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Instability analysis and shear band spacing in gradient-dependent thermoviscoplastic materials with finite speeds of thermal waves

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WE ANALYZE THE STABILITY of a homogeneous solution of coupled nonlinear equations governing simple shearing deformations of a strain-rate gradient-dependent thermoviscoplastic body in which thermal disturbances propagate at a finite speed. The homogeneous solution is perturbed by an infinitesimal amount and equations linear in the perturbation variables are derived. Conditions for these perturbations to grow are deduced. The shear band spacing, L_s, is defined as $L_s = \inf_{t_0 \geq 0} (2\pi/\xi_m(t_0))$ where ξ_m is the wave number of the perturbation introduced at time t_0 that has the maximum growth rate at time t_0. It is found that the thermal relaxation time (i.e. the ratio of the coefficient of the second time-derivative of the temperature in the heat equation to that of the first time-derivative) significantly affects the shear band spacing and the value of t_0 for which $\xi_m(t_0)$ is maximum.

Key words: Material characteristic length, strain-rate gradient, thermal relaxation time, dominant growth rate.

1. Introduction

ADIABATIC SHEAR BANDS are narrow regions, usually a few microns wide, of intense plastic deformation that form during high strain-rate plastic deformation of most metals and some polymers. Their study is important since they precede ductile fractures. Even though TRESCA [25] observed these a long time ago, their study has attracted considerable attention since 1944 when ZENER and HOLLOMON [30] observed 32 μm wide shear bands during the punching of a hole in a low-carbon steel plate. Subsequent experimental studies (e.g. see MARCHAND and DUFFY [19]) have focussed on delineating conditions for the initiation of a shear band and the evolution of the temperature and plastic strain within a band. Recently, NESTERENKO *et al.* [21] observed a series of parallel, nearly 1 mm apart, shear bands during the radial collapse of titanium and stainless steel hollow cylinders deformed due to explosive loads applied on their outer surfaces. The average strain-rate within the shear-banded region was estimated to be 10^4/s.

CLIFTON [15] used the perturbation method to study the stability of quasistatic simple shearing deformations of a thermoviscoplastic body. BAI [2] employed the perturbation method to analyze the stability of the time-dependent homogeneous solution of equations governing the dynamic deformations of a strain-hardening thermoviscoplastic body. He derived the conditions necessary for the homogeneous solution to become unstable, and also computed the characteristic length and the characteristic time of the deformation mode with the dominant growth rate at time t_0 when the homogeneous solution is perturbed. Linear perturbation analysis has also been employed to study the initiation of the instability by BURNS [10] among others; some of these works are summarized in the book by BAI and DODD [3]. The reader is referred to TOMITA [24], ZBIB *et al.* [29], ARMSTRONG *et al.* [1], PERZYNA [22], BATRA [7] and BATRA *et al.* [9] for the pertinent literature on adiabatic shear bands.

WRIGHT and OCKENDON [28] also studied the growth of infinitesimal perturbations superimposed on a homogeneous solution of the equations governing simple shearing deformations of a thermoviscoplastic body and postulated that, in an infinite body, perturbations growing at different sites will not merge and result in multiple shear bands. Thus the wavelength of the dominant instability mode with the maximum initial growth rate will determine the shear band spacing. Wright and Ockendon's definition of the shear band spacing seems to differ by 2π from Bai's definition of the characteristic length. MOLINARI [20] has extended Wright and Ockendon's work to strain-hardening materials, and has estimated the effect of the finite thickness of the plate upon the shear band spacing, L_s, defined as $L_s = \inf\limits_{t_0 \geq 0} 2\pi/\xi_m(t_0)$. Here ξ_m is the wavelength of the perturbation introduced at time t_0 that has the maximum growth rate at t_0. Note that Wright and Ockendon and Bai did not find the infimum of $2\pi/\xi_m(t_0)$. Molinari presumed that $\inf\limits_{t_0 \geq 0} 2\pi/\xi_m(t_0) \simeq 2\pi/\xi_m(t_0^s)$, where t_0^s corresponds to the time when the superimposed infinitesimal perturbation has the maximum growth rate. If $\bar{t}_0^s$ denotes the time when $2\pi/\xi_m(t_0)$ is infimum, BATRA and CHEN [8] found that for the titanium modeled by a power-law thermal softening, $\bar{t}_0^s$ corresponds to the time when the shear stress has dropped a little below its maximum value, but for the SAE 4340 and the S-7 tool steels modeled by an affine thermal softening, $\bar{t}_0^s$ equals the time when the shear stress has significantly dropped from the peak value. Batra and Chen did not consider work-hardening of the materials. CHEN and BATRA'S [13] work for work-hardening strain-rate gradient-dependent materials indicates that the shear band spacing rapidly increases with an increase in the work-hardening exponent, the material characteristic length, the thermal conductivity and the strain-rate hardening exponent.

In contrast to the perturbation method used in the above-mentioned studies, GRADY and KIPP [17] determined the shear band spacing by accounting for the momentum diffusion due to unloading within a shear band. They analyzed simple shearing deformations of a thermally softening rigid plastic material.

In all of the aforestated numerical and analytical studies, thermal waves were assumed to propagate at an infinite speed. However, *in real materials, thermal disturbances like mechanical discontinuities are expected to propagate at a finite speed.* Here we analyze the effect of the finite speed of thermal waves on the shear band spacing in strain-rate gradient-dependent thermoviscoplastic materials deformed in simple shear. SAAD and CHA [23] found that in heat transfer problems involving very short time intervals and/or very high heat fluxes, the hyperbolic heat equation gives significantly different results than the parabolic heat equation. It has been suggested (e.g. see the review paper by CHANDRASEKHARIAH [12]) that the hyperbolic heat equation should be considered when the duration of the loading pulse is less than 10 µs or when the heat flux equals about 10^5 W/cm^2. We note that temperature gradients across a shear band are extremely large resulting in high values of the heat flux, and times involved are of the order of a few microseconds. BATRA [4] considered higher-order spatial and temporal gradients of temperature, and for rigid heat conductors he found constitutive relations compatible with the Clausius-Duhem inequality. He showed that thermal disturbances can propagate with finite speed in such materials.

The present work shows that the shear band spacing in titanium deformed at a nominal strain-rate of 10^6/s decreases rapidly from 48 µm to 22 µm as the thermal relaxation time is reduced from 10^{-6} s to 10^{-8} s.

2. Formulation of the problem

We study overall adiabatic simple shearing deformations of a work-hardening, strain-rate hardening, strain-rate gradient hardening, thermally softening, isotropic and homogeneous body in which thermal disturbances propagate at a finite speed. In terms of non-dimensional variables, equations governing the thermomechanical deformations of the body are

$$\rho\dot{v} = (s - \ell\sigma_{,y})_{,y}, \tag{2.1}$$

$$\dot{\theta} + \tau\ddot{\theta} = k\theta_{,yy} + sv_{,y} + \ell\sigma v_{,yy}, \tag{2.2}$$

$$v_{,y} = \Lambda s, \qquad v_{,yy} = \frac{\Lambda\sigma}{\ell},$$

$$\dot{\psi}\left(1 + \frac{\psi}{\psi_0}\right)^n = sv_{,y} + \ell\sigma v_{,yy},$$

$$I \equiv \left[(v_{,y})^2 + (\ell v_{,yy})^2\right]^{1/2} = f(s, \sigma, \theta, \psi)$$

Here the effect of material elasticity has been neglected, and all of the plastic working is assumed to be converted into heating. This is justified since our interest is in studying large plastic deformations of the body bounded by the planes $y = \pm 1$ and sheared in the x-direction. Other investigators (e.g. BAI [2], WRIGHT and OCKENDON [28]) have also ignored the effects of material elasticity. In Eqs. (2.1) – (2.5), ρ is the mass density, v the velocity of a material particle in the x-direction, s the shear stress, σ the dipolar stress, θ the present temperature, τ the thermal relaxation time, k the thermal conductivity, ℓ the material characteristic length, Λ a plastic multiplier, and ψ the work-hardening parameter. Furthermore, a superimposed dot indicates the material time-derivative, and a comma followed by y signifies partial differentiation with respect to y. Equations (2.1) and (2.2) express, respectively, the balance of linear momentum and the balance of internal energy, Eq. (2.3) is the flow rule, (2.4) an evolution equation for the work-hardening parameter, and (2.5) characterizes the thermoviscoplastic material of the body. WRIGHT and BATRA [27] generalized GREEN *et al.* [18] theory of elastic-plastic materials to elastic-viscoplastic materials and proposed the afore-stated equations except that here Eq. (2.2) has been modified to account for the finite speed of thermal disturbances. For quasistatic deformations, a continuity argument for neutral loading discussed by GREEN *et al.* [18] requires that the plastic multiplier, Λ, in Eqs.$(2.3)_1$ and $(2.3)_2$ be the same. Here we have assumed that the plastic multiplier in these equations is the same even for transient deformations. As pointed out by WRIGHT and BATRA [27], the material characteristic length in each of the four Eqs. (2.1) – (2.4) could be different. However, not knowing how to estimate these lengths from microscopic considerations, and to simplify the work, we have set them equal to each other. CHANDRASHEKHARAIH [12] has discussed different forms of the balance of internal energy that give finite speeds of thermal waves; the form adopted here is due to CATTANEO [11] and VERNOTTE [26]. The thermal relaxation time τ in Eq. (2.2) represents the time required to establish a steady state of heat conduction in an element suddenly exposed to a temperature gradient. CHESTER [14] has estimated that

$$\hat{\tau} = 3\hat{k}/\left(\hat{\rho}\hat{c}\hat{V}_p^2\right)$$

where $\hat{V}_p$ is the speed of an elastic wave, and a superimposed hat over a quantity denotes the dimensional value of that quantity. The dimensional and non-dimensional variables are related as follows:

$$\begin{aligned}
&\hat{y} = Hy,\ \hat{\ell} = H\ell, \quad \hat{\psi} = \psi, \quad \hat{s} = s_0 s, \quad \hat{\sigma} = s_0\hat{\ell}\sigma, \quad \hat{v} = vH\dot{\gamma}_0 \\
&\hat{t} = t/\dot{\gamma}_0, \quad \hat{\theta} = \theta_r\theta, \quad \hat{\rho} = \rho s_0/(H^2\dot{\gamma}_0^2), \quad \hat{k} = k(\hat{\rho}\hat{c}\dot{\gamma}_0 H^2), \\
&\hat{\tau} = \tau/\dot{\gamma}_0, \quad \theta_r = s_0/(\hat{\rho}\hat{c}), \quad \dot{\gamma}_0 = V_0/H.
\end{aligned}$$

Here $2H$ equals the thickness of the layer, V_0 the shearing speed prescribed on the top and bottom surfaces of the layer, s_0 the yield stress at room temperature θ_0 of the material of the layer in a quasistatic simple shear test, and $\hat{c}$ is the specific heat. FRANCIS [16] has experimentally determined the value of $\hat{\tau}$ for some materials; these range from 10^{-10} s for gases to 10^{-14} s for metals.

