)m_2]\}$$
$$\check{C}^1_{1313} = p_4 + (1-f)m_1(r_1 + p_4 q_1),$$
$$\check{C}^2_{1313} = p_4\rho_\kappa(m_2 + fm_1)$$
$$\check{C}^3_{1313} = -\tfrac{1}{2}\rho_c[1+(1-f)m_1 q_1],$$
$$\check{C}^4_{1313} = -\tfrac{1}{2}\rho_c\rho_\kappa(m_2 + fm_1)$$
$$\check{C}^1_{2323} = p_4 + (1-f)m_2(r_1 + p_4 q_1),$$
$$\check{C}^2_{2323} = p_4\rho_\kappa(m_1 + fm_2)$$
$$\check{C}^3_{2323} = -\tfrac{1}{2}\rho_c[1+(1-f)m_2 q_1],$$
$$\check{C}^4_{2323} = -\tfrac{1}{2}\rho_c\rho_\kappa(m_1 + fm_2) \quad \text{(A21)}$$

$$\check{e}^1_{311} = 1 + 2(1-f)m_2[1-p_0+(1+p_0)m_1]p_{12}^-$$
$$\check{e}^2_{311} = \rho_c\{2-(1-f)m_2[1-p_0+(1+p_0)m_1](1-2p_{12}^-)\}$$
$$\check{e}^3_{311} = (\rho_c)^2\{1-(1-f)m_2[1-p_0+(1+p_0)m_1]\}$$
$$\check{e}^1_{322} = 1 + 2(1-f)m_1[1-p_0+(1+p_0)m_2]p_{12}^-$$
$$\check{e}^2_{322} = \rho_c\{2-(1-f)m_1[1-p_0+(1+p_0)m_2](1-2p_{12}^-)\}$$
$$\check{e}^3_{322} = (\rho_c)^2\{1-(1-f)m_1[1-p_0+(1+p_0)m_2]\}$$
$$\check{e}^1_{333} = p_3\{1+2(1-f)(3-p_0)m_1 m_2 p_{12}^-\}$$
$$\check{e}^2_{333} = \frac{\rho_c}{1-p_0}\{(1-p_0)p_3 - p_0 - (1-f)(3-p_0)m_1 m_2[(1-p_0)p_3 - 2p_0 p_{12}^-]\}$$
$$\check{e}^3_{333} = -\frac{p_0(\rho_c)^2}{1-p_0}[1-(1-f)(3-p_0)m_1 m_2] \quad \text{(A22)}$$

$$\check{k}^1_{11} = q_1 + 2(1-f)m_1(r_1 + p_4 q_1),$$
$$\check{k}^2_{11} = -\rho_\kappa[1+2(1-f)m_1 p_4]$$
$$\check{k}^3_{11} = \rho_c q_1(m_2 + fm_1),$$
$$\check{k}^4_{11} = -\rho_\kappa\rho_c(m_2 + fm_1)$$
$$\check{k}^1_{22} = q_1 + 2(1-f)m_2(r_1 + p_4 q_1),$$
$$\check{k}^2_{22} = -\rho_\kappa[1+2(1-f)m_2 p_4]$$
$$\check{k}^3_{22} = \rho_c q_1(m_1 + fm_2), \quad \check{k}^4_{22} = -\rho_\kappa\rho_c(m_1 + fm_2)$$
$$\check{k}^1_{33} = 1 + 2(1-f)(3-p_0)m_1 m_2 p_{12}^-,$$
$$\check{k}^2_{33} = \rho_c[1-(1-f)(3-p_0)m_1 m_2] \quad \text{(A23)}$$

REFERENCES

Aboudi, J. 2000. "Micromechanical Modeling of Finite Viscoelastic Multiphase Composites," *Zeitschrift Für Angewandte Mathematik und Physik*, 51(1):114–134.

Agbossou, A., Viet, H.N. and Pastor, J. 1999. "Homogenization Techniques and Application to Piezoelectric Composite Materials," *Int. J. Appl. Electromagn. Mech.*, 10(5):391–403.

Alberola, N.D. and Benzarti, K. 1998. "Modeling of the Relationship Between Morphology and Viscoelastic Behavior of Unidirectional Fiber-Reinforced Polymers," *Polymer Engg. Sci.*, 38(3): 429–439.

