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# Blast resistance of unidirectional fiber reinforced composites

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Dedicated to Professor Liviu Librescu who on 16 April 2007 sacrificed his life in order to save that of students in his class during the tragedy at Virginia Tech.

### Abstract

Three-dimensional transient deformations of a laminate comprised of several unidirectional fiber reinforced layers perfectly bonded to each other and subjected to a blast load are analyzed by the finite element method with an in-house developed, verified and fully validated code with rate-dependent damage evolution equations for anisotropic bodies. The continuum damage mechanics approach employing three internal variables is used to characterize damage due to fiber/matrix debonding, fiber breakage, and matrix cracking. The delamination between two adjoining layers is assumed to ensue when the stress state there satisfies a failure criterion, and may initiate simultaneously at several points. The relative sliding between adjoining layers is simulated by the nodal release technique. The interaction among fiber/matrix debonding, fiber breakage, matrix cracking and delamination, and the possibility of their initiating concurrently at one or more points in the composite are considered. The effect of different material, geometric and loading parameters on the damage development and propagation, and the energy absorbed in each one of the four failure modes have been examined. These results give preliminary information on composite structure's design for maximizing the energy absorption and hence increasing structure's resistance to blast loads. The paper is a sequel to Hassan and Batra's paper [Hassan NM, Batra RC. Modeling damage development in polymeric composites. Composites B, doi:10.1016/j.compositesb.2007.02.001] wherein details of the damage evolution equations, verification of the code, and the validation of the mathematical model are given.

Keywords: A. Polymer-matrix composites (PMCs); B. Impact behavior; C. Damage mechanics; C. Finite element analysis (FEA); Figure of merit

# 1. Introduction

The damage caused by severe shock loads to ship structures and marine vessels is of considerable interest to engineers working in defense and civilian industries [1,2]. In an attempt to enhance the operational efficiency and reduce the life time cost of these structures, more emphasis is being placed on designing as many components as possible of composites. Shock loads are induced by the underwater explosion of a mine or a torpedo, ignition of an explosive device in air, weapons effects, the structure striking a partially submerged object in water, and/or the slamming pressure that occurs at high sea states when the forefront of the vessel rises above the water surface and then rapidly reenters the water. These shock waves generally generate impulses of very high pressures but short durations, resulting in extremely high strain rates, which may cause severe structural damage. Loads considered here are typically induced by the ignition in air of an explosive located at a known distance from the composite structure.

The estimate of service life of a structural component requires knowledge of the progressive degradation of its strength due to growth of the internal damage that determines the energy absorbed by the structure. Factors influencing damage induced in a composite include fiber type, fabric and composite construction, thermomechanical

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properties of constituents, anisotropy, rate sensitivity, and the interaction among its constituents. The initiation and propagation of damage due to blast loads has been studied experimentally, analytically and numerically. Tests are generally performed by subjecting large composite panels (up to  $3 \text{ m} \times 3 \text{ m}$  in size) or full scale sections of a ship to increasing levels of shock loads and ascertaining in the laminate the extent of fiber breakage, matrix cracking, fiber/matrix debonding, and delamination. Mouritz [3] used the four-point bend test to measure the residual flexural strength of a glass reinforced polymer (GRP) laminate after it had been loaded by an underwater shock wave. The examination by a scanning electron microscope of the laminate after being exposed to a shock pressure of 8 MPa revealed damage in the form of matrix cracking and a small number of short delaminations; consequently, the flexural strength remained essentially unchanged. However, shock pressures exceeding 8 MPa caused severe cracking of the polymer and breakage of fibers on the back surface, buckling of fibers near the impacted surface, and large delamination zones. According to Will et al. [4], in high speed impacts a structure deforms locally and a little energy is used to deform fibers and the structure but a significant amount of energy is dissipated in mechanisms such as delamination, debonding and fiber pull-out. In [5] we have reviewed the literature and summarized known results for the effect of target thickness, fiber and matrix properties, fiber length and fiber volume fraction, fiber geometry, fiber/matrix interface properties, ply stacking sequence, stitching of layers, shock strength, and boundary conditions. Most of the works reviewed in [5] delineated how the change in one parameter affected the damage induced in a composite laminate and employed either numerical or experimental techniques. Here we use the mathematical tool and the computational algorithm developed, verified and validated in [6] to delineate the effect of various material and geometric parameters, different types of explosives, core materials for sandwich construction, composite laminates bonded to a steel plate, and boundary conditions on the blast resistance of the AS4/ PEEK composite laminate. This composite is studied because test data, from which values of material parameters can be deduced, are available [10]. This parametric study complements that reported in [5], and the two together provide a comprehensive data for the transient response of a composite laminate to blast loads.

# 2. Problem formulation

We describe lamina's deformations by using rectangular Cartesian coordinates with the  $X_1$ -axis aligned along fibers, and  $X_2$ - and  $X_3$ -axes perpendicular to fibers; these are usually called material principal directions. Deformations are governed by the balance of mass, the balance of linear momentum, the balance of moment of momentum, and the balance of internal energy supplemented by constitutive relations, damage evolution equations, initial conditions, and boundary conditions. We describe damage evolution in rate-dependent bodies with three internal variables denoted by  $\boldsymbol{\xi} = \{\phi^m, \phi^f, \phi^d\}$  and represent the corresponding conjugate forces by  $\boldsymbol{\omega} = \{Y^m, Y^f, Y^d\}$ .

We assume that the laminate is initially stress free and adopt the following constitutive relations: S = C + cF

$$\omega_{(i)} = Y^{(i)} = -\frac{1}{2} \frac{\partial C_{\alpha\beta\gamma\delta}}{\partial \phi^{(i)}} E_{\alpha\beta} E_{\gamma\delta}, \quad i = m, f, d,$$
<sup>(1)</sup>

for the second Piola-Kirchhoff stress tensor S and the conjugate force  $\omega$ . Here *E* denotes the Green-St. Venant strain tensor, and the elastic constants  $C_{\alpha\beta\gamma\delta} = C_{\gamma\delta\alpha\beta} = C_{\beta\alpha\gamma\delta}$  are functions of  $\xi$ . Eq. (1)<sub>1</sub> accounts for geometric nonlinearities and describes a neo-Hookean material. We assume that a lamina reinforced with unidirectional fibers can be modeled as transversely isotropic with the axis of transverse isotropy perpendicular to the plane of the lamina. Thus there are five independent elastic constants  $C_{\alpha\beta\gamma\delta}$  for each lamina. In terms of the more familiar elastic constants, components of matrix [C] are given in Refs. [7–9], and their values in terms of those of the matrix and the fiber, and their volume fractions derived by using the mechanics of materials approach are listed in [6]. These relations imply that elastic constants of the lamina degrade with the evolution of damage. Even though a damage variable affects only one of the elastic moduli in the material principal directions, it influences all moduli of the lamina when global axes do not coincide with the material principal directions. The matrix of elastic constants, the second Piola-Kirchhoff stress tensor, and the Green-St. Venant strain tensor are transformed to global coordinate axes by using tensor transformation rules [7–9].