If ψ is interpreted as the effective plastic strain, and $\sigma_e \equiv (s^2 + \sigma^2)^{1/2}$ as the effective stress, then $\sigma_e = (1 + \psi/\psi_0)^n$ describes the effective stress vs. the effective plastic strain curve for quasistatic deformations of the body. Equation (2.4) implies that the rate of evolution of ψ is proportional to the plastic working due to the shear stress s and the dipolar stress σ. For a strain-hardening, strain-rate hardening and thermally softening material,

$$\frac{\partial \sigma_e}{\partial \psi} > 0, \qquad \frac{\partial \sigma_e}{\partial I} > 0 \qquad \frac{\partial \sigma_e}{\partial \theta} < 0.$$

When constitutive relation (2.5) is written as $I = f(\sigma_e, \theta, \psi)$, we require that

$$\frac{\partial f}{\partial \psi} < 0, \qquad \frac{\partial f}{\partial \sigma_e} > 0, \qquad \frac{\partial f}{\partial \theta} > 0.$$

The constitutive relation (4.1) used herein satisfies inequalities (2.8) and (2.9).

In order to complete the formulation of the problem we also need initial and boundary conditions. The boundary conditions considered are

$$v|_{y=\pm 1} = \pm 1, \ \theta_{,y}|_{y=\pm 1} = 0.$$

That is, the shear speed is prescribed on the top and bottom surfaces of the plate and these two bounding surfaces are thermally insulated. The time t is measured from the instant when the steady state has reached. Thus $v(0) = y$, $\sigma(0) = 0$, and $\psi(0)$, $\theta(0)$ and $\dot{\theta}(0)$ need to be prescribed.

3. Instability analysis

The approach used to study the stability of a homogeneous solution of Eqs. (2.1) – (2.5) and (2.10) is similar to that of BAI [2]. Let $\bar{\mathbf{s}} = [\bar{v}, \bar{s}, \bar{\sigma}, \bar{\theta}, \bar{\psi}]^T$ be a homogeneous solution of these equations; clearly $\bar{v} = y$ and $\bar{\sigma} \equiv 0$. Values of $\bar{s}(t)$, $\bar{\theta}(t)$ and $\bar{\psi}(t)$ are obtained by simultaneously solving

$$\dot{\bar{\theta}} + \tau \ddot{\bar{\theta}} = \bar{s}, \ \dot{\bar{\psi}} \left(1 + \frac{\bar{\psi}}{\psi_0}\right)^n = \bar{s}, \ 1 = f(\bar{s}, 0, \bar{\theta}, \bar{\psi}) \tag{3.1}$$

We assume that the homogeneous solution $\bar{\mathbf{s}}$ at time $t = t_0$ is perturbed by an infinitesimal amount

$$\delta \mathbf{s} = e^{\eta(t - t_0)} e^{i\xi y} \delta \mathbf{s}^0, t \geq t_0.$$

where $\delta\mathbf{s}^0 = [\delta v^0, \delta s^0, \delta\sigma^0, \delta\theta^0, \delta\psi^0]^T$, ξ is a wave number and η its growth rate at time $t = t_0$. It is tacitly assumed here that all perturbations are admissible which is the case only if the block is of infinite thickness. Substitution of $\mathbf{s}(y,t,t_0) = \bar{\mathbf{s}}(y,t) + \delta\mathbf{s}(y,t,t_0)$ into Eqs. (2.1), (2.2), (2.4), (2.5) and

$$\sigma v_{,y} = \ell s v_{,yy}$$

obtained by eliminating Λ from $(2.3)_1$ and $(2.3)_2$, and linearization of these equations with respect to $\delta\mathbf{s}^0$, gives

$$\mathbf{A}(t_0,\eta,\xi)\delta\mathbf{s}^0 = \mathbf{0},$$

$$\mathbf{A}(t_0,\eta,\xi) = \begin{vmatrix} \rho\eta & -i\xi & -\ell\xi^2 & 0 & 0 \\ -is^0\xi & -1 & 0 & \tau\eta^2+\eta+k\xi^2 & 0 \\ -\ell s^0\xi^2 & 0 & -1 & 0 & 0 \\ -is^0\xi & -1 & 0 & 0 & \psi_1^0\eta \\ i\xi & -f^0_{,s} & 0 & -f^0_{,\theta} & -f^0_{,\psi} \end{vmatrix}$$

$$s^0 = \bar{s}(t_0), \quad f^0_{,s} = \partial f/\partial s|_{\mathbf{s}=\mathbf{s}^0} \text{ etc.}, \quad \psi_1^0 = \left(1 + \frac{\psi^0}{\psi_0}\right)^n$$

and we have set $\bar{v} = y$ and $\bar{\sigma} = 0$. Thus $f^0_{,\sigma} = 0$.

In order for Eq. (3.4) to have a nontrivial solution, $\det\mathbf{A} = 0$, which yields the following quartic equation for the initial growth rate η:

$$a\eta^4 + b\eta^3 + c\eta^2 + d\eta + e \quad 0,$$

where

$$\begin{aligned} a(t_0) &= \rho\tau\psi_1^0 f^0_{,s}, \\ b(\xi,t_0) &= \rho\tau f^0_{,\psi} + \rho\psi_1^0 f^0_{,s} + \tau\psi_1^0\xi^2 + \ell^2\tau\psi_1^0 s^0 f^0_{,s}\xi^4, \\ c(\xi,t_0) &= \rho f^0_{,\psi} + \rho\psi_1^0 f^0_{,\theta} + (\psi_1^0 + \rho k\psi_1^0 f^0_{,s} - \tau s^0 f^0_{,\psi})\xi^2 \\ &\qquad + \ell^2 s^0(\tau f^0_{,\psi} + \psi_1^0 f^0_{,s})\xi^4, \\ d(\xi,t_0) &= (\rho k f^0_{,\psi} - s^0 f^0_{,\psi} - s^0\psi_1^0 f^0_{,\theta})\xi^2 + (\ell^2 s^0(f^0_{,\psi} + \psi_1^0 f^0_{,\theta}) \\ &\qquad + k\psi_1^0)\xi^4 + k\ell^2 s^0\psi_1^0 f^0_{,s}\xi^6, \\ e(\xi,t_0) &= -ks^0 f^0_{,\psi}(1 - \ell^2\xi^2)\xi^4 \end{aligned}$$

Note that ρ, τ, k, ℓ, s^0, ψ_1^0, $f_{,s}^0$ and $f_{,\theta}^0$ are positive, and $f_{,\psi}^0$ is negative. For $\tau = 0$, Eq. (3.6) reduces to the cubic Eq. (17) of CHEN and BATRA [13].

If the spectral Eq. (3.6) has a root with a positive real part, then the homogeneous solution $\mathbf{s}^0 = \bar{\mathbf{s}}(t_0)$ will be unstable. Clearly, roots of (3.6) depend upon the wave number ξ and the time t_0 when the perturbation is introduced.

For large wavelengths, $\xi \to 0$, the solutions of the spectral Eq. (3.6) are

$$\eta = 0, 0, \left(-b_0 \pm \sqrt{b_0^2 - 4ac_0}\right)/2a$$

$$b_0 = b(0, t_0) = \rho(\tau f_{,\psi}^0 + \psi_1^0 f_{,s}^0), \quad c_0 = c(0, t_0) = \rho(f_{,\psi}^0 + \psi_1^0 f_{,\theta}^0). \tag{3.9}$$

Assuming that $b_0 > 0$, the solution will be unstable if $c_0 < 0$, and stable for $c_0 > 0$. The assumption $b_0 > 0$ appears reasonable since for $\tau = 0$, $b_0 > 0$. Thus $b_0 > 0$ as long as $\tau < -\psi_1^0 f_{,s}^0 / f_{,\psi}^0$ which is likely to be true for $\tau \ll 1$. When τ equals 0, Eq. (3.6) reduces to a cubic equation which for long wavelengths has roots $0, 0, -\left(\frac{1}{\psi_1^0} f_{,\psi}^0 + f_{,\theta}^0\right)/f_{,s}^0$, and the solution will be unstable only if $(f_{,\psi}^0 + \psi_1^0 f_{,\theta}^0) < 0$ since $\psi_1^0 \simeq 1 > 0$.

