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COLD ROLLING OF A LAMINATED COMPOSITE SHEET—A NUMERICAL SOLUTION*

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SUMMARY

The problem of cold rolling of a composite sheet which consists of alternate layers of viscoelastic and elastic materials is studied by the finite element method. Results computed and presented graphically include the pressure distribution at the contact surface, the normal stress distribution at the middle surface and the asymmetric deformation of the surface of the sheet in contact with the roller.

INTRODUCTION

Lynch's[1] study of cold rolling of a viscoelastic sheet by the finite element method was recently extended by Batra[2] to the case when heat generated due to viscous dissipation and the dependence of material moduli on the temperature are accounted for. Here we study the problem of cold rolling of a laminated composite sheet which consists of alternate laminae of viscoelastic and elastic materials. The elastic laminae are placed in positions symmetrical with respect to the central plane of the sheet. That the problem is of interest should be clear from the fact that one way to manufacture laminated composites (e.g. plywood) is to bond sheets of laminae and subsequently apply pressure by rolling. Experimental work on cold rolling of metallic composites has recently been reported by NASA.[3]

Viscoelastic rolling contact problems have been studied both analytically[4,5] and experimentally.[6] For other references on the related work, the reader is referred to the papers cited above and to the book by Oden.[7] As also noted in References 1, 2, the complexity of viscoelastic boundary value problems, especially of those in which the boundary surface where surface tractions are prescribed is not a material surface, forces one to use numerical methods to solve the problem. Here we solve the title problem by the finite element method.

Since the finite element formulations of viscoelastic boundary value problems are given in References 1, 2, we do not delve into the details of working out the problem. Rather we formulate the problem in the next section and, present and discuss results in section 'Results for a sample problem'.

FORMULATION OF THE PROBLEM

A schematic diagram of the system studied herein is shown in Figure 1. We assume that the rollers are made of a material considerably harder than the material of the constituents of the

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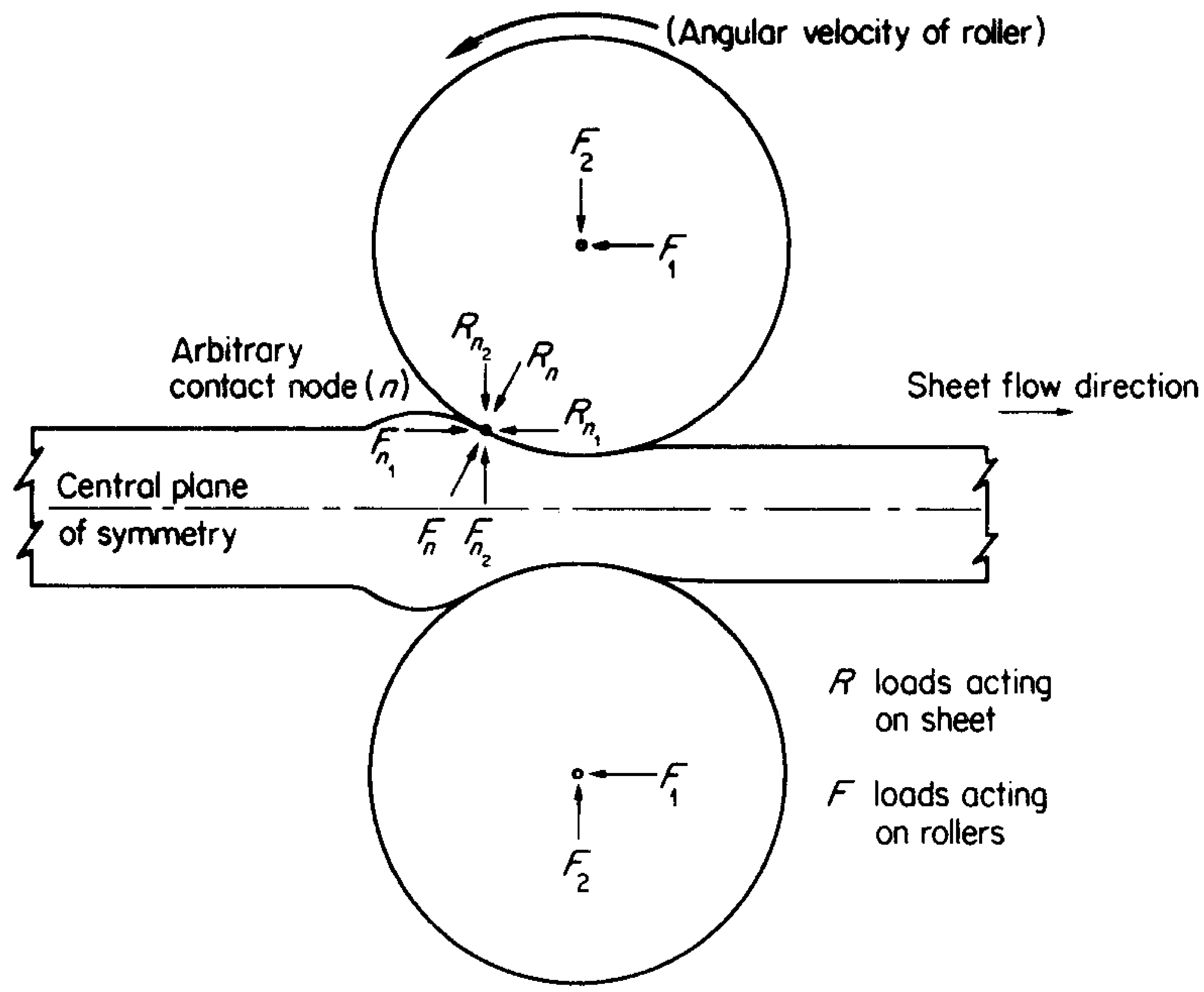

