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INDENTATION OF A VISCOELASTIC RUBBER COVERED ROLL BY A RIGID PLANE SURFACE

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Abstract

Deformations of a viscoelastic rubberlike layer bonded to a rigid cylinder and indented by a rigid plane surface are studied by the finite element method. The constitutive relation assumed for the viscoelastic rubber is that proposed by Boltzman. Some of the assumptions made to simplify the work are that the roll cover is rotating at a uniform angular speed, steady state has reached, the deformations of the rubberlike layer are infinitesimal and the effect of inertia forces is negligible. Results presented include the pressure distribution at the contact surface and the stress distribution near the bond surface.

Introduction

The problems of the smooth rolling contact between a viscoelastic cylinder and a rigid plane, and between a rigid cylinder and a viscoelastic half space have been studied by Hunter [1] and by Morland [2,3]. In each of these studies Boltzman's linear viscoelastic stress-strain relation was used. It was also assumed that plane strain state of deformation prevailed and the effect of inertia forces was neglected. For the problem of the viscoelastic cylinder the maximum principal strain found to occur was of the order of the semicontact angle. Harvey [4] has studied the problem of a linear viscoelastic cylinder rolling on a rigid plane on the assumption that the frictional force between the cylinder and the plane is sufficient to prevent slipping between the two over the entire contact area. Also the effect of inertia force was included in the analysis in an approximate way.

In the present work the viscoelastic cylinder is assumed to have a rigid cylindrical core and contact surfaces are presumed to be smooth. Other assumptions made to simplify the problem are that the steady state has reached, the effect of inertia forces is negligible and that plane strain state of deformation prevails. The possible applications we have in mind are in the paper, plastic and textile industry. We solve the problem by the finite element method and do not make any assumption on the thickness of the roll cover relative to the radius of the cylinder. Also in the numerical solution of the problem actual relaxation moduli, if available, can be used rather easily. We note that the finite element method has been used earlier [5-8] to solve viscoelastic contact problems. We refer the reader to References 5, 6, and 7 for details of the finite element formulation of the viscoelastic boundary value problem. Here we give the governing equations for our problem in the next section and then present and discuss results for a sample problem.

Formulation of the Problem

As shown in Figure 1, we use a fixed set of rectangular Cartesian axes (with origin at the point of contact of the undeformed roll cover and the plate) to describe the deformations of the rubberlike layer. We assume that the cylindrical core and the plate are made of materials considerably harder than rubber and regard these as being rigid. In the absence of body forces such as gravity, the mechanical deformations of the rubberlike layer are governed by

$$\dot{\rho} + \rho \dot{x}_{i,i} = 0, \qquad \sigma_{ij,j} = \rho \ddot{x}_i . \tag{1}$$

In (1) σ_{ij} is the Cauchy stress tensor, ρ is the present mass density, $\underset{\sim}{x}$ is the present position of a material particle that occupied place $\underset{\sim}{X}$ in the reference configuration, a superimposed dot indicates material time differentiation, a comma followed by an index j indicates partial differentiation with respect to x_j, and the usual summation convention is used. Before stating the constitutive relation for the rubber and the side conditions such as boundary conditions accompanying (1) we give below the assumptions made to simplify the problem.

We assume that the rubberlike layer is homogeneous and isotropic, the roll cover rotates at a uniform angular speed Ω, steady state has been reached, contact surfaces are smooth, and the deformations are small so that a constitutive relation linear in displacement gradients applies. Also as is often presumed for viscoelastic materials [5-9], we assume that the bulk behavior of the rubber is elastic. Furthermore, we assume that a plane strain state of deformation prevails and that the inertia effects are negligible. This last assumption appears reasonable since the mass density of rubber is quite low, being comparable to that of water. Under the foregoing assumptions the indices i and j in (1) range over 1,2, the problem becomes 2-dimensional quasistatic and one needs to solve for the $\underset{\sim}{x}$ only since

$$\rho = \rho_o(1 - u_{i,i}), \qquad (2)$$
$$u_i = x_i - X_i.$$

The constitutive relation for the rubber is taken as

$$\sigma_{ij} = \int_{-\infty}^{t} G_1(t - \tau)\,\frac{\partial\varepsilon_{ij}(\underset{\sim}{X},\tau)}{\partial\tau}\,d\tau + \frac{\delta_{ij}}{3}\int_{-\infty}^{t}[G_2 - G_1(t-\tau)\,\frac{\partial\varepsilon_{kk}(\underset{\sim}{X},\tau)}{\partial\tau}\,d\tau, \qquad (3)$$

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2.$$

Here G_1 and G_2 are, respectively, the shear and the bulk moduli of rubber and δ_{ij} is the Kronecker delta. Substitution from (3) into $(1)_2$ yields linear field equations for $\underset{\sim}{u}$ which are to be solved under the following boundary conditions.