For short wavelengths, $\xi \to \infty$, the finite solution of (3.6) is $(-f_{,\psi}^0)/(\psi_1^0 f_{,s}^0)$ which in view of inequalities (2.9) is always positive. Therefore, the shear deformation is always unstable for short wavelengths. Note that for a simple material (i.e. $\ell = 0$) with $\tau = 0$, BAI [2] has found that the homogeneous shear deformation is stable with respect to perturbations of short wavelengths. For $\ell = 0$ and $\tau \geq 0$, the finite root of Eq. (3.6) for short wavelengths is $s^0 f_{,\psi}^0 / \psi_1^0$ which is negative implying thereby that the shear deformation is stable. For $\tau = 0$ but $\ell \neq 0$, the finite solution of Eq. (3.6) is $(-f_{,\psi}^0/(\psi_1^0 f_{,s}^0))$ which is positive. Thus *for a strain-rate gradient-dependent thermoviscoplastic material, perturbations of infinitesimal wavelengths will always destabilize the homogeneous shear deformation.* This is counter-intuitive to the common belief that the consideration of higher-order gradients always has a stabilizing influence. However, the growth-rate of these perturbations need not be large as compared to that of the underlying homogeneous shear deformation.

Henceforth we assume that the root of Eq. (3.6) corresponding to the maximum growth rate is real which is usually the case if the homogeneous solution is perturbed after the shear stress has attained its peak value; this was verified through numerical experiments. For a fixed value of t_0, we are interested in seeking the wave number ξ_m for which the growth rate η_m at time t_0 is maximum. Thus (η_m, ξ_m) satisfy (3.6) and $\left.\frac{d\eta}{d\xi}\right|_{(\eta=\eta_m, \xi=\xi_m)} = 0$ which gives

$$\tau\psi_1^0(1 + 2\ell^2 s^0 f_{,s}^0 \xi_m^2)\eta_m^3 + [\psi_1^0(1 + \rho k f_{,s}^0) - \tau s^0 f_{,\psi}^0 \tag{3.10}$$

$$+ 2\ell^2 s^0(\tau f^0_{,\psi} + \psi^0_1 f^0_{,s})\xi^2_m]\eta^2_m + [(\rho k - s^0) f^0_{,\psi} - s^0\psi^0_1 f^0_{,\theta} \tag{3.10 [cont.]}$$

$$+ 2(\ell^2 s^0(f^0_{,\psi} + \psi^0_1 f^0_{,\theta}) + k\psi^0_1)\xi^2_m + 3k\ell^2 s^0\psi^0_1 f^0_{,s}\xi^4_m]\eta_m$$

$$- ks^0 f^0_{,\psi}(2 - 3\ell^2\xi^2_m)\xi^2_m = 0.$$

Substituting $\eta = \eta_m$ and $\xi = \xi_m$ into (3.6) and solving it with (3.10), we obtain the wave number ξ_m corresponding to the maximum growth rate η_m at time t_0. Note that both ξ_m and η_m depend upon t_0. We now consider two special cases: simple or nonpolar materials for which $\ell = 0$, and strain-rate gradient-dependent materials with $\tau = 0$.

3.1. Simple materials

For simple materials, $\ell = 0$, and Eq. (3.10) yields

$$\xi^2_m = -\eta_m[\tau\psi^0_1\eta^2_m + (\psi^0_1(1 + \rho k f^0_{,s}) - \tau s^0 f^0_{,\psi})\eta_m \tag{3.11}$$

$$\rho k f^0_{,\psi} - s^0(f^0_{,\psi} + \psi^0_1 f^0_{,\theta})]/(2k(\psi^0_1\eta_m - s^0 f^0_{,\psi}))$$

For locally adiabatic conditions, $k = 0$ and Eq. (3.11) gives either $\eta_m = 0$ or $\xi_m = \infty$. The second alternative implies that η_m is a monotonically nondecreasing function of ξ_m and takes on a maximum value at $\xi_m = \infty$. As discussed earlier, the simple shearing deformation of simple materials is stable with respect to perturbations of short wavelength.

We now consider heat-conducting simple materials. Assuming that $\eta_m > 0$, the condition $\xi^2_m > 0$ gives

$$\tau\psi^0_1\eta^2_m + (\psi^0_1(1 + \rho k f^0_{,s}) - \tau s^0 f^0_{,\psi})\eta_m + \rho k f^0_{,\psi} - s^0(f^0_{,\psi} + \psi^0_1 f^0_{,\theta}) < 0. \tag{3.12}$$

Since the first two terms on the left-hand side of (3.12) are positive, therefore, the condition for the instability to occur is that

$$A \equiv (\rho k f^0_{,\psi} - s^0 f^0_{,\psi} - s^0\psi^0_1 f^0_{,\theta}) < 0. \tag{3.13}$$

Recalling that $\xi^2_m \geq 0$, we obtain the following limits for the value of η_m

$$0 \leq \eta_m \leq \frac{-B + \sqrt{B^2 - 4\tau\psi^0_1 A}}{2\tau\psi^0_1} \equiv \eta^*_m \tag{3.14}$$

where $B = \psi^0_1 + \rho k\psi^0_1 f^0_{,s} - \tau s^0 f^0_{,\psi}$. Whenever condition (3.13) for instability is satisfied, the spectral Eq. (3.6) with $\ell = 0$ has a solution that lies between 0 and η_m^*. Note that the inequality (3.13) does not involve τ, but does contain the thermal conductivity k and the mass density ρ. So *the instability criterion for simple materials is the same, whether or not thermal disturbances propagate at a finite speed.* For a typical hard steel and $\dot{\gamma}_0 = 10^3/\mathrm{s}$, $\rho = O(10^{-5})$, $k = O(10^{-3})$, $s^0 = O(1)$, $\psi^1 = O(1)$, thus the instability criterion (3.13) reduces to $(f^0_{,\psi} + \psi^0_1 f^0_{,\theta}) > 0$. From $(2.7)_9$ and $(2.7)_{10}$ we conclude that $\rho k \propto \dot{\gamma}_0$, so the first term in the expression for A will make a negligible contribution even when $\dot{\gamma}_0 = 10^6/\mathrm{s}$.

For heat-conducting non-work-hardening simple materials, $\ell = 0$, $f^0_{,\psi} = 0$, the spectral Eq. (3.6) reduces to

$$\tau\rho f^0_{,s}\eta^3 + (\rho f^0_{,s} + \tau\xi^2)\eta^2 + [\rho f^0_{,\theta} + (1 + \rho k f^0_{,s})\xi^2]\eta + (k\xi^2 - s^0 f^0_{,\theta})\xi^2 = 0, \tag{3.15}$$

and the instability condition becomes

$$s^0 f^0_{,\theta} > k\xi^2 \tag{3.16}$$

A comparison of Eqs. (3.13) and (3.16) suggests that the product of the thermal conductivity and the square of the wave number now plays the role of the work-hardening of the material. The expression (3.14) for η_m^* simplifies to

$$\eta_m^* = \left[-(1 + \rho k f^0_{,s}) + \sqrt{(1 + \rho k f^0_{,s})^2 + 4\tau s^0 f^0_{,\theta}}\,\right]/2\tau.$$

For $0 < \tau \ll 1$, $\eta_m^* = s^0 f^0_{,\theta}/(1 + \rho k f^0_{,s}) - \tau(s^0 f^0_{,\theta})^2/(1 + \rho k f^0_{,s})^3$ implying thereby that *the thermal relaxation time decreases the upper limit for the growth rate of the perturbations for non-work-hardening simple materials.*

3.2. Strain-rate gradient-dependent materials with $\tau = 0$

In materials with $\tau = 0$, thermal disturbances propagate at infinite speed. The spectral Eqs. (3.6) and (3.10) for the growth rate at time t_0 to be maximum take the forms

$$\begin{aligned}\rho\psi^0_1 f^0_{,s}\eta^3 &+ [\rho(f^0_{,\psi} + \psi^0_1 f^0_{,\theta}) + \psi^0_1(1 + \rho k f^0_{,s})\xi^2 + \ell^2\psi^0_1 s^0 f^0_{,s}\xi^4]\eta^2 \\ &+ [((\rho k - s^0)f^0_{,\psi} - s^0\psi^0_1 f^0_{,\theta})\xi^2 + (k\psi^0_1 + \ell^2 s^0(f^0_{,\psi} + \psi^0_1 f^0_{,\theta}))\xi^4 \\ &+ k\ell^2 s^0\psi^0_1 f^0_{,s}\xi^6]\eta + ks^0 f^0_{,\psi}(-1 + \ell^2\xi^2)\xi^4 = 0,\end{aligned} \tag{3.18}$$

$3k\ell^2 s^0(f^0_{,\psi} + \psi^0_1 f^0_{,s}\eta_m)\xi^4_m + [2\ell^2 s^0\psi^0_1 f^0_{,s}\eta^2_m$

(3.19) [cont.]
$$+\ 2(k\psi_1^0 + \ell^2 s^0(f^0_{,\psi} + \psi_1^0 f^0_{,\theta}))\eta_m - 2ks^0 f^0_{,\psi}]\xi_m^2$$
$$+ [\psi_1^0(1+\rho k f^0_{,s})\eta_m^2 + ((\rho k - s^0)f^0_{,\psi} - s^0\psi_1^0 f^0_{,\theta})\eta_m] = 0.$$

Equation (3.18) with $\xi = \xi_m$, $\eta = \eta_m$, and Eq. (3.19) determine ξ_m and η_m. For locally adiabatic deformations of simple materials, $k = \ell = 0$, Eqs. (3.18) and (3.19) give

$$\eta_m = s^0(f^0_{,\theta} + f^0_{,\psi}/\psi_1^0), \tag{3.20}$$
$$\xi_m = \infty, \tag{3.21}$$

which implies that perturbations of infinitesimal wavelength grow the fastest provided that $(f^0_{,\theta} + f^0_{,\psi}/\psi_1^0) > 0$. This agrees with the result of BATRA and CHEN [8] who studied shear band spacing in three non-work-hardening strain-rate gradient-dependent materials.