### 2.1. Damage evolution equations

We postulate that  $\phi^f = \phi^f (Y^f)$ ,  $\phi^m = \phi^m(Y^m)$ ,  $\phi^d = \phi^d(Y^d)$ , and determine the functional dependence from the test data by assuming that the damage at a material point does not increase while the material there is unloading as indicated by a decrease in a suitable scalar measure of stresses and/or strains. For the AS4/PEEK with the volume fraction  $V^f$  of isotropic fibers equal to 0.6, values of material parameters of the fibers and the isotropic matrix taken from Kyriakides et al. [10] and used in this work are given in Table 1 of [5]. The least squares fit to the test data of Kyriakides et al. [10] resulted in the following expressions (see [6] for details).

$$\phi^f = A_f (1 - e^{(-B_f Y^f)}), \tag{2}$$

$$\phi^{m} = \left(\frac{A_{m}B_{m} + C_{m}(Y^{m})^{D_{m}}}{B_{m} + (Y^{m})^{D_{m}}}\right),\tag{3}$$

$$\phi^{d} = \left(\frac{A_{d}B_{d} + C_{d}(Y^{d})^{D_{d}}}{B_{d} + (Y^{d})^{D_{d}}}\right).$$
(4)

Values of constants  $A_f$ ,  $B_f$ ,  $A_m$ ,  $B_m$ ,  $C_m$ ,  $D_m$ ,  $A_d$ ,  $B_d$ ,  $C_d$ , and  $D_d$  are listed in Table 2 of [5]. Constants in Eq. (2) expressing damage due to fiber breakage have different values for fibers loaded in tension and compression. When fibers are not

aligned with a global coordinate axis, we analyze the problem in the global coordinate system and compute the axial strain in the fiber by using the tensor transformation rules. The sign of the axial strain in a fiber dictates which values of  $A_f$ ,  $B_f$  and  $Y^f$  to use in Eq. (2). We follow a similar procedure for selecting appropriate values of  $A_m$ ,  $B_m$ ,  $C_m$ ,  $D_m$  and  $Y^m$  in Eq. (3) and of  $A_d$ ,  $B_d$ ,  $C_d$ ,  $D_d$  and  $Y^d$  in Eq. (4). Even though  $\phi^f$ ,  $\phi^m$  and  $\phi^d$  depend only upon  $Y^f$ ,  $Y^m$  and  $Y^d$ respectively, under a general loading, one damage mode may influence the other two damage modes, and also the delamination discussed below. It is because a change in  $\phi^f$ ,  $\phi^m$  and  $\phi^d$  affects all elastic moduli of the laminate with fibers in at least one layer not oriented along the  $X_1$ -axis. Thus interaction among the four failure modes is implicitly considered. Also, at a given instant, different failure modes may ensue simultaneously at one or more points in the body.

### 2.2. Failure criterion

It is assumed that the failure due to fiber breakage, matrix cracking and fiber/matrix debonding occurs when  $Y^f$ ,  $Y^m$  and  $Y^d$  reach their critical values  $Y^f_{crit}$ ,  $Y^m_{crit}$  and  $Y^d_{crit}$  respectively. Values of  $Y^f_{crit}$ ,  $Y^m_{crit}$  and  $Y^d_{crit}$  depend upon materials of the fiber and the matrix, sizing or functionalizing of fibers, and possibly on the fabrication process; these are to be determined from the experimental data. Values for the AS4/PEEK composite found from the test data [10] and used herein are listed in Table 2 of [5]. We refer the reader to [6] for details of simulating delamination between two adjoining layers.

# 2.3. Strain rate effect

As discussed in [6], the experimental data for the AS4/ PEEK reported in [10] at different strain rates suggests the following functional dependence of conjugate variables  $Y^m$  and  $Y^d$  upon  $\dot{E}_{22}$  and  $\dot{E}_{12}$  respectively:

$$Y^{m} = \left(\frac{B_{m}\left(-A_{m} + \phi^{m}\left(1 - s^{m}\log_{10}\left(\frac{\dot{E}_{22}}{E_{22}^{0}}\right)\right)\right)}{C_{m} - \phi^{m}\left(1 - s^{m}\log_{10}\left(\frac{\dot{E}_{22}}{E_{22}^{0}}\right)\right)}\right)^{1/D_{m}},$$
(5)

$$Y^{d} = \left(\frac{B_{d}\left(-A_{d} + \phi^{d}\left(1 - s^{d}\log_{10}\left(\frac{\dot{E}_{12}}{E_{12}^{0}}\right)\right)\right)}{C_{d} - \phi^{d}\left(1 - s^{d}\log_{10}\left(\frac{\dot{E}_{12}}{E_{12}^{0}}\right)\right)}\right)^{1/D_{d}}.$$
(6)

Here  $\dot{E}_{22}^0$  and  $\dot{E}_{12}^0$  represent, respectively, values of the reference transverse and the reference shear strain rates.  $Y^{f}$  is assumed to be independent of strain rate because the experimental stress–strain curve for AS4/PEEK in longitudinal tension and compression is insensitive to the axial strain rate.