Figure 1. System to be studied

sheet and, therefore, regard the rollers as being rigid. Furthermore, we do not consider the effect of gravity. The equations governing the deformations of the sheet are

$$\dot{\rho} + \rho \dot{x}_{i,i} = 0,$$
$$\sigma_{ij,j} = \rho \ddot{x}_i, \tag{1}$$

in which σ_{ij} is the Cauchy stress tensor, ρ is the present mass density, $\mathbf{x}$ is the present position in rectangular Cartesian co-ordinates of a material particle $\mathbf{X}$ at time t, a superposed dot indicates material time derivative, a comma followed by an index j indicates partial differentiation with respect to x_j, and the usual summation convention is used. Equation (1) is supplemented by the constitutive equation for σ_{ij} and by side conditions such as the boundary conditions and the conditions at the interface between two dissimilar materials. Before outlining these, we state the assumptions made to simplify the problem.

We assume that the material of each laminate is homogeneous and isotropic, the sheet moves at a constant velocity v, steady state has been reached, contact between rigid rollers and the sheet is frictionless, the deformations are small so that a constitutive law linear in displacement gradients applies, and that the bulk behaviour of the viscoelastic material is elastic. We note that the last assumption is often made for viscoelastic materials.[1,2,8,9] Furthermore, we assume that a plane strain state of deformation prevails and that the inertia effects are negligible. Thus the indices i and j in (1) range over 1,2 and the problem becomes 2-dimensional quasi-static.

The Cauchy stress σ_{ij} for the viscoelastic and the elastic constituents of the sheet is assumed to be given by

$$\sigma_{ij}(\mathbf{X}, t) = \int_{-\infty}^{t} G_1(t-\tau)\frac{\partial \varepsilon_{ij}(\mathbf{X}, \tau)}{\partial \tau}\mathrm{d}\tau + \frac{\delta_{ij}}{3}\int_{-\infty}^{t} [G_2 - G_1(t-\tau)]\frac{\partial \varepsilon_{kk}(\mathbf{X}, \tau)}{\partial \tau}\mathrm{d}\tau,$$

$$\sigma_{ij}(\mathbf{X}, t) = g_1\varepsilon_{ij}(\mathbf{X}, t) + \frac{\delta_{ij}}{3}(g_2 - g_1)\varepsilon_{kk}(\mathbf{X}, t), \tag{2}$$

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2,$$

$$u_i = x_i - X_i.$$

Here G_1 and G_2 are, respectively, the shear and the bulk moduli of the viscoelastic material, g_1 and g_2 are the shear and the bulk moduli of the elastic material, δ_{ij} is the Kronecker delta, and $\mathbf{u}$ stands for the displacement of a material particle. Substitution of (2) into (1) gives field equations for $\mathbf{x}$ and $\mathbf{u}$ which are to be solved under the following boundary conditions.