We denote the value (3.20) of η_m by η_{m0}. For $k = 0$, Eqs. (3.18) with $\xi = \xi_m$, $\eta = \eta_m$, and (3.19) give

$$\rho f^0_{,s}\eta_m^2 + [\rho\eta_{mo}/s^0 + \xi_m^2(1+\ell^2 s^0 f^0_{,s}\xi_m^2)]\eta_m + \xi_m^2(\ell^2\xi_m^2 - 1)\eta_{m0} = 0, \tag{3.22}$$
$$\xi_m^2 = (\eta_{m0} - \eta_m)/2\ell^2(s^0 f^0_{,s}\eta_m + \eta_{m0}). \tag{3.23}$$

Since $\xi_m^2 > 0$, therefore $\eta_m < \eta_{m0}$. Thus *positive values of ℓ decrease the growth rate of perturbations, and the decrease in the growth rate is proportional to ℓ^2.* Also, the wavelength of the perturbation corresponding to the maximum growth rate is finite.

For locally adiabatic deformations of non-work-hardening gradient-dependent viscoplastic materials, $\tau = k = f_{,\psi} = 0$, and Eq. (3.18) evaluated at $\xi = \xi_m$, $\eta = \eta_m$ and Eq. (3.10) reduce to the following two equations:

$$\rho f^0_{,s}\eta_m^2 + [\rho f^0_{,\theta} + \xi_m^2(1+\ell^2 s^0 f^0_{,s}\xi_m^2)]\eta_m + s^0 f^0_{,\theta}(-1+\ell^2\xi_m^2)\xi_m^2 = 0, \tag{3.24}$$
$$\eta_m = \frac{s^0 f^0_{,\theta}(1-2\ell^2\xi_m^2)}{1+2\ell^2 s^0 f^0_{,s}\xi_m^2}. \tag{3.25}$$

Hence the stability condition is

$$\xi_m < \frac{1}{\sqrt{2}\ell} \equiv \xi_m^*,$$

and only perturbations with wavelength greater than $(2\sqrt{2}\pi)$ times the material characteristic length can have the positive maximum growth rate at time t_0. The requirement $\xi_m^2 > 0$ and (3.25) yield

$$0 \le \eta_m \le s^0 f^0_{,\theta} \equiv \eta_m^*.$$

Thus the maximum growth rate at time t_0 of the perturbations is set by the present value of the shear stress and the thermal softening characteristics of the material.

4. Shear band spacing

We consider materials for which

$$f(s,\sigma,\theta,\psi) = \mu_0^{-\frac{1}{m}}\theta^{-\frac{\nu}{m}}\left(1+\frac{\psi}{\psi_0}\right)^{-\frac{n}{m}}(s^2+\sigma^2)^{\frac{1}{2m}}$$

where μ_0 is a strength parameter, m describes the strain-rate hardening of the material and $\nu < 0$ its thermal softening. The relation between nondimensional μ_0 and dimensional $\hat{\mu}_0$ is

$$\mu_0 = \frac{\dot{\gamma}_0^m s_0^{\nu-1}}{(\hat{\rho}\hat{c})^\nu}\hat{\mu}_0. \tag{4.2}$$

For f given by (4.1), a homogeneous solution of Eqs. (2.1) – (2.5) under boundary conditions (2.10) is

$$\begin{aligned}
\bar{v} &= y,\ \bar{\sigma} = 0,\\
\bar{s} &= \mu_0\bar{\theta}^\nu[\tilde{\psi}_0(\tau\dot{\bar{\theta}} + \bar{\theta} - \tilde{A})]^{\tilde{n}},\\
\bar{\psi} &= \psi_0\left[\tilde{\psi}_0^{\frac{1}{1+n}}(\tau\dot{\bar{\theta}} + \bar{\theta} - \tilde{A})^{\frac{1}{1+n}} - 1\right]
\end{aligned}$$

where

$$\tilde{\psi}_0 \quad \frac{1+n}{\psi_0} \qquad \tilde{n} = \frac{n}{1+n} \qquad \tilde{A} = \tau\dot{\bar{\theta}}(0) + \bar{\theta}(0) - \tilde{\psi}_0^{-1}, \tag{4.4}$$

and $\bar{\theta}$ is found by numerically solving

$$\tau\ddot{\bar{\theta}} + \dot{\bar{\theta}} = \mu_0\tilde{\psi}_0^{\tilde{n}}\bar{\theta}^\nu(\tau\dot{\bar{\theta}} + \bar{\theta} - \tilde{A})^{\tilde{n}}$$

Furthermore.

$$f_{,s} = \frac{sf}{m(s^2+\sigma^2)} > 0, \qquad f_{,\sigma} = \frac{\sigma f}{m(s^2+\sigma^2)}$$

$$f_{,\theta} = -\frac{\nu f}{m\theta} > 0, \qquad f_{,\psi} = -\frac{n}{m}\frac{1}{\psi_0}\frac{f}{\left(1+\dfrac{\psi}{\psi_0}\right)} < 0$$

Thus inequalities (2.9) are satisfied

In computing numerical results, we assigned the following values to the material and geometric parameters:

$$\begin{aligned}
\hat{\rho} &= 4510\ \mathrm{kg/m^3}, \quad m = 0.033, \quad \nu = -1.7, \quad \hat{\mu}_0 = 6.0 \times 10^{12},\\
\psi_0 &= 0.01, \quad \dot{\gamma}_0 = 10^5/s, \quad \hat{c} = 528\ \mathrm{J/kg\ K}, \quad \hat{k} = 19\ W/m\ K,\\
s_0 &= 405\ \mathrm{MPa}, \quad \dot{\bar{\theta}}(0) = 1.441, \quad \ell = 0.001, \quad n = 0.15,\\
H &= 2.5\ \mathrm{mm}, \quad \bar{\psi}(0) = 0, \quad \bar{\theta}(0) = 300\ \mathrm{K}, \quad \hat{\tau} = 10^{-10}\ \mathrm{s}.
\end{aligned}$$

These values, except possibly those of ℓ and $\hat{\tau}$, are for titanium. We will investigate the effect of different values of ℓ and $\hat{\tau}$ upon the shear band spacing. For an aluminum alloy, FRANCIS [16] has experimentally determined $\hat{\tau}$ to be 10^{-11} s. The value of H is used to nondimensionalize the variables. The value of $\dot{\bar{\theta}}(0)$ is estimated from Eq. (4.3) by setting $\tau = 0$, $\dot{\gamma}_0 = 10^6/\mathrm{s}$, and numerically solving the equations. The results presented below are for a layer of infinite thickness. Thus the effect of boundary conditions has been neglected.

Figure 1 shows, for homogeneous deformations of the body at nominal strain-rates of $10^5/\mathrm{s}$ and $10^6/\mathrm{s}$, the shear stress (or the effective stress) vs. the average shear strain curves for $\hat{\tau} = 0$, 10^{-7} and 10^{-11} s; these represent solutions of the

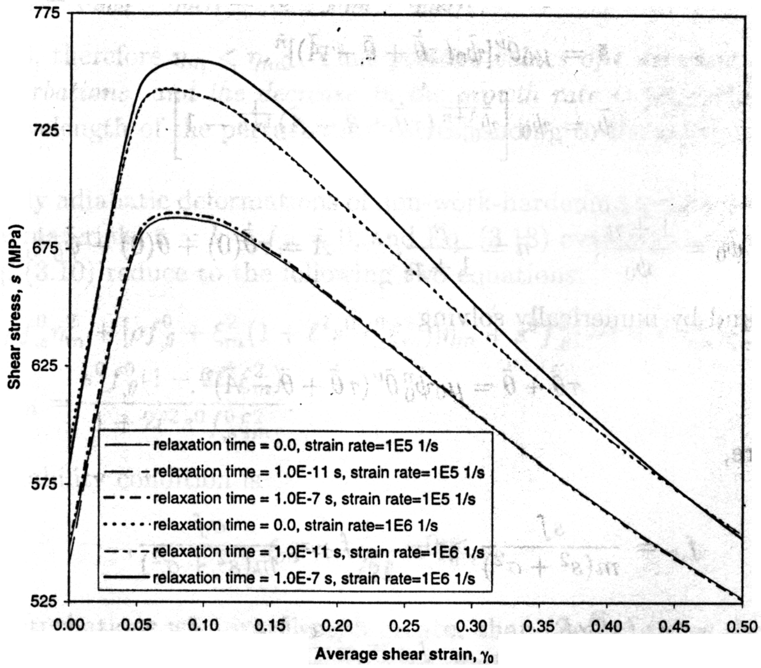


FIG. 1. Shear stress vs. shear strain curves at nominal strain-rates of $10^5/\mathrm{s}$ and $10^6/\mathrm{s}$ for three different values of the relaxation time τ. Shear stress vs. shear strain curves for thermal relaxation time $\tau = 0$ and 10^{-11} s coincide with each other

initial-value problem represented by Eqs. (4.3) – (4.5). For homogeneous deformations, both mechanical and thermal waves have died out. These equations were integrated by the Runge-Kutta method. The algorithm was validated by comparing analytical and numerical solutions of Eq. (4.5) for $\nu = 0$. It is clear that at a nominal strain-rate of 10^5/s, the stress-strain curves for homogeneous deformations of the body are essentially unaffected by the value of $\hat{\tau}$. However, for $\dot{\gamma}_0 = 10^6$/s, the values of $\hat{\tau}$ influence the shear stress vs. the average shear strain curves. Because of the strain-rate sensitivity of the material, at a given value of the shear strain, the shear stress for $\dot{\gamma}_0 = 10^6$/s is higher than that for $\dot{\gamma}_0 = 10^5$/s. For $\dot{\gamma}_0 = 10^5$/s, values of $\bar{\psi}$ and $\bar{\theta}$ are virtually the same for the three values of $\hat{\tau}$ which span over a wide range.