At the inner surface, $\underset{\sim}{u} = \underset{\sim}{0}$ and at the outer surface

$$e_i\sigma_{ij}n_j = 0,$$
$$n_i\sigma_{ij}n_j = 0, \qquad |x_1 + c\ell| \geq \ell, \qquad (4)$$
$$x_2 = D, \qquad |x_1 + c\ell| < \ell.$$

Here $\underset{\sim}{n}$ is an outward directed unit normal to the outer surface, $\underset{\sim}{e}$ is an unit tangent vector, D is the depth of indentation, 2ℓ is the contact width and $c\ell$ is the distance between the center of the contact width and the center line of the roll cover. The fact that $\sigma_{ij}n_j = 0$ at $|x_1 + c\ell| = \ell$ implies that $n_i\sigma_{ij}n_j = 0$ and this ensures that the normal stress is continuous across the arc of contact and that a contact problem rather than a punch problem is being solved. We note that of the three constants, c, ℓ and D appearing in (4) only one can be taken to be known and the other two are to be determined as a part of the solution.

In solving the problem by the finite element method we have found it convenient to replace $(4)_3$ by

$$\begin{aligned} n_i \sigma_{ij} n_j &= p(x_1), & |x_1 + c\ell| &\leq \ell, \\ p(x_1) &\to 0 \quad \text{as} & |x_1 + c\ell| &\to \ell. \end{aligned} \tag{5}$$

Here p stands for the normal pressure between the roll and the plane surface Of course p is unknown apriori and is to be determined as a part of the solution. A boundary condition such as (5) was used in References, 6, 7 and 8 wherein the load p was replaced by equivalent normal loads at the nodal points on the contact surface.

We refer the reader to References 5, 6 and 7 for details of the finite element formulation of the problem and how to solve the resulting system of equations. As is done in these references, we presume 2ℓ and iterate on the estimated pressure profile until the deformed surface of the roll cover matches with the profile of the indentor to within a prespecified tolerance. The value of c is then found graphically from the plot of pressure distribution on the contact surface. In practice, the total load P given by

$$P = \int_{|x_1 + c\ell| \leq \ell} p \, dx_1$$

is prescribed. Taking P as known and finding c, ℓ and D that will satisfy all the pertinent equations though feasible takes considerable amount of computing time.

Results for a Sample Problem

The finite element program developed by Batra [7] was suitably modified to solve the present contact problem. We add that unlike a linear elastic problem the stiffness matrix in the resulting system of equations for the viscoelastic problem has a very large band width and is asymmetric. Whereas Batra [7] studied a thermoviscoelastic problem we study here a homothermal problem. That is, the temperature is uniform throughout the rubberlike layer and it stays the same for all times.

In order to solve the problem by the finite element method, the region of the rubberlike layer lying within six times the estimated arc of contact is divided into subregions. Fig. 2 depicts the region studied and its subdivision in the reference configuration. The region considered is sufficient since the stresses have been shown to decay rather rapidly in somewhat similar problems studied earlier [5-8]. The end faces of the region shown in Fig. 2 are presumed to be traction free.

In the results presented herein for a sample problem we have taken the following values of various geometric and material parameters.

$$G_1 = 150(1 + e^{-t/.2})\ \text{psi},\ G_2 = 20{,}000\ \text{psi}$$

$$\Omega = .244\ \text{rad/sec.},$$

$$R_0 = 10.25\ \text{in.},\ R_i = 10.05\ \text{in.},\ D = 4.59 \times 10^{-3}\ \text{in.}$$

For the elastic problem the value of G_1 is taken as 300 psi. These values of G_1 and G_2 correspond to nearly incompressible rubbers. The indented surface is assumed to conform to the plane profile of the indentor if, in the deformed position, each nodal point lies within .01D of the plane surface. The normal loads on the presumed contact surface are iterated until such is the case.