$$\sigma_{ij}(x_1, 0)n_j(x_1, 0) = 0, \qquad |x_1 + cl| \geq l,$$

$$\left.\begin{array}{l} u_2(x_1, 0) = d - \dfrac{x_1^2}{2R}, \\ e_i(x_1, 0)\sigma_{ij}(x_1, 0)n_j(x_1, 0) = 0, \end{array}\right\} \qquad |x_1 + cl| \leq l, \tag{3}_{1,2,3}$$

$$u_2(x_1, D/2) = 0, \qquad \sigma_{12}\left(x_1, \frac{D}{2}\right) = 0, \tag{3}_{4,5,6}$$

$$|\sigma_{ij}n_j| \to 0 \quad \text{as} \quad |\mathbf{x}| \to \infty,$$

and across the interface between layers of different materials

$$[\![\sigma_{ij}n_j]\!] = 0, \qquad [\![u_i]\!] = 0. \tag{4}$$

In writing (3), we have used the fact that the problem is symmetrical about the middle surface of the sheet and, consequently, we study the deformation of the lower half of the sheet. In (3), $\mathbf{n}$ is an outward directed unit normal to a bounding surface, $\mathbf{e}$ is a unit vector trangent to a bounding surface, D is the thickness of the sheet, d is the depth of indentation at $x_1 = 0$, R is the radius of the roller, cl is the distance between the contact centre and the centre line of the rollers as shown in Figure 2, $2l$ is the width of the contact surface, and $[\![u_i]\!]$ denotes the jump in u_i across a surface. Thus (4) states that surface tractions and displacements are continuous across the interface between two layers of different materials. $(3)_1$ implies that $n_i(x_1, 0)\ \sigma_{ij}(x_1, 0)\ n_j(x_1, 0) = 0$ at $|x_1 + cl| = l$ and this ensures that the normal stress is continuous across the arc of contact and that a contact problem rather than a punch problem is being solved. Of the three constants c, l and d appearing in (3), only one can be taken to be known and the other two are to be determined as a part of the solution.

We note that for the purpose of solving the problem by the finite element method, one may replace $(3)_2$ by

$$n_i(x_1, 0)\sigma_{ij}(x_1, 0)n_j(x_1, 0) = p(x_1), \qquad |x_1 + cl| \leq l,$$
$$p(x_1) \to 0 \quad \text{as} \quad |x_1| \to |x_1 + cl|. \tag{5}$$