The integration of Eq. $(4.3)_3$ yields

$$\bar{\theta}(t) = \bar{\theta}(0)e^{-t/\tau} + \frac{1}{\tau}\int_0^t \left(1 + \frac{\bar{\psi}}{\psi_0}\right)^{n+1} e^{-(t-\xi)/\tau} d\xi + \tilde{A}(1 - e^{-t/\tau}$$

Thus for $t \gg \tau$, values of the temperature and hence of the stress depend upon τ through the dependence of the second and third terms upon τ. Since $\bar{\psi}(t)$ may depend upon τ, it is difficult to characterize how $\bar{\theta}(t)$ should vary with τ. We note that the computed results also depend upon the value of $\dot{\bar{\theta}}(0)$ through the dependence of $\tilde{A}$ upon $\dot{\bar{\theta}}(0)$.

For assigned values of the time t_0 and the wave number ξ, Eq. (3.6) is solved for the growth rate η. Figure 2 depicts, for $\dot{\gamma}_0 = 10^5$/s and $\ell = 0$, the initial dominant growth rate η (i.e. the growth rate at time t_0 with the largest positive real part) vs. the wave number ξ for four different values of the average strain γ_0 or the time t_0 when the homogeneous solution is perturbed. For each value of γ_0, the initial dominant growth rate η first increases, reaches a maximum value and subsequently decreases with an increase in ξ. We call the maximum value of η the initial critical growth rate and denote it and the corresponding value of ξ by η_m and ξ_m respectively; clearly η_m and ξ_m depend upon t_0 or equivalently γ_0, and η_m is not a monotonically increasing function of t_0. According to WRIGHT and OCKENDON'S [28] postulate, i.e., the wavelength of the dominant instability mode with the maximum growth rate at time t_0 determines the shear band spacing L_s, we have

$$L_s = 2\pi/\xi_m(t_0^s).$$

However, the definition

$$L_s = \inf_{t_0 \geq 0} \frac{2\pi}{\xi_m(t_0)}, \tag{4.8}$$

will give the least possible spacing between adjacent shear bands. We note that for thermal softening described by an affine function of the temperature, definitions (4.7) and (4.8) of the shear band spacing give quite different results, e.g. see BATRA and CHEN [8]. MOLINARI [20] pointed out that for the CRS1018 steel modeled by a power-law type relation, the two definitions give essentially the same value of the shear band spacing.

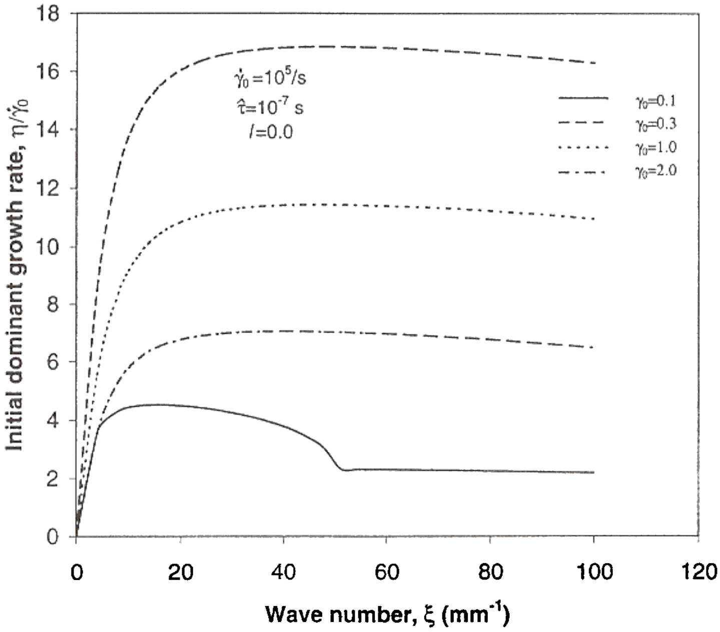
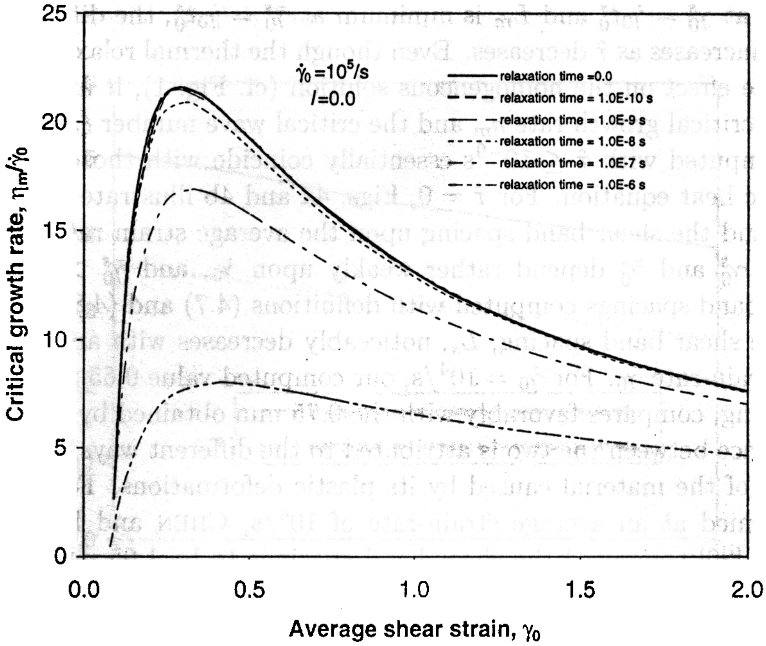


FIG. 2. Initial dominant growth rate vs. the wave number for four different values of the average shear strain, γ_0, when the homogeneous solution corresponding to $\dot{\gamma}_0 = 10^5/\mathrm{s}$ is perturbed.

Equations (3.11) and (3.21) and either one of the two definitions of the shear band spacing imply that *the shear band spacing equals zero in locally adiabatic deformations of simple materials whether or not thermal waves propagate at a finite speed in these materials.* This generalizes earlier similar result of BATRA and CHEN [8] for non-work-hardening to work-hardening simple materials. Equation (3.23) implies that *the shear band spacing in strain-rate gradient-dependent materials is proportional to the material characteristic length ℓ.* However, ξ_m and η_m depend upon the time t_0 when the homogeneous solution is perturbed. Taking into account this dependence, CHEN and BATRA [13] derived an approximate expression for the shear band spacing in locally adiabatic deformations of gradient-dependent materials. They found that the *shear band spacing is proportional to the square root of the material characteristic length.* Numerical experiments of BATRA and KIM [5] gave strong dependence of the shear band width upon ℓ.

Figures 3a and 3b exhibit, for $\dot{\gamma}_0 = 10^5/\mathrm{s}$, $\ell = 0$ and six different values of the thermal relaxation time $\hat{\tau}$, the dependence of η_m and the corresponding critical

a)



b)

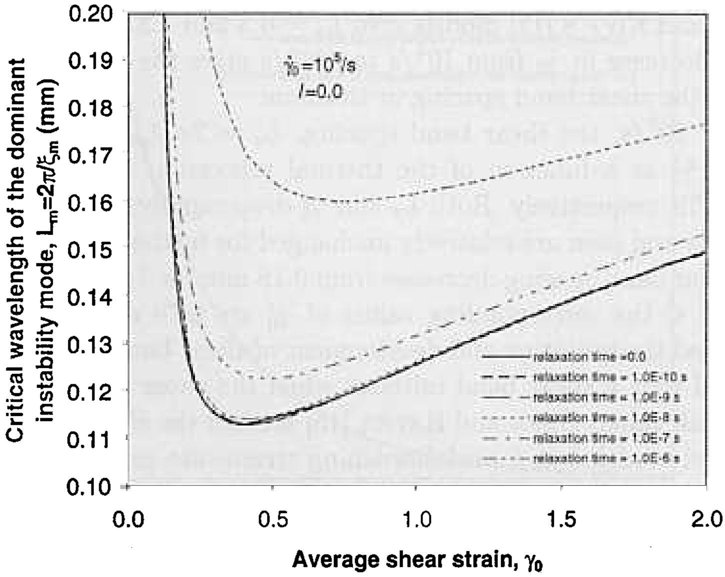


FIG. 3. Dependence of (a) the critical growth rate, and (b) the critical wavelength of the dominant instability mode, upon the average shear strain, γ_0, when the homogeneous solution for $\dot{\gamma}_0 = 10^5$/s is perturbed.

wavelength, $L_m = 2\pi/\xi_m$, upon the average shear strain γ_0. Recall that the coefficients a, b and c in (3.6) depend upon τ. For each value of $\hat{\tau}$, η_m is maximum at $\gamma_0^s = \dot{\gamma}_0 t_0^s$ and L_m is minimum at $\bar{\gamma}_0^s = \dot{\gamma}_0 \bar{t}_0^s$; the difference between γ_0^s and $\bar{\gamma}_0^s$ increases as $\hat{\tau}$ decreases. Even though the thermal relaxation time $\hat{\tau}$ has a negligible effect on the homogeneous solution (cf. Fig. 1), it affects noticeably the initial critical growth rate η_m and the critical wave number ξ_m. However, the results computed with $\hat{\tau} \leq 10^{-9}$s essentially coincide with those for $\hat{\tau} = 0$, i.e., a parabolic heat equation. For $\hat{\tau} = 0$, Figs. 4a and 4b illustrate the dependence of γ_0^s, $\bar{\gamma}_0^s$ and the shear band spacing upon the average strain rate $\dot{\gamma}_0$. It is clear that both γ_0^s and $\bar{\gamma}_0^s$ depend rather weakly upon $\dot{\gamma}_0$, and $\bar{\gamma}_0^s > \gamma_0^s$. However, the shear band spacings computed with definitions (4.7) and (4.8) are nearly the same. The shear band spacing, L_s, noticeably decreases with an increase in the average strain-rate $\dot{\gamma}_0$. For $\dot{\gamma}_0 = 10^4$/s, our computed value 0.65 mm of the shear band spacing, compares favorably with the 0.75 mm obtained by MOLINARI [20]; the difference between the two is attributed to the different ways of modeling the hardening of the material caused by its plastic deformations. For the CRS1018 steel deformed at an average strain-rate of 10^4/s, CHEN and BATRA [13] and MOLINARI [20] computed the shear band spacings to be 1.05 mm and 1.4 mm, respectively. NESTERENKO *et al.* [21] measured $L_s = 1$ and 0.85 mm in the CRS1018 steel and titanium respectively, and estimated the strain-rate in the band to be 10^4/s. For the titanium studied here, WRIGHT and OCKENDON'S [28] and GRADY and KIPP'S [17] models give $L_s = 0.3$ and 1.8 mm, respectively. We note that a decrease in $\dot{\gamma}_0$ from 10^4/s to 6850/s gives the experimental value of 0.85 mm for the shear band spacing in titanium.