#### 2.4. Governing equations

Substitution from Eq. (1) into the balance of linear momentum gives the following nonlinear field equation for the determination of the displacement **u**.

$$\rho_R \ddot{u}_i = \left[ (\delta_{i\alpha} + u_{i,\alpha}) (C_{\alpha\beta\gamma\delta} E_{\gamma\delta}) \right]_{,\beta} + \rho_R b_i, 
u_{i,\alpha} = \partial u_i / \partial X_{\alpha}.$$
(7)

Let  $\Omega$  be the region occupied by the body in the reference configuration at time t = 0. A general form of initial and boundary conditions is

$$\begin{aligned} x_{i}(\mathbf{X}, 0) &= X_{\alpha} \delta_{i\alpha}, \\ \dot{x}_{i}(\mathbf{X}, 0) &= v_{i}^{0}(\mathbf{X}), \\ x_{i}(\mathbf{X}, t) &= \bar{x}_{i}(\mathbf{X}, t), \quad \mathbf{X} \in \partial_{X} \Omega, \quad t \in (0, \widetilde{T}), \\ T_{i\alpha}(\mathbf{X}, t) N_{\alpha}(\mathbf{X}) &= f_{i}(\mathbf{X}, t), \quad \mathbf{X} \in \partial_{f} \Omega, \quad t \in (0, \widetilde{T}). \end{aligned}$$

$$\tag{8}$$

Here  $\partial_X \Omega$  and  $\partial_f \Omega$  are parts of the boundary  $\partial\Omega$  of  $\Omega$  where final positions (or equivalently, displacements) and surface tractions are prescribed, respectively, as **X** and **f**, and **T** is the first Piola–Kirchhoff stress tensor. Note that  $\partial_X \Omega$  and  $\partial_f \Omega$  need not be disjoint since linearly independent components of displacements and surface tractions may be specified at the same point on  $\partial\Omega$ . However, for the sake of simplicity, they are assumed to be disjoint in Eq. (8). Initial values of internal variables representing the fiber breakage, fiber/matrix debonding, and matrix cracking are taken to be zeros.

We note that the elastic constants matrix C in Eq. (7) depends upon the damage variables. Thus as the body is deformed and damage is induced, the elastic constants change.

# 3. Numerical solution

We have developed a 3-D finite element (FE) code in Fortran using 8-node brick elements to solve numerically the aforestated initial-boundary-value problem. The code has been verified by using the method of fictitious body forces described in [19], see comments following eqn. (30) of [19]. It has been validated by showing that results computed from it for different initial-boundary-value problems match well with test findings [6]. Values of material parameters are found from one set of experiments, and results computed for a totally different set of loadings and by different investigators are compared with experimental results. After having found nodal displacements, values of conjugate variables and damage parameters (or internal variables)  $\phi^f$ ,  $\phi^m$  and  $\phi^d$  at each integration point are determined, and are used to account for the degradation in the material strength by updating values of elastic constants.

When an internal variable  $\phi^f$ ,  $\phi^m$  and/or  $\phi^d$  equals 1.0 or the corresponding conjugate variable  $Y^f$ ,  $Y^m$  and/or  $Y^d$ equals its critical value at an integration point, the material there is taken to have failed due to fiber breakage, matrix cracking and/or fiber/matrix debonding respectively. Even if the material at all eight integration points within an element has failed, that element is not removed from the analysis. Once all elastic constants at each one of the eight integration points in an element have been reduced to zero, all stress components in that element will subsequently be zero, and for all practical purposes that element will represent a hole or a void. However, the mass of the element still contributes to the kinetic energy of the body. The relative sliding between adjoining delaminated layers is simulated by the nodal release technique.

In order to assess the structure's resistance to impact loads, the following quantities are computed by using Eqs. (9)–(15): work  $W^{\text{ef}}$  done by applied loads; energies  $E^{\text{fb}}$ ,  $E^{\text{mc}}$ ,  $E^{\text{db}}$  and  $E^{\text{dl}}$  dissipated, respectively, due to fiber breakage, matrix cracking, fiber/matrix debonding, and delamination; work  $W^{\text{def}}$  done to deform the body, and the kinetic energy  $K_{\text{e}}^{\text{t}}$  of the body at the terminal value of the time *t*.

$$W^{\rm ef} = \int_0^t \sum_n F_3^n \frac{\mathrm{d}u_3^n}{\mathrm{d}t} \,\mathrm{d}t,\tag{9}$$

$$E^{\rm fb} = \int_0^t \int_{\Omega} Y^f \frac{\mathrm{d}\phi^f}{\mathrm{d}t} \,\mathrm{d}V \,\mathrm{d}t = \int_0^{\phi f} \int_{\Omega} Y^f \,\mathrm{d}\phi^f \,\mathrm{d}V, \tag{10}$$

$$E^{\rm mc} = \int_0^t \int_{\Omega} Y^m \frac{\mathrm{d}\phi^m}{\mathrm{d}t} \,\mathrm{d}V \,\mathrm{d}t = \int_0^{\phi^m} \int_{\Omega} Y^m \,\mathrm{d}\phi^m \,\mathrm{d}V, \tag{11}$$

$$E^{\rm db} = \int_0^t \int_{\Omega} Y^d \frac{\mathrm{d}\phi^d}{\mathrm{d}t} \,\mathrm{d}V \,\mathrm{d}t = \int_0^{\phi^d} \int_{\Omega} Y^d \,\mathrm{d}\phi^d \,\mathrm{d}V, \tag{12}$$

$$W^{\text{def}} = \int_0^t \int_\Omega S_{\alpha\beta} \frac{\mathrm{d}E_{\alpha\beta}}{\mathrm{d}t} \,\mathrm{d}V \,\mathrm{d}t, \tag{13}$$

$$K_{\rm e}^{\rm t} = \int_{\Omega} \frac{\rho}{2} (v_x^2 + v_y^2 + v_z^2) \,\mathrm{d}V, \tag{14}$$

$$E^{\rm dl} = W^{\rm ef} - (W^{\rm def} + K_{\rm e}^{\rm t} + E^{\rm fb} + E^{\rm mc} + E^{\rm db}).$$
(15)

The summation in Eq. (9) is over all nodes on the top surface of the laminate where pressure is applied. Eq. (15) follows from the balance of energy.

### 4. Results and discussion

The mathematical model described above is applied to the problem schematically shown in Fig. 1. A 220 mm × 220 mm × 10 mm AS4/PEEK composite panel has the fiber volume fraction  $V^{\rm f}$  equal to 0.6. These dimensions equal those of the test specimen of Turkmen and Mecitolu [11] who studied the dynamic response of a laminated composite subjected to air blast loads, and are of the same order of magnitude as those employed in other tests [1,12,13]. Values of material parameters for the fiber and the matrix



Fig. 1. Schematic sketch of the problem studied.



Fig. 2. Discretization of the structure into a FE mesh of  $20 \times 20 \times 4$  elements.

and those of parameters in damage relations (2)–(4) are given in Tables 1 and 2 of [5]. The 4-ply panel, clamped at all four edges, is divided into 8-node brick elements with finer elements in the central portion. A FE discretization of the domain, exhibited in Fig. 2, has smaller elements near the center of the panel and coarser elements elsewhere. The load due to air blast explosion is simulated by applying a time-dependent pressure field P(r, t) on the top surface of the specimen; the exponential decay of the peak pressure at r = 0 with time t, and at any instant its spatial variation are



Fig. 3a. Variation at r = 0 of the pressure with time for 64 kg TNT ignited at a distance of 10 m from the target.