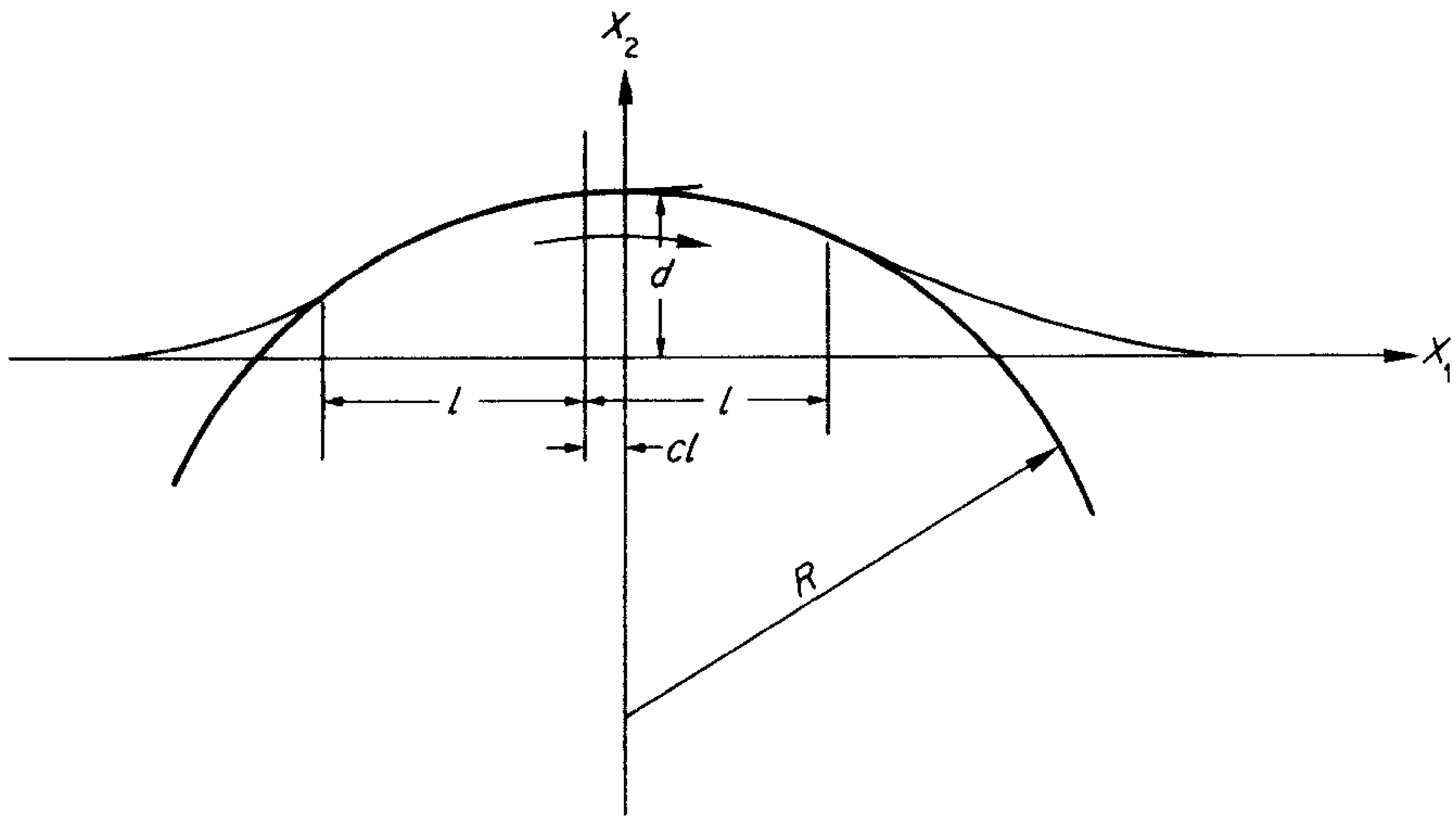

Figure 2. A configuration of the indented surface

Here p stands for the normal pressure between the roller and the sheet. A boundary condition such as (5) was used in References 2 and 8 wherein the distributed load p was replaced by equivalent normal loads at the nodal points on the contact surface.

RESULTS FOR A SAMPLE PROBLEM

We study the deformation caused by cold rolling of a laminated composite sheet consisting of five laminates which when considered from the top are made alternatively of a viscoelastic and an elastic material and have thicknesses equal to $0{\cdot}2D$, $0{\cdot}1D$, $0{\cdot}4D$, $0{\cdot}1D$ and $0{\cdot}2D$ respectively. In order to solve the problem by the finite element method, the lower half of the sheet is divided into uniform rectangular elements as shown in Figure 3. In this figure, $D = 0{\cdot}25$ in. and the middle strip is elastic whereas the strips surrounding it are viscoelastic. This grid is the same as that used in Reference 2.

We took the following values of various geometric and material parameters.

$$\begin{aligned} &G_1 = 12(0{\cdot}43 + 0{\cdot}57\,e^{-t/0{\cdot}46})\ \text{ksi}, \\ &G_2 = 600\ \text{ksi}, \\ &D = 0{\cdot}25\ \text{in.}, \qquad d = 0{\cdot}00575\ \text{in.}, \\ &R = 1{\cdot}25\ \text{in.}, \qquad v = 0{\cdot}4\ \text{in./sec.} \end{aligned} \tag{6}$$

To get an idea of the effect of material properties of the elastic layer upon the deformation of the composite sheet, we solved the problem for the cases when $g_1/G_1(0) = g_2/G_2 = 5$, 20 and 100. We identify these as problems 2, 3 and 4, and call the case when the entire sheet is made of a homogeneous viscoelastic material having moduli given by $(6)_{1,2}$ as problem number 1. Results for the last problem are taken from Reference 2 and for each of the other three problems are obtained by using the finite element program developed therein. These are presented in Figures 4, 5 and 6. We add that these problems differ only in the values of g_1 and g_2. We recall that the boundary condition (5) is used to solve the problems and that the indented surface is assumed to