Figure 3 shows the pressure distribution at the contact surface for the elastic and the viscoelastic roll covers. As expected the pressure distribution for the viscoelastic roll cover is asymmetric. For the same value of the indentation, the peak pressure and the total load for the elastic problem are, respectively, 1.57 and 1.65 of that for the viscoelastic roll cover. Also the contact width, as found from Fig. 3, for the elastic problem is 7.5 percent more than that for the viscoelastic problem. The value of the parameter c appearing in equation (4), also determined from this figure, is .023 for the viscoelastic problem. Note that c equals zero for the elastic problem.

When the arc of semicontact width is .03 radians, the maximum principal strain found to occur was .034 and the maximum shear strain to be .068. This seems to agree qualitatively with that obtained by Hunter [1] who studied the rolling contact between a homogeneous viscoelastic cylinder and a rigid plane.

In Fig. 4 is plotted the variation of $-\sigma_{22}$ at the center of the outermost layer of the roll cover with the distance from the center line. The point where $-\sigma_{22}$ is maximum for the viscoelastic problem has shifted to the left of the point where peak pressure occured at the contact surface. The ratio of the maximum values of $-\sigma_{22}$ for the elastic roll cover to that for the viscoelastic roll cover is 1.56 which is essentially the same as that for the peak pressures on the outermost surface. Also the plot of $-\sigma_{22}$ at the center of the layer glued to the cylindrical core (Fig. 5) seems to indicate that this ratio of the maximum values of $-\sigma_{22}$ for the elastic and visco-elastic problems remains virtually unchanged through the thickness of the roll cover. The results plotted in Figures 4 and 5 support the assumption that stresses decay rapidly with the distance from the center line.

Acknowledgement

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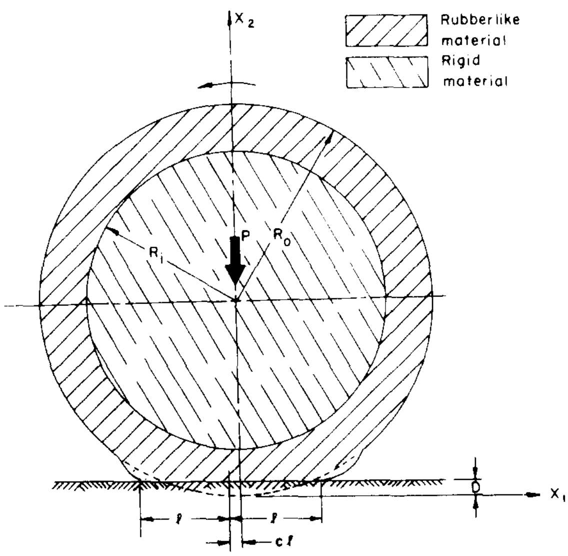