Fig. 3b. For a fixed value of time *t*, pressure distribution over specimen's top surface.

exhibited in Figs. 3a and b. Here r equals the distance of a point from the center of the plate's top surface. The peak pressure and its decay rate depend upon the charge weight, charge type, and the distance of the point of detonation from the target [14,15]. The spatial variation of P derived from the experimental work of Turkmen and Mecitolu [11] is taken to be

$$P(r,t) = (-0.0005r^4 + 0.01r^3 - 0.0586r^2 - 0.001r + 1)P(0,t),$$
(16)

where r is the distance, in cm, from the specimen center. The load considered here, unless otherwise specified, is due to 64 kg of TNT detonated in air at a distance of 10 m from the target plate.

Effect of mesh size: For one loading, we used the following four FE meshes:  $20 \times 20 \times 4$  (1600 elements, 2205 nodes),  $20 \times 20 \times 8$  (3200 elements, 3969 nodes),  $40 \times 40 \times 10^{-10}$ 4 (6400 elements, 8405 nodes), and  $40 \times 40 \times 8$  (12,800 elements, 15,129 nodes), with each mesh being fine in the central portion of the specimen; e.g. see Fig. 2. Results were computed until 220 µs. The maximum tensile and compressive principal stresses found with the four meshes differed at most by 11% and 13.7% respectively. Values of  $W^{def}$  calculated using Eq. (13) were found to be 378.13, 408.19, 397.91 and 405.15 J and differ at most by 7%. These numbers give an estimate of the error in results presented and discussed below with the  $20 \times 20 \times 4$  FE mesh. Because of the fewer computational resources needed for analysis with this mesh, one can quickly find variables to which the impact damage is most sensitive. If desired, subsequent high fidelity computations can be performed with successively finer meshes to improve the design of damage resistant panels.

In [5] we focused on delineating effects of fiber orientation on the damage induced in the laminate. Here we study how the lay-up sequence, target thickness, elastic moduli of the fiber and the matrix, fiber volume fraction, boundary conditions, sandwich construction, hybrid structures, and explosive type influence damage induced in the composite laminate.

### 4.1. Lay-up sequence

We first analyze deformations of two-ply, and then fourply laminates. For the two-ply laminated composite, we have studied the following eight lay-up sequences:  $0^{\circ}/90^{\circ}$ ,  $90^{\circ}/0^{\circ}$ ,  $45^{\circ}/-45^{\circ}$ ,  $-45^{\circ}/45^{\circ}$ ,  $0^{\circ}/45^{\circ}$ ,  $45^{\circ}/0^{\circ}$ ,  $90^{\circ}/45^{\circ}$ , and  $45^{\circ}/90^{\circ}$ ; these are called lay-ups 1 through 8, respectively, in the following discussion.

### 4.1.1. Two-ply laminates

For the eight stacking sequences, Fig. 4a shows the energy dissipated in the four failure modes; it is evident that for each lay-up of the two plies most of the input energy due to work done by applied loads is dissipated in delaminating the two layers, and the least in breaking the fibers. The  $45^{\circ}/0^{\circ}$  sequence has the most energy dissipated due to delamination, and the energy dissipated due to matrix cracking is essentially the same for the  $0^{\circ}/90^{\circ}$ ,  $90^{\circ}/0^{\circ}$ ,  $45^{\circ}/-45^{\circ}$  and  $-45^{\circ}/45^{\circ}$  laminates. However, the energy dissipated due to matrix cracking in the  $0^{\circ}/45^{\circ}$ ,  $45^{\circ}/0^{\circ}$ ,  $90^{\circ}/45^{\circ}$  and  $45^{\circ}/90^{\circ}$  laminates is nearly 20% of that in the other four laminates studied.

For the eight lay-ups of plies, we have plotted bar charts in Fig. 4b for the  $W^{\text{ef}}$ ,  $W^{\text{def}}$  and the  $K_{\text{e}}^{\text{t}}$  at the terminal value 220 µs of the time *t*. The  $W^{\text{ef}}$  in each one of the first four



Fig. 4a. Energy dissipated in the four failure modes for different stack up sequences.



Fig. 4b. Total work done, strain energy, and kinetic energy for different stack up sequences.



Fig. 4c. Evolution of energy dissipation due to delamination for [45/90] stack-up sequence.

lay-ups is essentially the same, and is a little higher than that in each one of the last four sequences. The W<sup>def</sup> for the first two lay-up of plies is slightly higher than that for the remaining six lay-ups, and the  $K_e^t$  for the  $45^\circ/-45^\circ$  and the  $-45^\circ/45^\circ$  laminates is a little higher than that for the other six lay-ups of plies. Fig. 4c depicts time history of the  $E^{dl}$  for the  $45^\circ/90^\circ$  laminate. After the initial parabolic rise, the energy dissipated increases essentially linearly.

### 4.1.2. Four-ply laminates

For the four-ply laminates, we examined the following nine sequences:  $[0^{\circ}/45^{\circ}/90^{\circ}-45^{\circ}]$ ,  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$ ,  $[90^{\circ}/-45^{\circ}/0^{\circ}/45^{\circ}]$ ,  $[45^{\circ}/90^{\circ}-45^{\circ}/0^{\circ}]$ ,  $[0^{\circ}/45^{\circ}/-45^{\circ}/90^{\circ}]$ ,  $[0^{\circ}/90^{\circ}/-45^{\circ}/45^{\circ}]$ ,  $[0^{\circ}/-45^{\circ}/45^{\circ}/90^{\circ}]$  and  $[0^{\circ}/-45^{\circ}/90^{\circ}/45^{\circ}]$ ; fiber orientations in plies are given in going from the top layer to the bottom layer. Time histories of the deflection of the specimen centroid, not included herein but given in [16], indicated that deflections for the  $[0^{\circ}/90^{\circ}/45^{\circ}/-45^{\circ}]$  and  $[0^{\circ}/90^{\circ}/-45^{\circ}/45^{\circ}]$  laminates are

nearly the same, and they are smaller than those for the other seven lay-ups. The centroidal deflections for the  $[0^{\circ}/45^{\circ}/-45^{\circ}/90^{\circ}]$  and  $[0^{\circ}/-45^{\circ}/45^{\circ}/90^{\circ}]$  laminates are larger than those for the remaining seven laminates.