For $\dot{\gamma}_0 = 10^5$/s, the shear band spacing, $L_s = 2\pi/\xi_m(\bar{t}_0^s)$, and the average shear strain $\bar{\gamma}_0^s$ as a function of the thermal relaxation time $\hat{\tau}$ are plotted in Figs. 5a and 5b, respectively. Both L_s and $\bar{\gamma}_0^s$ drop rapidly as $\hat{\tau}$ is decreased from 10^{-6} to 10^{-8} s and then are relatively unchanged for further decrease in the value of $\hat{\tau}$. The shear band spacing decreases from 0.16 mm for $\hat{\tau} = 10^{-6}$ s to 0.114 mm for $\hat{\tau} = 10^{-8}$ s; the corresponding values of $\bar{\gamma}_0^s$ are 0.76 and 0.41. BATRA and KIM [6] studied the initiation and development of shear bands in twelve materials and proposed that a shear band initiates when the shear stress has dropped to 90% of its peak value. CHEN and BATRA [13] studied the shear band spacing in a CRS1018 steel modeled as a work-hardening strain-rate gradient-dependent material. They found that corresponding to the times when perturbations of the homogeneous solution resulted in the shear band spacing, $s/s_{\max}$ equalled about 0.95. Here $s/s_{\max}$ varies from 0.64 to 0.82 as the relaxation time $\hat{\tau}$ is decreased from 10^{-6}s to 10^{-10}s, and stays at 0.82 for smaller values of $\hat{\tau}$. The difference in the two sets of values is primarily due to the fact the nominal strain-rate in the present problem is ten times that considered by Chen and Batra, and to a less extent due to the difference in the thermomechanical response of the two materials.

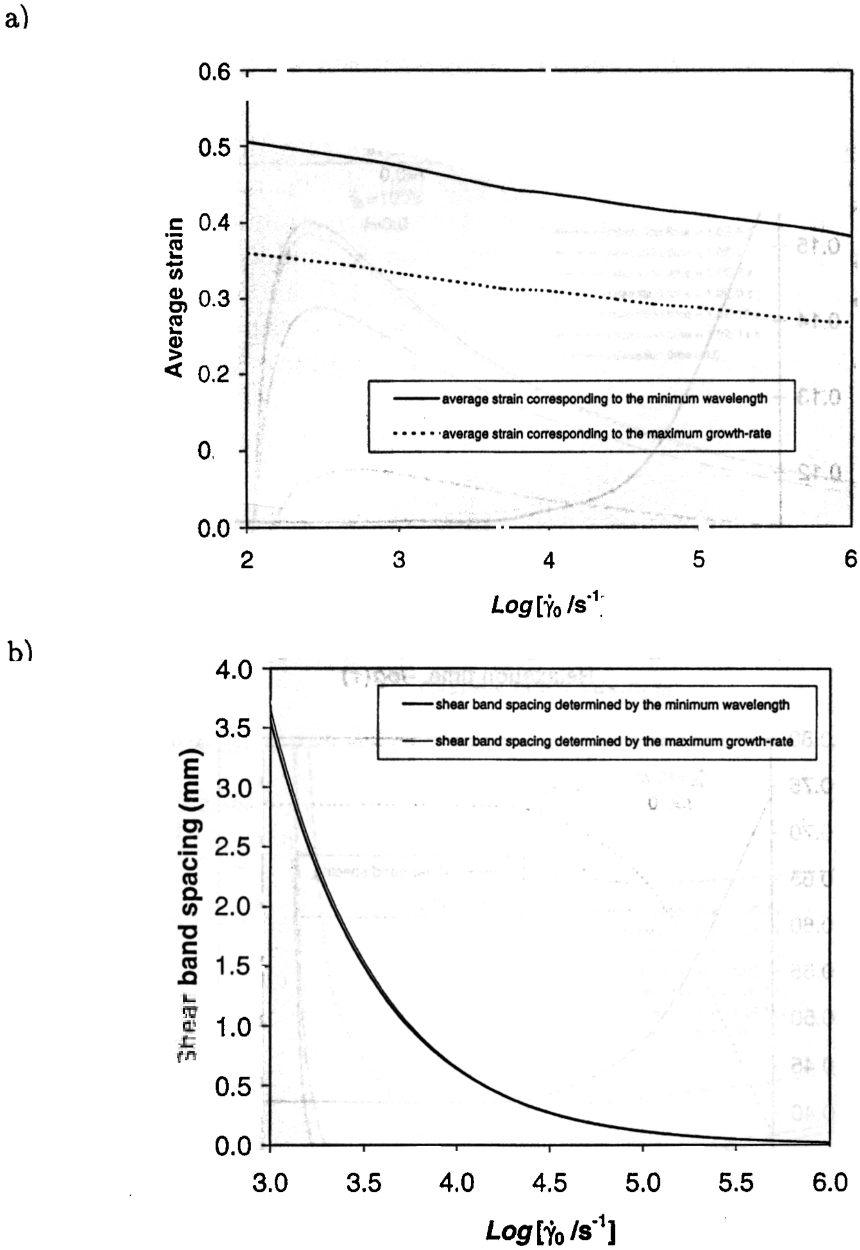
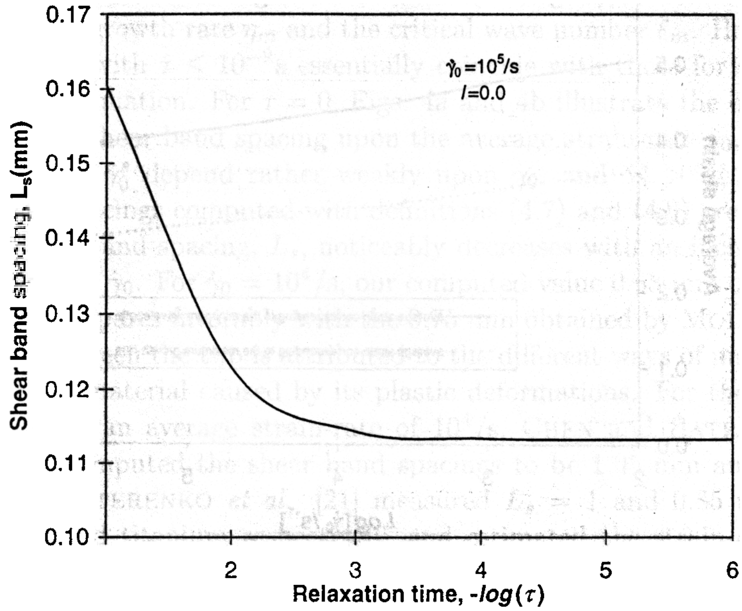


FIG. 4. Dependence upon the nominal strain-rate of (a) the average strains corresponding to the minimum wavelength and the maximum growth rate, and (b) the shear band spacing.

a)



b)

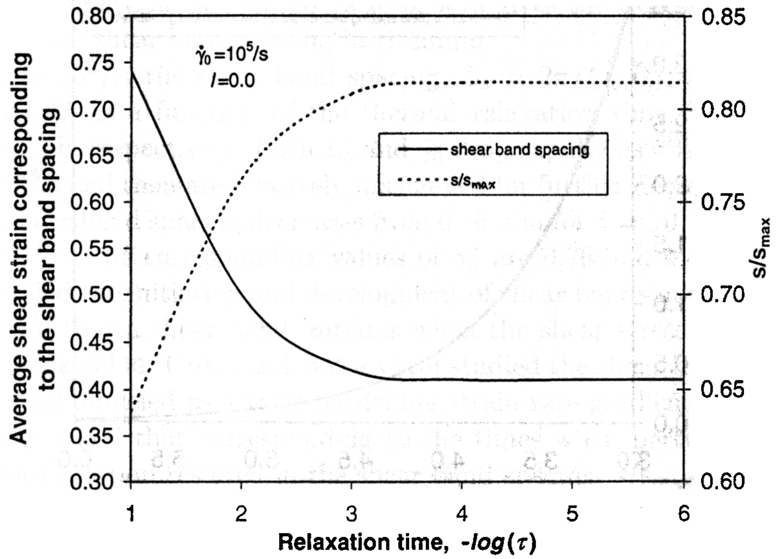


FIG. 5. Dependence upon the relaxation time of (a) the shear band spacing and (b) the average shear strain and s/s_{max} corresponding to the shear band spacing ($\dot{\gamma}_0 = 10^5$/s).