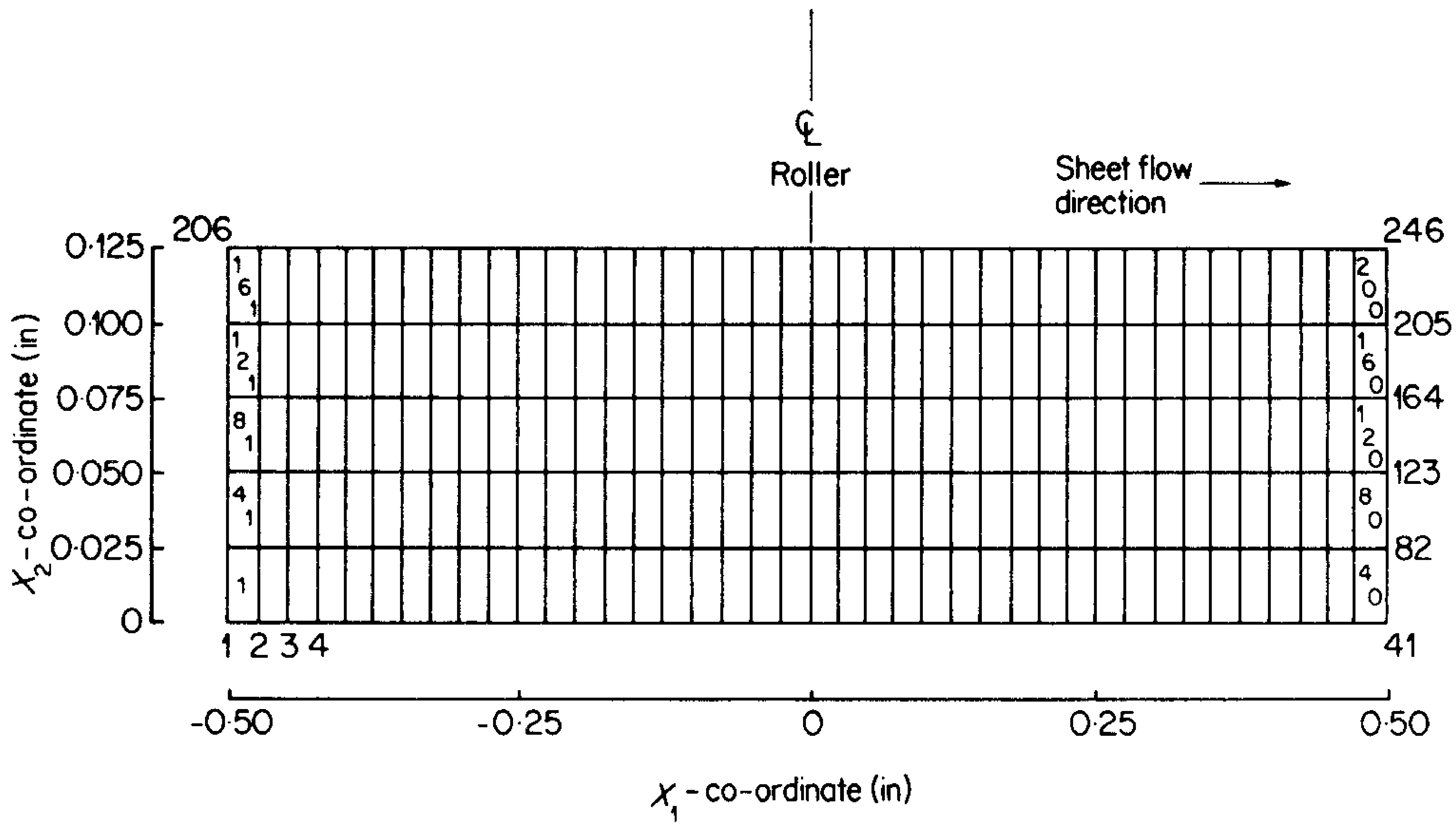

Figure 3. The grid used in the solution of the problem

conform to the circular profile of the indentor if, in the deformed position, each nodal point lies within $0{\cdot}01d$ of the circular profile of the roller. The normal loads on the assumed contact surface are iterated until such is the case.

