On the Directional Stability of a Propagating Crack

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Directional stability and the related dynamic crack curving have received considerable attention, e.g., see Yoffe (1951), Cotterell (1966), Cotterell and Rice (1980), Ramulu and Kobayashi (1983), Freund (1990), Gao (1993), and Marder and Gross (1994). The common assumption made by many authors is that the crack grows in the direction of the maximum hoop stress at the crack tip. Cotterell (1966) further stated that the crack is stable if the deviated crack tip will return to the original path. Here we supplement these two by postulating that for a directionally stable crack path, the absolute value of the angle of deviation of the crack should never increase. The critical speed for crack curving predicted by this criterion is found to agree well with the test data.

We consider a crack that has deviated by an infinitesimal angle, θ_0, from the original horizontal direction of propagation, as shown in Fig. 1. We assume that further extension of the deviated crack will be along a direction inclined clockwise at an angle θ^* to its current path. Cotterell (1966) and Cotterell and Rice (1980) have derived the following relationships for a quasi-static crack:

$$-\theta^* = \beta\theta_0 \text{ with } \beta = 1 - \frac{8T}{K_I}\sqrt{\frac{dL}{2\pi}}$$

$$\text{and } \frac{K_{II}}{K_I} = \left(\frac{1}{2} - \frac{4T}{K_I}\sqrt{\frac{dL}{2\pi}}\right)\theta_0 \quad (1)$$

where dL is the length of the deviated crack path, T the remote transverse loading, and K_I and K_{II}, respectively, are

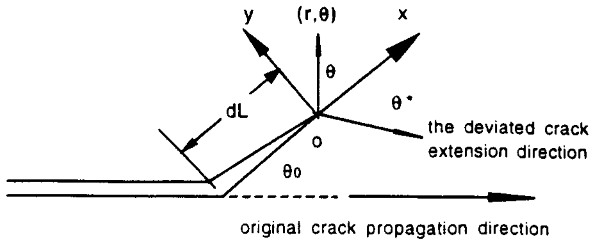

Fig. 1 Deviated semi-infinite propagating crack tip

the Mode-I and Mode-II stress intensity factors at the deviated crack tip. It is readily seen that, if the T-term in Eq. (1) is neglected, then $\beta = 1$ and the crack-path is neutrally stable. This is why the sign of the T-term determines the directional stability of quasi-static crack paths even when the absolute value of this term is significantly less than unity. However, such is not the case for a propagating crack. Experiments (Ramulu and Kobayashi (1983) and Ravi-Chandar and Knauss (1984)) have shown that the Cotterell criterion is not valid for a propagating crack path. It is well known, see, e.g., Williams and Ewing (1972) and Kitagawa et al. (1975), that the large angle of deviation will lead to crack curving. Therefore, we assume that the directional stability should be associated essentially with the decay of the deviation angle. Thus, in addition to Cotterell's stability condition $(-\theta^*/\theta_0) > 1$ we postulate that $(-\theta^*/\theta_0) < 2$ for the stability of the propagating crack. That is, the absolute value of the angle of deviation will never grow for a directionally stable crack-path and, therefore, the absolute value of the new deviation angle must be less than the initial deviation angle $|\theta_0|$. This certainly holds for a quasi-static crack.

Following Freund (1990), Ramulu et al. (1982) and Xu and Keer (1992), and using the notations of Ramulu et al. (1982) in the coordinate frame attached to the crack-tip as shown in Fig. 1, the hoop stress $\sigma_{\theta\theta}$ at the crack tip for infinitesimal deviations of the crack path can be expressed as

$$\sigma_{\theta\theta} = \frac{K_I B_I}{\sqrt{2\pi r}}\left\{\frac{C}{2}\theta^2 + \frac{4S_1S_2}{\gamma} - \gamma\right\} - \frac{K_{II}B_{II}\theta}{2S_2\sqrt{2\pi r}}B + T\theta^2;$$

$$\gamma = \left(1 + S_2^2\right). \quad (2)$$

Here B_I, B_{II}, S_1, and S_2 are functions of the crack-tip speed c, the dilatational wave speed c_1, shear wave speed c_2 and the Rayleigh wave speed c_R, $B = \gamma(S_2^2 - S_1S_2) + 8S_1S_2 - 2\gamma^2$ and $C = 4 - (14 + 3S_2^2)S_1S_2/\gamma + 4S_1S_2 + \gamma(3S_1^2 - 2)/4$. The crack is assumed to extend in the direction for which the hoop stress $\sigma_{\theta\theta}$ attains its maximum value. Thus we determine angle θ^* by $\sigma_{\theta\theta,\theta} = 0$ with $\sigma_{\theta\theta,\theta\theta} < 0$, and $1 < (-\theta^*/\theta_0) < 2$. The result is