From plots of Fig. 5a, we deduce that the  $E^{dl}$  is maximum for the  $[0^{\circ}/45^{\circ}/90^{\circ}/-45^{\circ}]$  laminate, and for this laminate it is significantly more than  $E^{fb}$ ,  $E^{mc}$  and  $E^{db}$ . However, for the  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$  laminate, the  $E^{dl}$  is negligible as compared to that in other modes of failure, and is the least of the  $E^{dl's}$  for the nine laminates. The  $E^{mc}$  is very high for the  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$  and  $[45^{\circ}/90^{\circ}/-45^{\circ}/90^{\circ}]$  laminates; it is comparable to the  $E^{dl}$  for several other laminates.

For the nine laminates, we have plotted in Fig. 5b the  $W^{\text{ef}}$ ,  $W^{\text{def}}$ , and the  $K_{\text{e}}^{\text{t}}$  at  $t = 220 \,\mu\text{s}$ . It is evident that the  $W^{\text{ef}}$  is maximum for the  $[0^{\circ}/45^{\circ}/90^{\circ}/-45^{\circ}]$  laminate, and the  $K_{\text{e}}^{\text{t}}$  is minimum for the  $[0^{\circ}/90^{\circ}/-45^{\circ}/45^{\circ}]$  laminate.

All four failure modes can be delayed by placing the 45° plies at the bottom of the laminate to resist fiber fracture,



Fig. 5a. For the nine stack-up sequences of four-ply laminates, energy dissipated in different failure modes.



Fig. 5b. For the nine stack-up sequences of the four-ply laminates, total work done by external forces, strain energy required to deform the body, and the change in the kinetic energy.

and the 0° or the 90° plies at the top of the laminate to resist delamination. The fiber/matrix debonding damage developed the last for the  $[0^{\circ}/90^{\circ}/-45^{\circ}/45^{\circ}]$  and the  $[0^{\circ}/90^{\circ}/45^{\circ}/-45^{\circ}]$  laminates, and the earliest for the  $[0^{\circ}/-45^{\circ}/45^{\circ}/90^{\circ}]$  composite (Fig. 6a). The time of initiation of the matrix cracking damage variable for the  $[0^{\circ}/90^{\circ}/-45^{\circ}/45^{\circ}]$  and  $[0^{\circ}/90^{\circ}/45^{\circ}/-45^{\circ}]$  laminates is the maximum, and it is minimum for the  $[45^{\circ}/90^{\circ}/-45^{\circ}/0^{\circ}]$  and the  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$  laminates (Fig. 6b). The fiber breakage damage variable developed the last for the  $[0^{\circ}/45^{\circ}/-45^{\circ}/90^{\circ}]$  and  $[0^{\circ}/-45^{\circ}/45^{\circ}/90^{\circ}]$  laminates and soonest for the  $[0^{\circ}/-45^{\circ}/90^{\circ}/45^{\circ}]$  and  $[0^{\circ}/45^{\circ}/90^{\circ}-45^{\circ}]$  laminates (Fig. 6c). In each case, except for delamination, the failure

mode that initiated first caused the maximum energy dissipation.

# 4.2. Target thickness

When studying the effect of target thickness in resisting a shock load, we assumed that it is comprised of four uniform 0° plies, and varied the thickness of each layer. In each case results were computed for load duration of 220 µs, and the maximum centroidal deflection occurred at t = 220 µs. It is clear from the maximum centroidal deflection for different target thicknesses plotted in Fig. 7a that the centroidal deflection decays exponentially



Fig. 6. For different stack-up sequences, time histories at the specimen centroid of (a) fiber breakage, (b) matrix cracking, and (c) debonding damage variable.

with an increase in the target thickness. Results plotted in Fig. 7b show that the  $E^{dl}$  decreases exponentially and is

maximum for the thinnest target; however, the  $W^{\text{ef}}$  is not a monotonic function of the target thickness – it first



Fig. 7a. Vertical displacement of the specimen centroid versus the normalized target thickness.



Fig. 7b. Energy dissipated in different failure modes versus the target thickness.

increases with an increase in the target thickness, and then decreases as the target is made thicker. For the 25-mm and thicker targets, the applied load is not sufficient to cause rapid deformations of material particles; thus the total  $K_e^t$  of the target is miniscule as compared to the  $W^{ef}$  nearly all of which is used to deform the laminate.

# 4.3. Elastic moduli of the fiber and the matrix

We analyze deformations of a  $[0^{\circ}]_{4}$  composite, and assess the effect of changing only one of the following material moduli at a time: (i) fiber's Young's modulus, (ii) matrix's Young's modulus, (iii) fiber's shear modulus, and (iv) matrix's shear modulus. Recall that the elastic moduli of the composite are computed from those of its constituents; thus one or more of five elastic constants of a layer will alter even if one of the elastic moduli of either the fiber or the matrix is modified. We note that values of parameters in the damage evolution Eqs. (2)–(6) and their critical values at failure are taken to be unaffected by a change in fiber's and matrix's elastic moduli. An increase in the elastic modulus will enhance the wave speed, help propagate quickly the impact load, and more of the laminate material will contribute to resisting the applied load.

Time histories of specimen's centroidal deflection, plotted in Fig. 8a, reveal that an increase in fiber's Young's modulus decreases noticeably the vertical displacement of laminate's centroid. With an increase in fiber's elastic



Fig. 8. For two values of fiber's elastic modulus, time histories of evolution at the laminate centroid of (a) the deflection, (b) the debondnig damage variable, and (c) the matrix cracking damage variable.

modulus to five times its original value, the shape of the centroidal deflection versus time curve changes from concave downward to concave upwards. Also, for the stiffer fibers the vertical deflection plateaus at about  $180 \ \mu s$  as opposed to continuing to increase with time for the regular fibers.

Time histories of evolution at laminate's centroid of the damage variables corresponding to the fiber breakage, fiber/matrix debonding (Fig. 8b), and matrix cracking (Fig. 8c) reveal the following. The two curves for the fiber breakage are essentially identical till ~170 µs when fibers with the lower Young's modulus broke but those with the higher one did not. The increase in fiber's Young's modulus gives slightly different rates of evolution of the  $\phi^d$ , and delays the initiation of the matrix cracking damage by about 25 µs. These results are in qualitative agreement

with those of Sierakowski and Chaturvedi [17] who stated that the  $\phi^d$  is mainly related to fiber's properties.