Fig. 1
System to be Studied

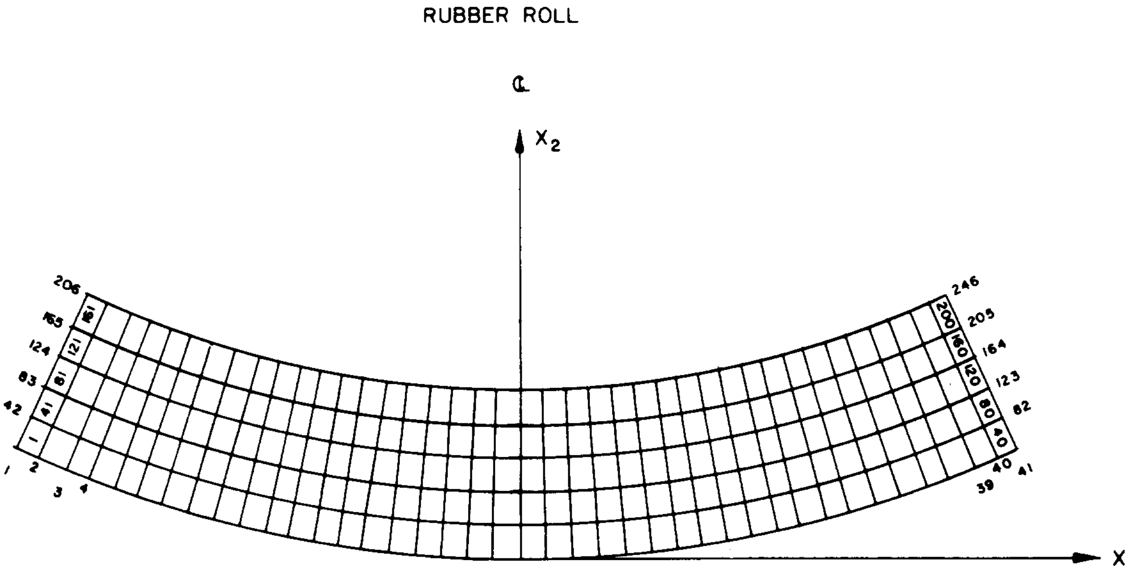

Fig. 2 The Finite Element Grid

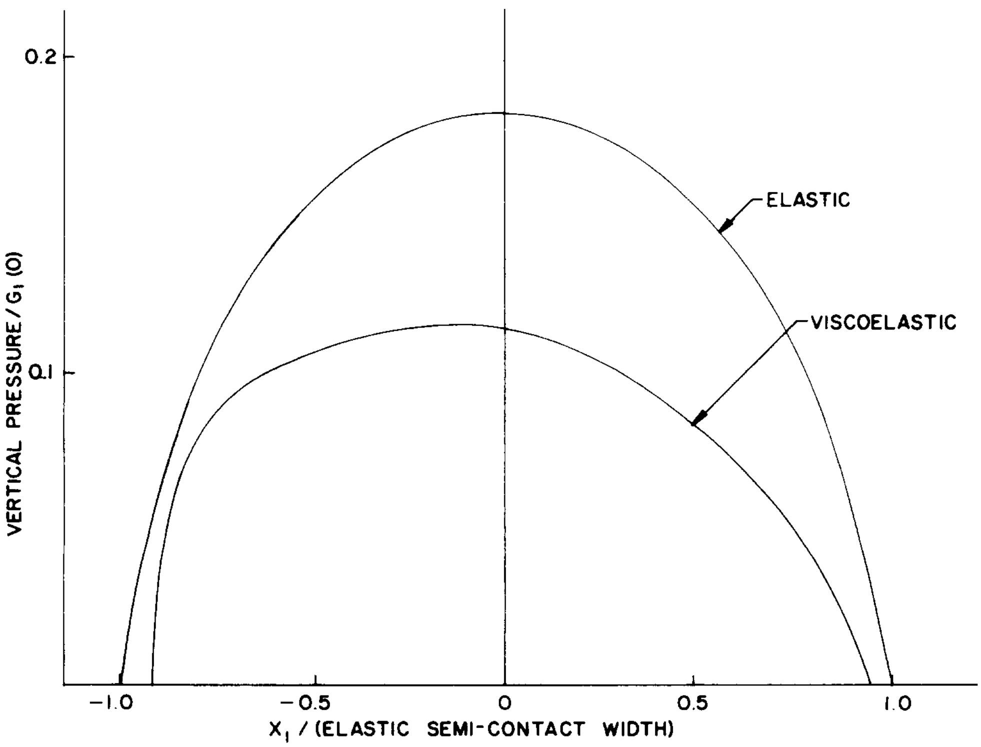

Fig. 3 Pressure Distribution at the Contact Surface