Avellaneda, M. and Swart, P.J. 1998. "Calculating the Performance of 1-3 Piezoelectric Composite for Hydrophone Application: A Effective Approach," *J. Acoustical Society America*, 103(3): 1449–1467.

Chan, H.L.W. and Unsworth, J. 1989. Simple Model for Piezoelectric Ceramic/Polymer 1-3 Composites Used in Ultrasonic Transducer Applications," *IEEE Trans. On Ultrasonic Ferroelectric and Frequency Control*, 36:434–441.

Christensen, R.M. 1982. *Theory of Viscoelasticity, An Introduction*, Academic Press, New York.

Dias, C.J. and Das-Gupta, D.K. 1996. "Inorganic Ceramic/Polymer Ferroelectric Composite Electrets," *IEEE Trans. on Dielectrics and Electrical Inclusions*, 3(5):706–733.

Dunn, M.L. and Taya, M. 1993. "Micromechanics Predictions of the Effective Electroelastic Moduli of Piezoelectric Composites," *Int. J. Solids Structures*, 30:161–175.

Ferry, J.D. 1970. *Viscoelastic Properties of Polymers*, JohnWiley and Sons, Inc., New York.

Hashin, Z. 1965. "Viscoelastic Behavior of Heterogeneous Media," *Journal of Applied Mechanics*, 28(11):1111–1120.

Hashin, Z. 1966. "Viscoelastic Fiber Reinforced Materials," *AIAA Journal*, 4:1411–1417.

Hedvig, P. 1977. *Dielectric Spectroscopy of Polymer*, Akademiai, Budapest.

Hornsby, J.S. and Das-Gupta, D.K. 2000. "Finite-Difference Modeling of Piezoelectric Composite Transducers," *J. Appl. Phys.*, 87(1):467–473.

Jiang, B., Fang, D.N. and Hwang, K.C. 1999a. "A Unified Model for the Multiphase Piezocomposites with Ellipsoidal Inclusions," *Int. J. Solids and Structures*, 36(18):2707–2733.

Jiang, B., Fang, D.N. and Hwang, K.C. 1999b. "Constitutive Models of Ferroelectric Composites with a Viscoelastic and Dielectric Relaxation Matrix I: Theory," *Sci. in. China Series A*, 42(11):1193–1200.

Jiang, B., Fang, D.N. and Hwang, K.C. 2000. "Constitutive Models of Ferroelectric Composites with a Viscoelastic and Dielectric Relaxation Matrix II: Experimenting and Calculating," *Sci. in China Series A*, 43(6):647–654.

Jiang, B. and Batra, R.C. 2001. "Effective Electroelastic Moduli of Piezocomposite with Elliptic Cylinder Piezoelectric Inclusion," submitted for publication.

Kaliske, M. 2000. "A Formulation of Elasticity and Viscoelasticity For Fiber-Reinforced Material at Small and Finite Strains," *Computer Methods in Applied Mechanics and Engineering*, 185(2–4): 225–243.

Li, J. and Weng, G.J. 1994. "Effective Creep Behavior and Complex Moduli of Fiber- and Ribbon-Reinforced Polymer-Matrix Composites," *Composites Science and Technology*, 52:615–629.

Lovinger, A. 1983. "Ferroelectric Polymers," *Science*, 220:1115–1121.

Mori, T. and Tanaka, K. 1973. "Average Stress in Matrix and Average Elastic Energy of Materials with Misfitting Inclusion," *Acta Metallurgica*, 21:571–574.

Papanicolaou, G.C., Zaoutsos, S.P. and Cardon, A.H. 1999. "Prediction of the Non-linear Viscoelastic Response of Unidirection Fiber Composite," *Composites Science and Technology*, 59(9):1311–1319.

Shapery, R.A. 1967. "Stress Analysis of Composite Materials," *Journal of Composite Materials*, 1:228–267.

Shapery, R.A. 1969. "On the Characterization of Non-linear Viscoelastic Materials," *Polymer Engineering and Science*, 9(4): 295–310.

Skurda, A.M. and Auzukalns, YaV. 1970. "Creep and Long-Term Strength of Unidirectional Reinforced Plastics in Compression," *Polymer Mech.*, 6:718–722.

Tressler, J.F., Alkoy, S. and Newnham, E.E. 1998. "Piezoelectric Sensors and Sensor Materials," *J. Electroceramics*, 2(2):257–272.