Decreasing Young's modulus of the matrix material to one-fifth of its value had virtually no effect on the time history of evolution of specimen's centroidal deflection, and evolution there of the  $\phi^f$  and the  $\phi^d$  (Fig. 9a). However, it delayed the initiation of the  $\phi^m$  (cf. Fig. 9b). These results agree qualitatively with the experimental observations of Mouritz [2] who found that a toughened polymer–matrix increases the interlaminar strength of the laminate but does not increase much its resistance to impact loads. We note that Sierakowski and Chaturvedi [17] found that the matrix properties influence delamination and matrix cracking.

Increasing fiber's shear modulus by a factor of five has virtually no effect on the centroidal deflection of the laminate, and the  $\phi^m$ . However, from results plotted in



Fig. 9. For two values of the matrix elastic modulus, time histories of evolution at the specimen's centroid of (a) the fiber/matrix debonding, and (b) the matrix cracking damage variable.

Fig. 10a and b we conclude that it delays the initiation of the  $\phi^{f}$  and enhances the initiation of the  $\phi^{d}$ .

Increasing the matrix shear modulus by a factor of five reduces noticeably specimen's centroidal deflection as depicted in Fig. 11a, and enhances both the time of initiation and the rate of growth of the  $\phi^d$  as documented in Fig. 11b. It has virtually no effect on the evolution of the  $\phi^m$ , but it delays the initiation of the damage due to fiber breakage as exhibited in Fig. 3.64 of [16].

### 4.4. Fiber volume fraction

For the  $[0^{\circ}]_4$  composite, Figs. 12a and b present the effect of the fiber volume fraction  $V^{\text{f}}$  on the  $W^{\text{ef}}$ ,  $W^{\text{def}}$ ,  $K_e^{\text{t}}$ ,  $E^{\text{fb}}$ ,  $E^{\text{mc}}$ 

and  $E^{db}$ . It is evident that with an increase in  $V^{f}$  the  $W^{ef}$  decreases affinely, the  $K_{e}^{t}$  quadratically, and the  $W^{def}$  stays virtually unchanged. The  $E^{mc}$  is essentially zero except when  $V^{f}$  equals 0.8. Whereas the  $E^{db}$  increases monotonically with an increase in the  $V^{f}$ , the  $E^{dl}$  attains a minimum value when  $V^{f} = 0.6$ , and then increases. The  $E^{fb}$  has a maximum value at  $V^{f} = 0.45$  and has a parabolic variation.

### 4.5. Boundary conditions

In an attempt to assess the effect of boundary conditions on deformations of a composite laminate, we have analyzed the response of  $[0^{\circ}]_4$  and  $[45^{\circ}]_4$  laminates that are either clamped at the edge surfaces or have null deflections



Fig. 10. For two values of fiber's shear modulus, time histories of evolution at specimen's centroid of (a) the fiber breakage damage, and (b) the debonding damage variable.



Fig. 11. For two values of the matrix shear modulus, time histories at specimen's centroid of evolution of (a) the centroidal deflection, and (b) the fiber/matrix debonding damage variable.

at the boundaries of the bottom surface (i.e.,  $u_3 = 0$  for points on the plane  $X_3 = 0$  that are on the lines  $X_1 = 0$ ,  $X_1 = 220$  mm,  $X_2 = 0$ , and  $X_2 = 220$  mm); the latter are called below simply supported (SS) edges. For clamped edges the delamination begins at the edges and subsequently occurs at the laminate centroid. However, for a SS laminate, delamination starts at the specimen centroid. Furthermore, delamination initiates earlier for the clamped edges as compared to that for the SS edges. For each one of the two laminates and the two boundary conditions, the  $W^{\text{ef}}$ ,  $W^{\text{def}}$  and the  $K_{\text{e}}^{\text{t}}$  at the terminal time of 220 µs are depicted in Fig. 13a as bar charts. These reveal that the  $W^{\text{ef}}$  and the final  $K_{\text{e}}^{\text{t}}$  are lower while the  $W^{\text{def}}$  is higher for a SS composite than those for a clamped one. The energy dissipated in each one of the four failure modes for a SS laminate is lower than that for a clamped laminate; e.g. see Fig. 13b. Also, in each case, the  $E^{del}$  is considerably higher than  $E^{fb}$ ,  $E^{mc}$  and  $E^{db}$ . Only about  $0.12W^{ef}$  is dissipated in the four failure modes.

# 4.6. Sandwich structures

We now delineate benefits, if any, of sandwich construction for resisting impact loads by studying the response of four isotropic core materials embedded between  $0^{\circ}$  fiber reinforced face sheets; the four core materials are low carbon, aluminum, high carbon and a hypothetical one. For comparison purposes, we also study the response of a  $[0^{\circ}]_4$ 



Fig. 12a. Total work done, strain energy and kinetic energy for different fiber volume fractions.



Fig. 12b. Energy dissipated due to different failure modes versus the fiber volume fraction.

laminate, and a hypothetical foam with negative Poisson's ratio. All edge surfaces of the specimen are clamped. Properties of materials of the core, taken from the literature, are obtained from test data derived from either split Hopkinson pressure bar tests or uniaxial compression tests conducted at different strain rates. Axial stress versus axial strain curves for the three foams deformed in uniaxial compression are exhibited in Fig. 14a. Due to lack of information about the behavior of the foams in tension and under other loading conditions that is needed to find values of material parameters of anisotropic foams, we assume that the foam is homogeneous and isotropic. Poisson's ratio for each foam to be given by  $E = E_1 = E_2 = E_3 = E^m(1-\phi^m)$  where values of  $E^m$  and  $\phi^m$  at different values of the axial strain are determined from the slope of the axial stress-axial strain curve depicted in Fig. 14a. The variation of E with the axial strain is evinced in Fig. 14b. For numerical simulations, the core structure is modeled as a composite material with  $V^f = 0$ , and the degradation in its modulus is due to the evolution of the (fictitious) matrix cracking variable. Note that the axial strain at failure for two of the three foams is greater than 0.6. However, we have modeled structure's response till an axial strain of 0.6 and assumed that the foam then fails. The strain energy density till failure of the high-strength carbon foam is much higher than that of the other two foams. We also



Fig. 13. For two types of boundary conditions and for the two laminates, (a) total work done, strain energy and kinetic energy, (b) energy dissipated in different failure modes.



Fig. 14a. Axial stress-axial strain curves of three foams for compressive axial loading.



Fig. 14b. Variation of Young's modulus with axial strain for three foams.