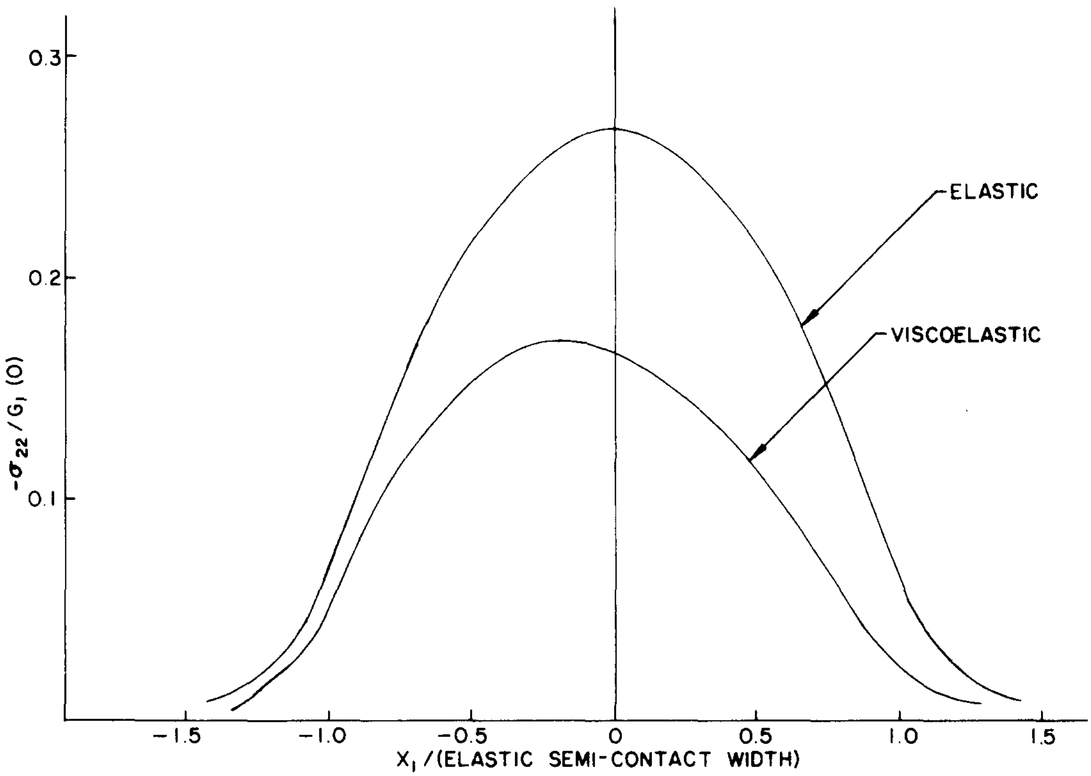

Fig. 4 Distribution of $-\sigma_{22}$ near the Contact Surface

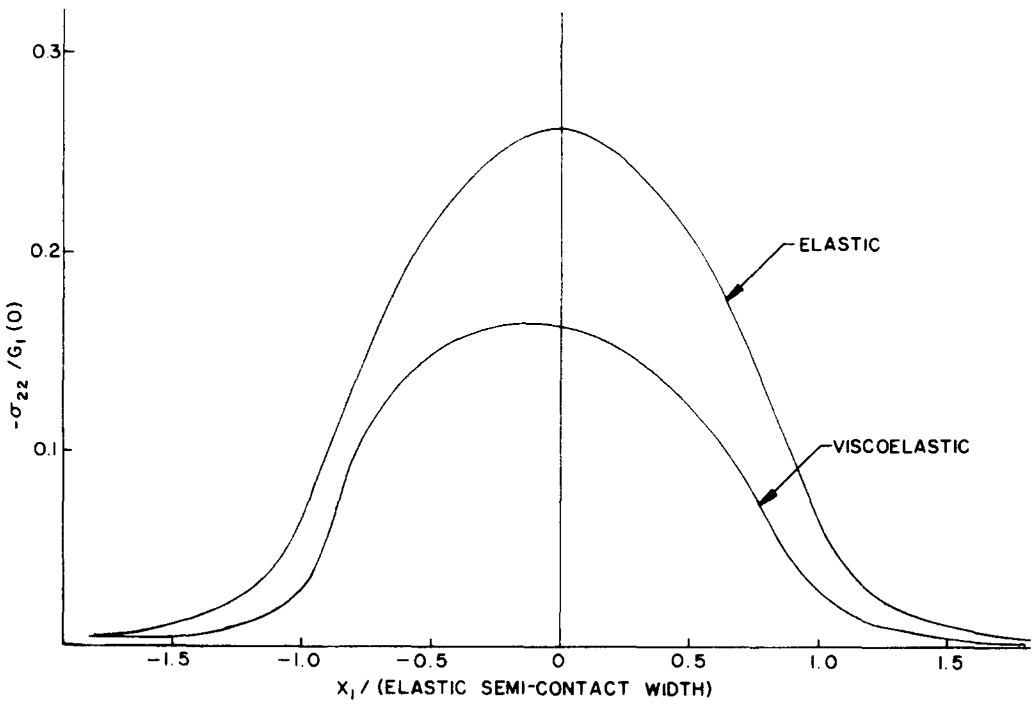

Fig. 5 Distribution of $-\sigma_{22}$ near the Bond Surface