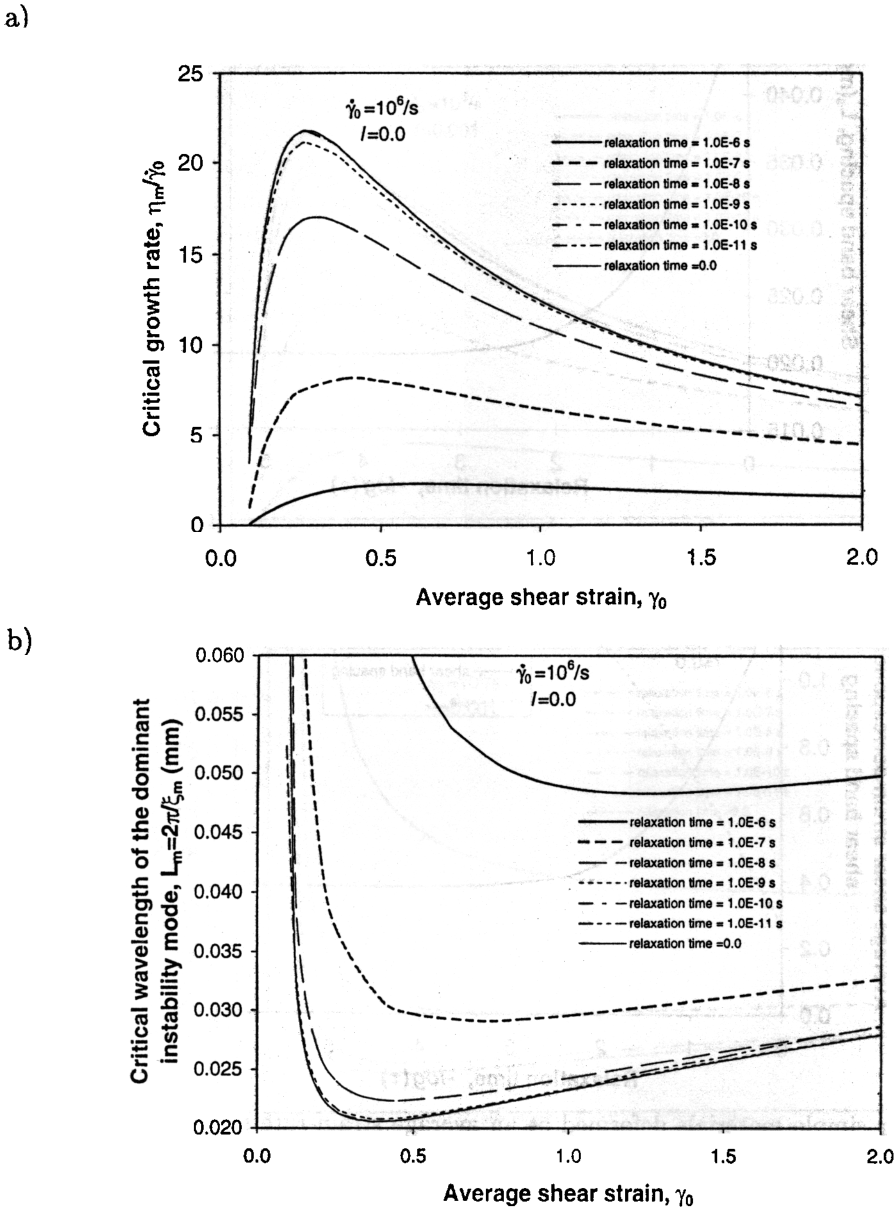
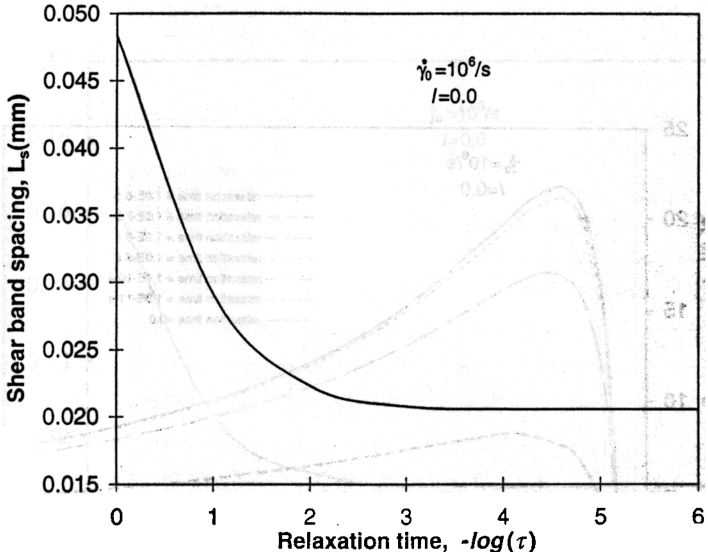

a)
$\dot{\gamma}_0$=10^6/s
l=0.0
relaxation time = 1.0E-6 s
relaxation time = 1.0E-7 s
relaxation time = 1.0E-8 s
relaxation time = 1.0E-9 s
relaxation time = 1.0E-10 s
relaxation time = 1.0E-11 s
relaxation time =0.0
Critical growth rate, $\eta_m/\dot{\gamma}_0$
Average shear strain, γ_0
b)
$\dot{\gamma}_0$=10^6/s
l=0.0
relaxation time = 1.0E-6 s
relaxation time = 1.0E-7 s
relaxation time = 1.0E-8 s
relaxation time = 1.0E-9 s
relaxation time = 1.0E-10 s
relaxation time = 1.0E-11 s
relaxation time =0.0
Critical wavelength of the dominant instability mode, $L_m=2\pi/\xi_m$ (mm)
Average shear strain, γ_0

FIG. 6

c)



d)

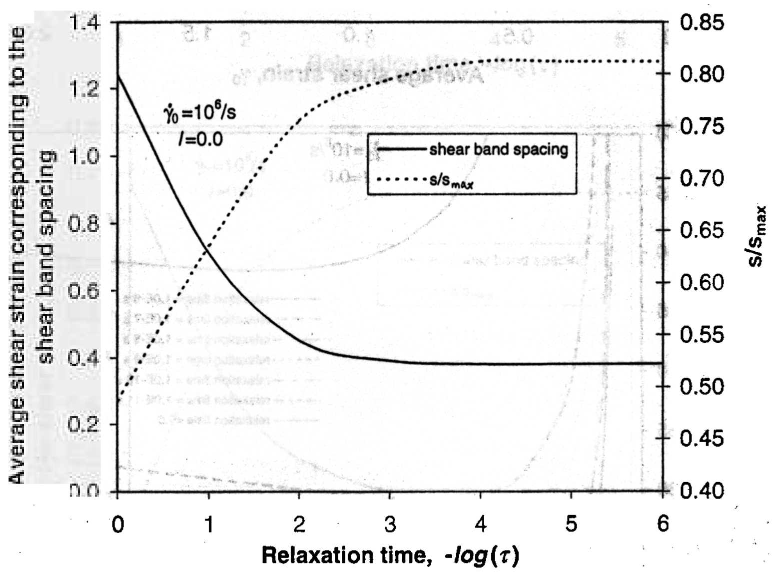
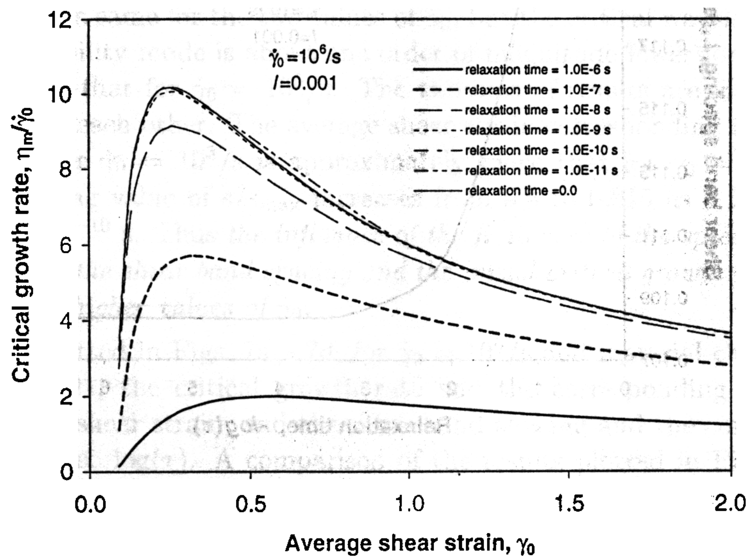


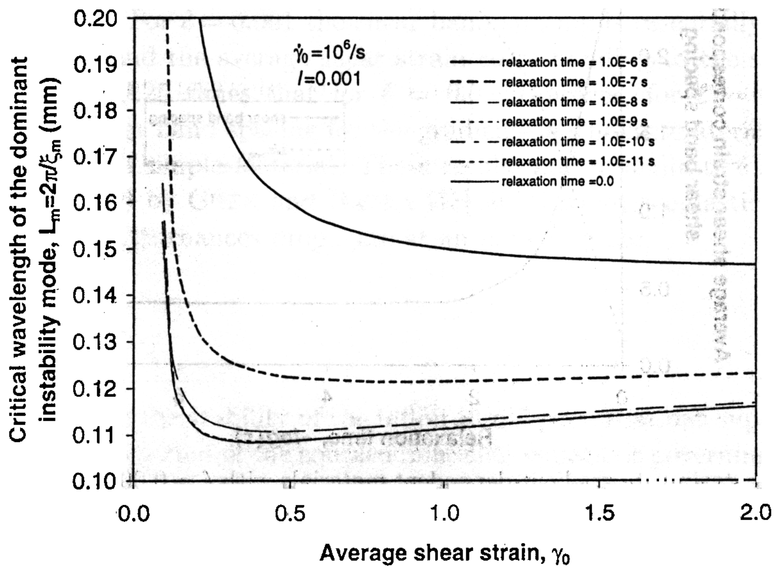
FIG. 6. For simple materials deformed at an average strain-rate of 10^6/s, dependence upon the average shear strain, γ_0, of (a) the critical growth rate, (b) the critical wavelength of the dominant instability mode; dependence upon the relaxation time of (c) the shear band spacing, and (d) the average shear strain and s/s_{max} corresponding to the shear band spacing.