Fig. 14c. For the five structures, time histories of evolution of the matrix cracking damage variable at structure's centroid.

consider a hypothetical isotropic foam having E = 0.1 GPa, and v = -0.6. We assume that the delamination between a face sheet and the core occurs at a point on their interface when the state of stress there satisfies the same failure envelope as that for the laminated composite. Furthermore, values of strength parameters in the failure envelope are taken to be the same as those between two adjoining layers of a laminate. Thus, computed results are approximate and provide preliminary information regarding the performance of different cores.

Time histories of specimen's centroidal deflection till  $t = 220 \ \mu s$  showed that the final deflection is the same for the four sandwich structures, and it is lower than that for

the  $[0^{\circ}]_4$  laminate. However, as can be seen from time histories of evolution of the  $\phi^m$  at the specimen centroid plotted in Fig. 14c, the core material noticeably influences the growth rate of the matrix cracking and also when it initiates. Whereas the  $\phi^m$  suddenly increases to 1.0 at  $t = 130 \,\mu$ s for the  $[0^{\circ}]_4$  laminate, it grows gradually in the sandwich structures. It was found (e.g. see [16]) that the  $W^{\text{ef}}$ ,  $W^{\text{def}}$  and the  $K_{\text{e}}^{\text{t}}$  are the same for all five structures. The balance of energy implies that the total energy dissipated in the four failure modes is also the same for the five structures. However, as documented in Fig. 14d, energies dissipated in different failure modes vary widely for them. For the structure without a core, and for structures with



Fig. 14d. Energy dissipated in different failure modes for five sandwich structures.



Fig. 15a. Work done, strain energy, and the final kinetic energy for laminates bonded to a steel plate.

low and high-strength carbon cores, most of the energy is dissipated in delamination, while for the structure with the aluminum foam core,  $E^{del}$  and  $E^{mc}$  are nearly the same.

# 4.7. Composite laminates bonded to a steel plate

In an attempt to see how deformations of the laminate and energy dissipated in it are affected in applications where it is likely to be bonded to a metallic plate, we have investigated response of a hybrid structure comprised of either the  $[0^{\circ}/90^{\circ}/45^{\circ}/-45^{\circ}]$  or the  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$  laminate bonded to a homogeneous and isotropic steel plate having E = 200 GPa, and v = 0.3. The afore-given results show that the  $[0^{\circ}/90^{\circ}/45^{\circ}/-45^{\circ}]$  laminate had the maximum and the  $[-45^{\circ}/0^{\circ}/45^{\circ}/90^{\circ}]$  the minimum energy dissipated in all failure mechanisms. The total thickness of the hybrid structure was taken to be either 10 mm or 30 mm with the ratio of the thickness of the steel plate to that of the composite laminate equal to 2. It was assumed that the delamination strength between the composite and the steel equaled that between two plies of the laminate. There is no damage evolution in the steel plate. Fig. 15a shows that for the same applied pressure load, adding a steel plate to the bottom layer of the laminate decreases the  $W^{\text{ef}}$ , the  $W^{\text{def}}$ , and the  $K_{\text{e}}^{\text{t}}$ , and increasing the target thickness further reduces them. The energy dissipated in different failure modes, plotted in Fig. 15b, also decreases with the addition



Fig. 15b. Energy dissipated in different failure modes for laminates bonded to a steel plate.

of the steel plate. This is because the steel plate drastically reduces deflections of the hybrid structure, and energy required to deform the steel plate consumes most of the  $W^{\rm ef}$  which is also substantially decreased because of very little displacements of points of application of loads.

# 4.8. Explosive type

We now investigate transient deformations of the clamped [0°]<sub>4</sub> laminate exposed to blast loads induced by three other explosives, HBX, PETN and nuclear, each having a mass of 64 kg, and detonated at the same standoff distance R = 10 m and angular position  $\theta = 0^{\circ}$ . Referring the reader to [5,14-16] for details, we have used Eqs.

(18) and (19) of [5] and values of parameters in Table 3 of [5] to plot in Fig. 16a the decay with time of the pressure acting at the centroid of the top surface of the laminate for the TNT, the HBX and the PETN explosives. For the nuclear explosive the pressure rises to 3.8 GPa in about 5 µs and essentially stays unchanged for the duration of the computation. The peak pressure induced by the nuclear explosive is nearly 200 times that for the other three explosives.

Computed time histories of specimen's centroidal deflection and of the evolution of the three damage variables there (e.g. see Fig. 16b and c) show that the response of the laminate to loads produced by the HBX and the PETN are the same but are quite distinct from that produced by the nuclear explosion. For loads produced by the HBX,



Fig. 16a. The decay with time of the pressure acting at the centroid of the top surface of the laminate for TNT, HBX, and PETN explosives with W = 64 kg and R = 10 m.



Fig. 16b. Time histories of evolution of the debonding damage variable at laminate's centroid for three explosives.



Fig. 16c. Time histories of evolution of the matrix cracking damage variable at laminate's centroid for three explosives.

the PETN and the TNT, Fig. 16d compares the  $W^{\text{ef}}$ ,  $W^{\text{def}}$ and the  $K_{\text{e}}^{\text{t}}$ . Whereas the  $W^{\text{ef}}$  is the same for the three explosives, the other two energies are different. The  $K_{\text{e}}^{\text{t}}$ and the  $W^{\text{df}}$  are equal to each other for the TNT explosive. However, for the HBX and the PETN explosives, the final  $K_{\text{e}}^{\text{t}}$  equals about 75% of the  $W^{\text{def}}$ . For deformations caused by the explosion of the nuclear device, the composite fails very quickly due to complete delamination coupled with early initiation and rapid growth of the fiber/matrix debonding and matrix cracking, and very little centroidal deflection and fiber breakage. For the laminate subjected to loads produced by the nuclear explosion, Fig. 16e

depicts at two times the delaminated area and fringe plots of the matrix cracking damage variable.

# 4.9. Stand off distance

The standoff distance, R, represents how far the explosive is from the target, and for a given weight of an explosive, affects the peak pressure exerted on the target and the decay rate of the pressure. Even though the shock wave profile also varies with the distance R, we have presumed that it is planar for R = 1, 10, 100 and 1000 m. For R = 1 and 100 m, the centroidal deflection equals



Fig. 16d. Total work done by external forces, strain energy, and the final kinetic energy for three explosives.



Fig. 16e. Delaminated area, and fringe plots of the matrix cracking damage variable in the laminate subjected to loads caused by the nuclear explosion.