One can read off from Figure 4 that as the moduli of the elastic laminate increase, as expected, we need larger and larger pressure to cause the same indentation. Asymmetry in the pressure

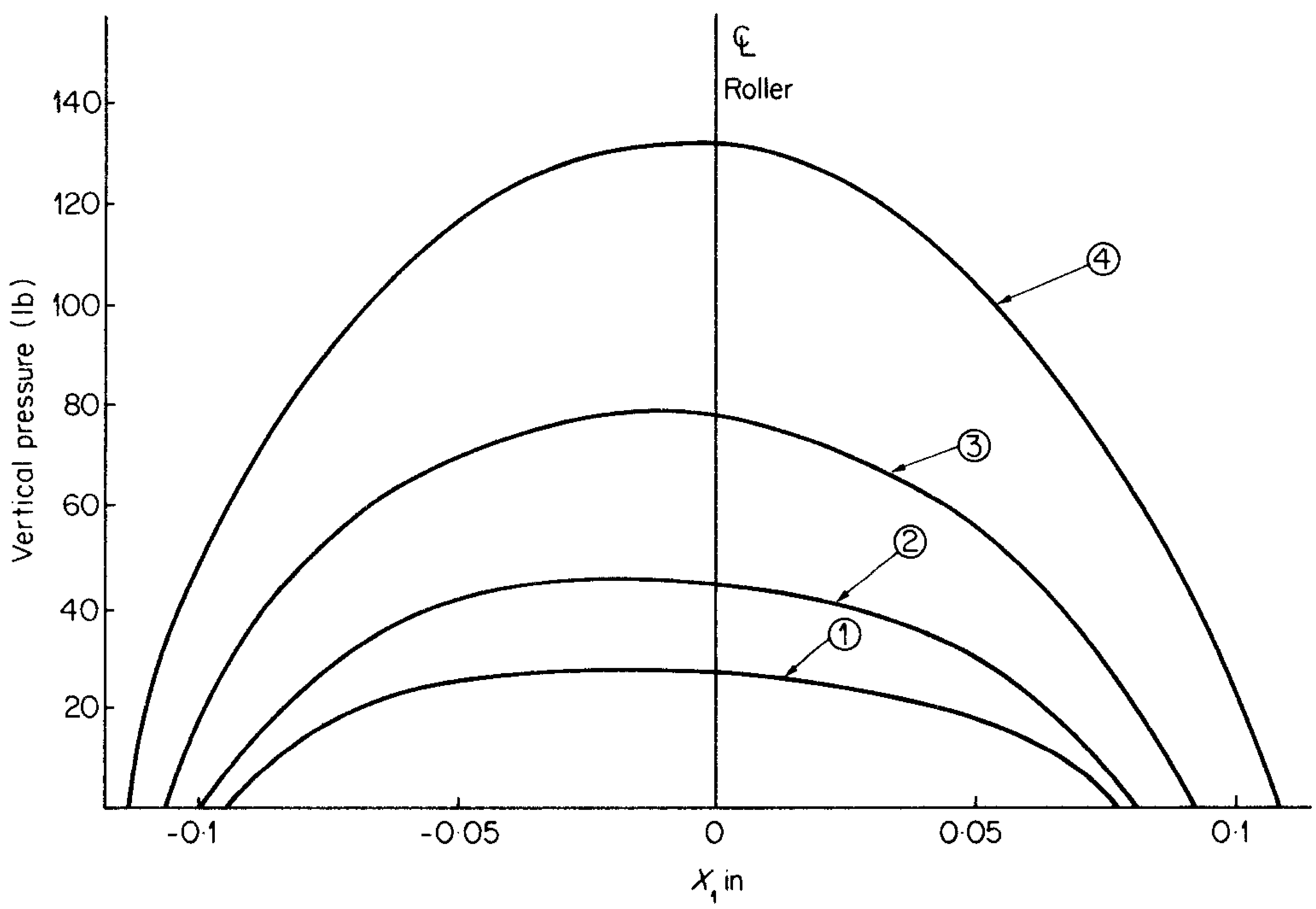

Figure 4. Pressure distribution over the contact surface

distribution causes a moment to act on the roller which opposes its motion. To overcome this, power has to be supplied. The ratio of power required for problems 2, 3 and 4 to that for problem 1 is found to be 1·54, 2·97 and 3·15 respectively. We note that increasing the value of $g_1/G_1(0)$ to 100 from 20 does not increase the power requirement that much as does the increase from 5 to 20. The semicontact width l for problems 1–4 as found from this figure is 0·17 in., 0·18 in., 0·2 in., and 0·22 in. respectively. The value of the parameter c appearing in (3), also determined from this figure, is 0·115, 0·107, 0·069 and 0·025. Note that c is zero for an elastic sheet. Thus the ratio of difference in the contact width on either side of the centre line of the rollers to the semi contact width decreases with the increase in the value of the moduli of the elastic layer. The same result should be apparent from Figure 5 in which is presented the deformed shape of the indented surface in various cases. The surface of the sheet near the ends of the arc of contact bulges out more with the rise in the values of g_1 and g_2. In Figure 6 is