a)



$\dot{\gamma}_0$=10^6/s
l=0.001
relaxation time = 1.0E-6 s
relaxation time = 1.0E-7 s
relaxation time = 1.0E-8 s
relaxation time = 1.0E-9 s
relaxation time = 1.0E-10 s
relaxation time = 1.0E-11 s
relaxation time =0.0
Critical growth rate, $\eta_m/\dot{\gamma}_0$
Average shear strain, γ_0

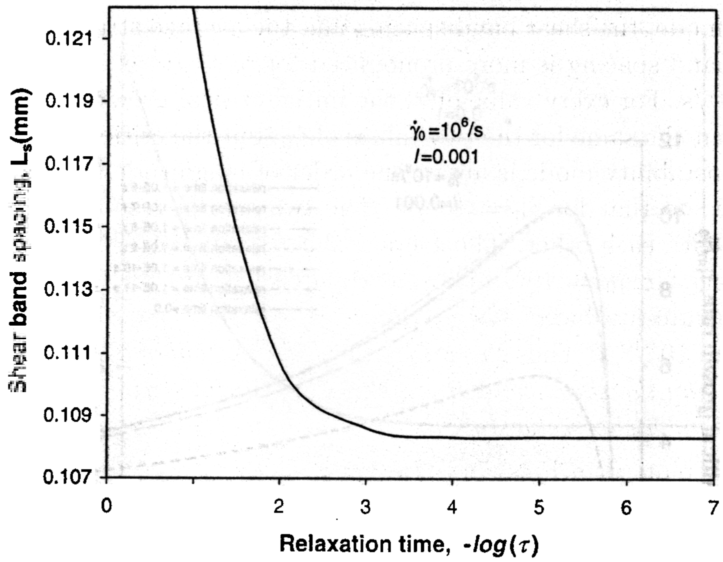
b)



$\dot{\gamma}_0$=10^6/s
l=0.001
relaxation time = 1.0E-6 s
relaxation time = 1.0E-7 s
relaxation time = 1.0E-8 s
relaxation time = 1.0E-9 s
relaxation time = 1.0E-10 s
relaxation time = 1.0E-11 s
relaxation time =0.0
Critical wavelength of the dominant instability mode, $L_m=2\pi/\xi_m$ (mm)
Average shear strain, γ_0

FIG. 7

c)



d)

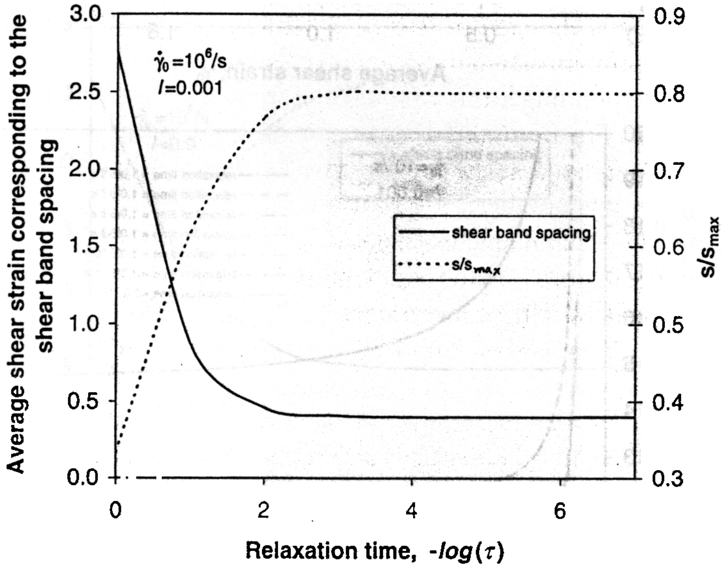


FIG. 7. For strain-rate gradient-dependent materials with $\ell = 0.001$ and nominal strain-rate of 10^6/s, (a) dependence upon the relaxation time of the critical growth rate, (b) the dependence of the critical wavelength of the dominant instability mode upon the average shear strain; dependence upon the relaxation time of (c) the shear band spacing, and (d) the average shear strain and $s/s_{\max}$ corresponding to the shear band spacing.

Figures 6a through 6d exhibit for $\dot{\gamma}_0 = 10^6/s$ the results corresponding to those plotted in Figs. 2, 3 and 5 for $\dot{\gamma}_0 = 10^5/s$. The influence of the relaxation time $\hat{\tau}$ on the initial critical growth rate, the critical wavelength of the dominant instability mode, the shear band spacing and the average strain corresponding to the shear band spacing is more pronounced for $\dot{\gamma}_0 = 10^6/s$ as compared to that for $\dot{\gamma}_0 = 10^5/s$. For every value of $\hat{\tau}$, the initial critical growth rate as a multiple of $\dot{\gamma}_0$ is about the same for the two values of $\dot{\gamma}_0$, but the critical wavelength of the dominant instability mode is about one order of magnitude lower for $\dot{\gamma}_0 = 10^6/s$ as compared to that for $\dot{\gamma}_0 = 10^5/s$. The two sets of results are in qualitative agreement with each other. The average shear strain corresponding to the shear band spacing for $\dot{\gamma}_0 = 10^5/s$ is approximately twice that for $\dot{\gamma}_0 = 10^6/s$, and the corresponding value of s/s_{max} increases from 0.5 to 0.815 as $\hat{\tau}$ is decreased from 10^{-6} to 10^{-10} s. Thus *the influence of the finiteness of the speed of thermal disturbances on the shear band spacing and the initial critical growth rate is more pronounced for higher values of* $\dot{\gamma}_0$.

We have plotted in Figs. 7a – 7d, for $\dot{\gamma}_0 = 10^6/s$ and material characteristic length $\ell = 0.001$, the critical growth rate and the corresponding wavelength vs. the average shear strain, and the shear band spacing and the corresponding average strain vs. $\log(\tau)$. A comparison of the results plotted in Figs. 6 and 7 reveals that when ℓ is changed from 0 to 0.001 and $\tau = 10^{-6}$ s, the initial critical growth rate is approximately halved and the corresponding wavelength is nearly tripled. The corresponding value of s/s_{max} increases from 0.32 for $\hat{\tau} = 10^{-6}$ s to 0.8 for $\hat{\tau} \leq 10^{-9}$ s. For $\ell = 0.001$ the shear band spacing is essentially increased by a factor of 2.5 and the average shear strain corresponding to the shear band spacing is almost 2.25 times that for $\ell = 0.0$. However, for $\ell = 0.001$ and $\hat{\tau} \leq 10^{-9}$ s, the shear band spacing for the gradient-dependent material is nearly five times that for a simple material. These results are in qualitative agreement with those obtained by CHEN and BATRA [13] for thermoviscoplastic materials in which thermal disturbances propagate at an infinite speed.

5. Conclusions

We have studied the stability of the infinitesimal perturbations superimposed on a homogeneous solution of the coupled nonlinear equations governing the thermomechanical simple shearing deformations of a strain-rate gradient-dependent viscoplastic body in which thermal waves propagate at a finite speed, and we have derived conditions for these perturbations to grow. For simple materials, the instability criterion (3.13) is independent of the thermal relaxation time. However, the growth rate of the perturbations is influenced by the thermal relaxation time $\hat{\tau}$. For non-heat-conducting simple materials, the growth rate of perturbations at time t_0 is a monotonically increasing function of the wavelength implying thereby that the shear band spacing in these materials equals zero. This generalizes a similar result of BATRA and CHEN [8] for non-work-hardening materials to work-hardening materials. In strain-rate gradient-dependent materials, perturbations of infinitesimal wavelength will always grow implying thereby that the homogeneous shear deformation is always unstable. However, the wavelength of perturbations with the maximum growth rate is about 16μm for an average shear strain-rate of 10^6/s and material characteristic length equal to 2.5 μm. For $\ell = 0$, i.e., in simple materials, the homogeneous simple shear is stable in the limiting case of perturbations of infinitesimal wavelength. In non-work-hardening simple materials, the thermal relaxation time decreases the maximum growth rate of the perturbations.

For the titanium alloy modeled as a simple material, the shear band spacing decreases rapidly from 160 μm to 114 μm as $\hat{\tau}$ is varied from 10^{-6} to 10^{-8} s and the nominal strain-rate $\dot{\gamma}_0$ equals 10^5/s. For $\dot{\gamma}_0 = 10^6$/s, the shear band spacing decreases from 48 μm to 21 μm when $\hat{\tau}$ decreases from 10^{-6} to 10^{-8} s. When the same alloy is modeled by a strain-rate gradient-dependent theory with material characteristic length $\ell = 2.5$ μs, the shear band spacing drops from 122 μm to 108.5 μm as $\hat{\tau}$ decreases from 10^{-6} to 10^{-8}s. It follows from Eq. (3.23) that the shear band spacing is proportional to ℓ, and that the growth rate of the perturbations for $\ell > 0$ is smaller than that for $\ell = 0$. CHEN and BATRA [13] have shown that the shear band spacing varies as the square-root of ℓ. NESTERENKO *et al.* [21] estimated the average strain-rate in the shear banded material to be 10^4/s, and measured the shear band spacing to be 0.85 mm. The computed shear-band spacing equals 3.65, 0.65 and 0.113 mm for $\dot{\gamma}_0 = 10^3$, 10^4 and 10^5/s respectively, and equals 0.85 mm for $\dot{\gamma}_0 = 6850$/s.

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