$$1 < \frac{-\theta^*}{\theta_0} = -\frac{(m/\theta_0)B_{II}B}{2S_2\left[B_IC + 2\sqrt{2\pi r_c}\,T/K_I\right]} < 2 \quad (3)$$

where r_c is a certain critical distance (Williams and Ewing (1972) and Ramulu and Kobayashi (1983)) and $m = K_{II}/K_I$.

For the sake of simplicity, we consider the case for which the effect of T-related term is less important and therefore set $r_c = 0$. Thus, according to Cotterell (1966) for the quasi-static case, and Ma and Burgers (1987) for the dynamic case, we have $m = 1/2\theta_0 + O(\theta_0\sqrt{dL})$ for the quasi-static case, and $\alpha = m/\theta_0 > 1/2$ for the dynamic case.

The numerical analysis shows that critical values for conditions $(-\theta^*/\theta_0) > 1$, $\sigma_{\theta\theta,\theta\theta} < 0$, and $C(c) = 0$, coincide with each other and are significantly higher than the experimental value of $c \approx 0.5c_R$. For example, for $\nu = 0.25$ and $1/3$, the critical values are $(c/c_2) = 0.63$ and 0.67, respectively. However, it is found that the critical value given by $(-\theta^*/\theta_0) < 2$

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Table 1 Value of α for $\theta_0 = 0.196$, $\nu = 0.25$ and the examination of the stability condition (3)

c/c_R	0.1	0.3	0.5	0.7
α	0.66	0.82	0.91	1.09
$(-\theta^*/\theta_0) < 2$	valid	valid	violated	violated

Table 2 Dependence of the critical speed c^* on Poisson's ratio ν

Poisson's ratio ν	0	0.10	0.15	0.20	0.25	0.30	1/3	0.35	0.40	0.50
Critical speed c^*/c_R	0.45	0.47	0.48	0.49	0.51	0.53	0.54	0.55	0.57	0.58
c^*/c_2	0.39	0.42	0.43	0.45	0.47	0.49	0.50	0.51	0.53	0.56

is significantly lower than that given by $(-\theta^*/\theta_0) > 1$. In other words, the stability condition $(-\theta^*/\theta_0) < 2$ is first violated as the propagation speed c increases.

To examine it, we must know the value of α. We consider the stress-wave loading. Cotterell and Rice (1980) have shown that the linearized analysis gives a reasonable estimate for $|\theta_0| < 15$ deg, therefore we use values of K_I and K_{II} for dynamic kinking under stress-wave loading with the deviation angle $\theta_0 \approx 0.196 \approx 11.24$ deg and $\nu = 0.25$ given by Burgers (1983) and Ma and Burgers (1987). The corresponding values of α and the result of applying condition $(-\theta^*/\theta_0) < 2$ are listed in Table 1. Therefore, the critical speed c^* satisfies $0.3c_R < c^* < 0.5c_R$ for $\nu = 0.25$ which agrees well with the experimental data for Homalite-100 given in Table 1 of Ravi-Chandar and Knauss (1984). A narrower range of bounds for c^* could not be obtained because of a lack of test data.

We now consider the time-independent constant loading on the crack surface. Results given in Burgers (1983) show that the relations between the static and dynamic SIFs for Mode-I and Mode-II given by Freund (1990) may be used to arrive at the approximate results for these cases within five percent error when $|\theta_0| < \pi/4$. Thus we follow Freund (1990) and arrive at $\alpha = \sqrt{((1 - c/c_1)/(1 - c/c_2))}\,/2 > 1/2$ and evaluate the dependence of the critical speed on Poisson's ratio ν; the results are given in Table 2. These agree well with the experimental data for Glass and Plexiglass listed in Table 1 of Ravi-Chandar and Knauss (1984). The increase in the critical speed with the increase in Poisson's ratio is in qualitative agreement with the results of Xu and Keer (1992); however, the value of the critical speed given here is significantly lower than that given by them for the same value of Poisson's ratio. In conclusion, we have proposed a criterion for determining the directional stability of a propagating crack. Neglecting the effects of T-stress, the critical speed predicted by the proposed stability condition (3) is found to be in good agreement with the experimental value of $0.5c_R$.

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