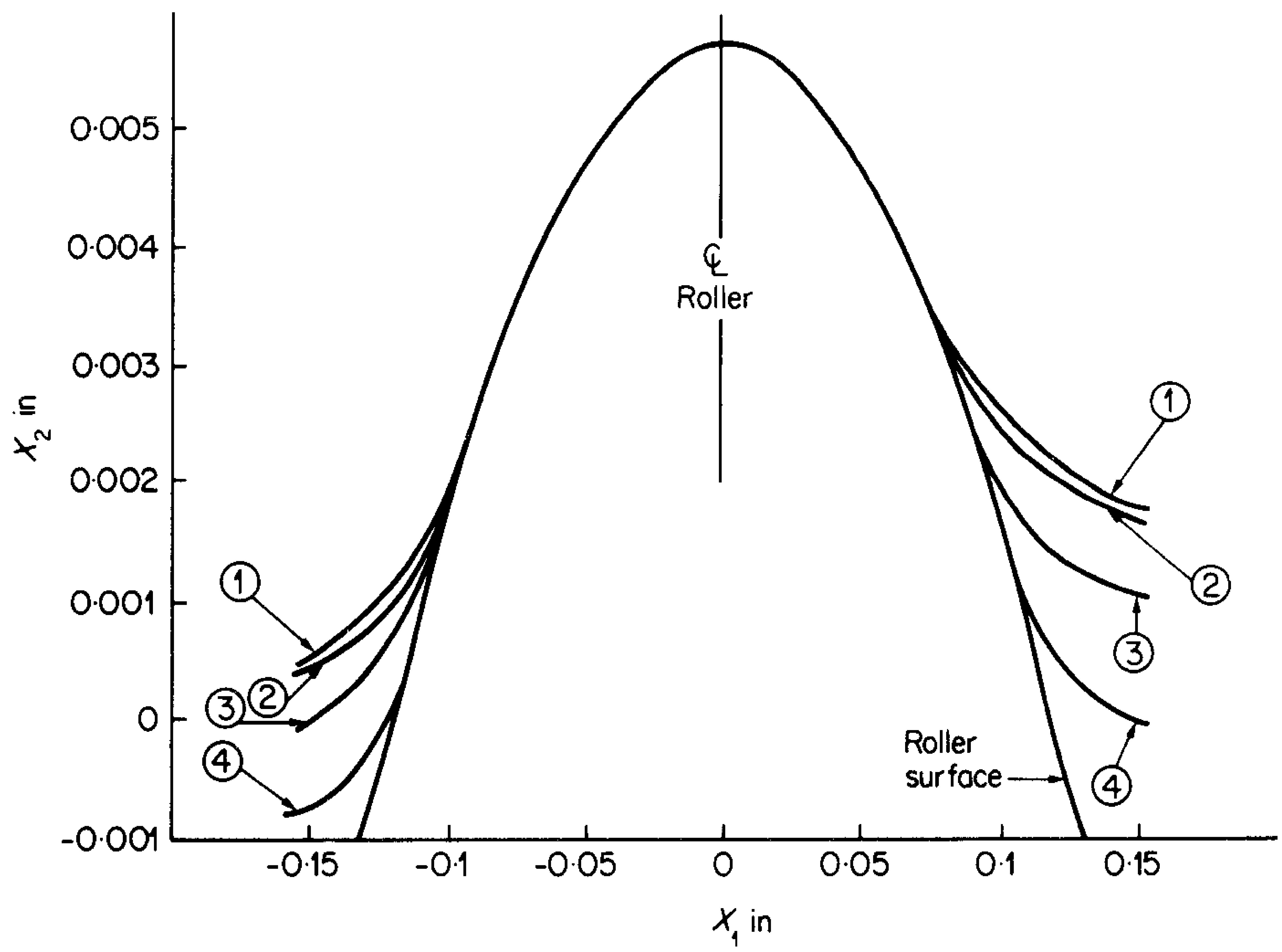

Figure 5. Asymmetric deformation of the indented surface

plotted the normal stress at the middle surface. With the increase in the values of g_1 and g_2, the maximum normal stress induced at the middle surface increases and the point where maximum normal stress occurs shifts closer to the centre line of the rollers.