10 mm and 0.05 mm respectively. Except for R = 10 m, the damage due to fiber breakage was negligible either because other three failure modes were dominant or the load was not high enough (e.g. for R = 1 km) to cause any damage. Time histories of  $\phi^d$  and the  $\phi^m$  plotted in Figs. 17a and b show that the value of R significantly affects when and how fast the damage variables evolve. For R = 1 and 10 m, the damage due to the TNT explosive is similar to that produced by the nuclear explosive discussed above.

### 4.10. Explosive mass

For R = 10 m, peak pressures produced by 2.5, 64, 550, 3000 and 17500 kg of TNT equal 5, 40, 80 and 160 MPa respectively. Deformations of a clamped laminate due to these loads have been studied. With an increase in the mass of the explosive, damages due to fiber/matrix debonding and matrix cracking overtake that due to fiber breakage; e.g. see Fig. 18. For a large mass of the TNT explosive that



Fig. 17. For four values of the stand-off distance *R* time histories of the (a) fiber/matrix debonding, and (b) matrix cracking damage variable at laminate's centroid.

generates peak pressures greater than 0.04 GPa, the failure modes are similar to those induced by nuclear loading for which the structure failed primarily due to delamination. This agrees qualitatively with Iannucci et al.'s [18] observation that as the impact energy increases to ballistic levels, generally only local delamination occurs.

# 5. Limitations of the model

We have used three internal variables to simulate matrix cracking, fiber breakage, and fiber/matrix debonding, and a failure envelope to model the initiation of delamination between two adjoining layers. Thus failures due to fiber pull out, fiber kinking, fiber buckling, and matrix crushing have not been considered. Furthermore, composites with unidirectional fiber reinforcements have been studied. Thus results are not applicable to stitched composites, woven composites, and to composites reinforced with either curved and/or randomly distributed short fibers. The theory of internal variables employed here homogenizes the damage, enables one to compute easily energies dissipated in different failure modes, and allows for interactions among various failure modes. However, it does not simulate cracked surfaces and fiber/matrix debonded areas as traction free surfaces; cracked surfaces are modeled as traction free in [20,21]. Effects of friction between two sliding surfaces subsequent to delamination have also not been considered.

### 6. Generalization of the work

The work presented here can be easily extended to include thermal effects and thus consider coupling among thermal and mechanical deformations. The consideration of microstructural effects in foams used as reinforcements in sandwich structures is more challenging and will require multiscale analysis. The evolution of porosity in the matrix can be simulated by using damage relations given in [22].



Fig. 18. For five values of the charge mass, time histories of the (a) centroidal deflection, (b) matrix cracking, and (c) debonding damage variable at laminate's centroid.

For thick composites having numerous plies, the fiber orientation angle can be varied slowly from one ply to the next resulting in a functionally graded (FG) laminate; e.g. see [23] where natural frequencies of such a laminate have been computed. It still remains to be investigated whether or not a FG laminate will have higher energy dissipation than an equally thick one but with discontinuous variation of the fiber orientation angle.

The analysis of transient deformations of thick laminates by the FEM can be computationally expensive both because of the effort required to generate the FE mesh and pre- and post-process the data. It may be more economical to use a theory of thick plates, e.g. see [24], and use a meshless method like that employed in [25,26] to analyze transient thermomechanical deformations of a plate. The challenging tasks here are to incorporate effects of the failure modes in the plate theory.

### 7. Conclusions

The mathematical model developed previously for analyzing transient deformations of a composite subjected to shock loads, and a modular computer code, in Fortran, to find numerically an approximate solution of the pertinent initial-boundary-value problem have been used to analyze transient deformations of a laminated composite. The problem formulation includes evolution of damage due to fiber breakage, fiber/matrix debonding, matrix cracking, and delamination. Values of material parameters for the AS4/PEEK composite were determined from the test data available in the literature. Energies dissipated in these failure modes are computed, and the effect on them of various geometric, material, and loading parameters has been examined. Following conclusions can be drawn from this work.

- Approximately 15% of the total work done by external forces is dissipated in the four failure modes.
- Both for the [0°]<sub>4</sub> and the [45°]<sub>4</sub> laminated composites, the energy dissipated due to delamination for clamped edges is nearly twice of that for simply supported edges. About 43% of the energy input into the structure is used to deform it, and the remaining 42% is converted into the kinetic energy. For clamped laminates, these proportions strongly depend upon the fiber orientation angle.
- The fiber orientation influences when and where each failure mode initiates and its direction of propagation.
- Debonding between fibers and the matrix occurs along the fibers rather than in a direction perpendicular to the fibers. For clamped edges, the debonding damage variable starts from the edges perpendicular to the fibers and propagates, along the fibers, towards the center; it propagates in the thickness direction instantaneously most likely due to thin laminates studied herein.
- Matrix cracking damage variable initiates first at the center of the back surface, where there are high tensile

stresses developed, and propagates faster along the fibers than in the transverse direction.

- Fiber breakage is concentrated at points near the specimen's centroid that are along the X<sub>2</sub>-axis.
- The energy dissipated due to matrix cracking is miniscule as compared to that in any of the other three damage modes. The fraction of the total work done by external forces dissipated due to various failure mechanisms has the maximum value of nearly 22% for fiber orientations of 30° and 60°, of which ~10% is due to delamination.
- The stacking sequence strongly influences energies dissipated in different failure modes.
- The target thickness determines the dominant failure mode. The fraction of energy dissipated due to delamination failure mode decreases exponentially with an increase in the target thickness, and has the maximum value for the thinnest target.
- Increasing fiber's Young's modulus results in slightly different rates of evolution of the fiber/matrix debonding damage variable, and delays the initiation of the matrix cracking damage. Increasing fiber's shear modulus delays the initiation of the fiber breakage variable and enhances the initiation of the fiber/matrix debonding damage variable. Increasing the matrix shear modulus reduces noticeably specimen's centroidal deflection, enhances both the time of initiation and the rate of growth of the fiber/matrix debonding damage variable, and delays the initiation of the damage due to fiber breakage.
- An increase in the fiber volume fraction decreases affinely the total work done by external forces, decreases parabolically the kinetic energy, and has virtually no effect on the energy required to deform the body.
- For deformations caused by the explosion of a nuclear device, the composite fails very quickly due to complete delamination coupled with early initiation and rapid growth of the fiber/matrix debonding and matrix cracking, and very little centroidal deflection and fiber breakage.
- Laminate's deformations for non-nuclear explosives detonated close to it are similar to those induced by a nuclear explosion.

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