We remark that as in References 2, 8, here also, we do not account for the change in the geometry of the deformed surface of the sheet. This is consistent with the use of the linear theory. Also, remarks made in Reference 2 concerning the possible extension of the work so as to include frictional force at the contact surface etc. apply here.

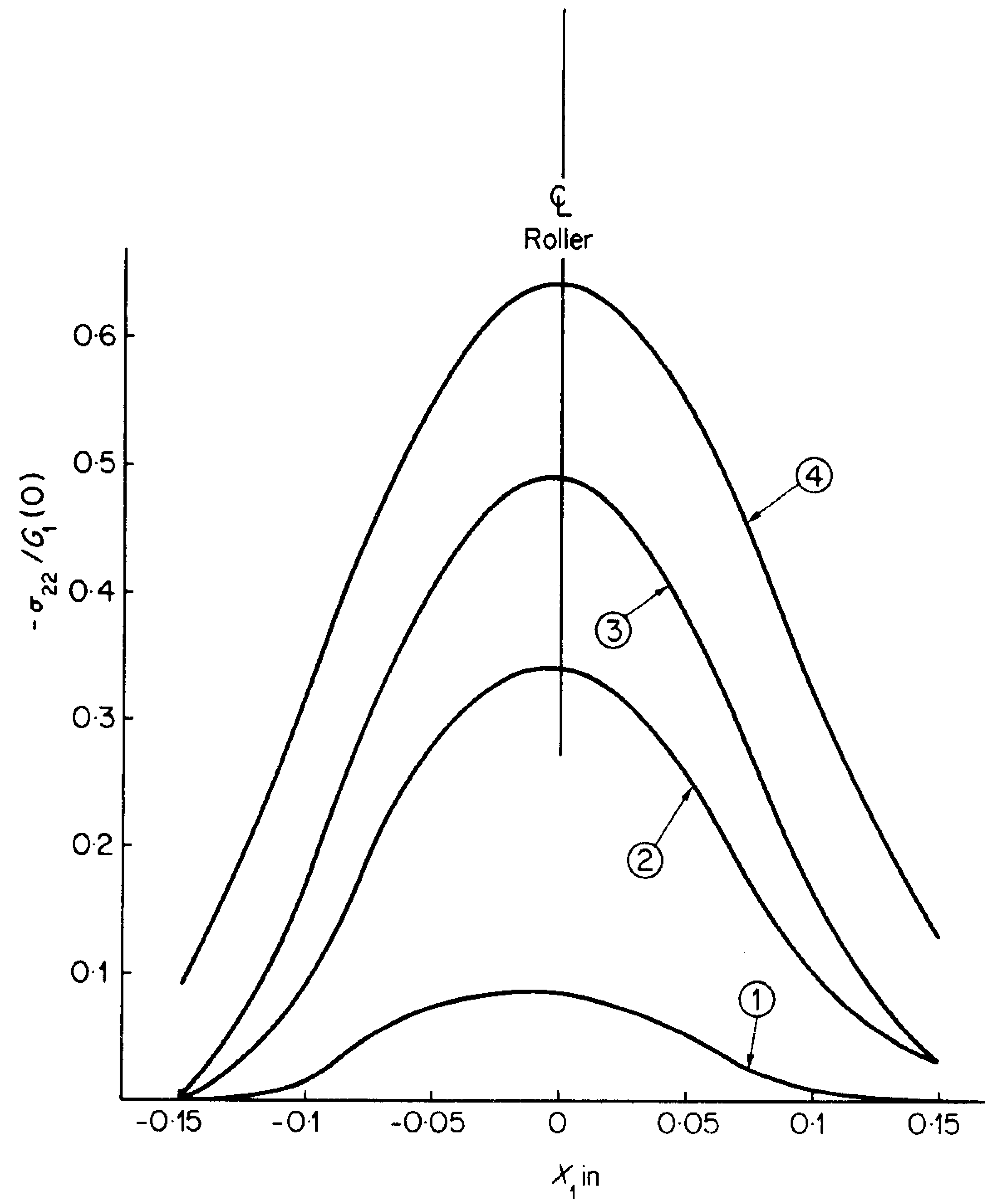

Figure 6. Normal stress distribution at the middle surface

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