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Steady state penetration of thermoviscoplastic targets

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Abstract. Steady state thermomechanical deformations of a target hit by a rigid cylindrical penetrator with an ellipsoidal nose are studied. The material of the target is assumed to be thermally softening but strain and strain-rate hardening. Results computed and presented graphically include the pressure distribution on the nose of the penetrator, dependence of the axial resisting force upon the speed of the penetrator, and the variation of field quantities such as the temperature and strain-rate in the target. Computed results show that the ratio of the major to minor axes of the ellipsoidal nose has a significant effect on the deformations of the target particles in the vicinity of the penetrator nose.

1 Introduction

In an attempt to shed some light on the validity of the approximations made in simple theories of penetration due to Alekseevskii (1966) and Tate (1967, 1969), Batra and Wright (1986) studied in detail the penetration problem that simulates the following situation. Suppose that the penetrator is in the intermediate stages of penetration so that the active target/penetrator interface is at least one or two penetrator diameters away from either target face, and the remaining penetrator is much longer than several diameters and is still traveling at a uniform speed. Thus steady state deformations of the target, presumed to be made of a rigid/perfectly plastic material, and being penetrated by a long cylindrical rigid rod with a hemispherical nose were analysed. Subsequently Batra (1987) showed that the axial resisting force experienced by the rigid penetrator is considerably reduced if its nose shape is ellipsoidal rather than hemispherical and also investigated the effect of the dependence of the flow stress upon the strain-rate. Herein we study the steady state penetration problem when the target material is thermally softening but strain and strain-rate hardening.

Pidsley (1984) has recently given a detailed numerical solution of the penetration problem in which both materials are considered to be deformable and rigid/perfectly plastic. We refer the reader to his paper for more references on this subject. Even though we study a somewhat simpler situation, our material model is more general in that we account for the effect of strain and strain-rate hardening and thermal softening. We note that the peak strains and strain-rates encountered during steady state deformations of the target are of the order of 10 and 10^5 sec^{-1} respectively. Also the temperature at target points may rise to as much as half of the melting point of the target material. We study the effect of these competing factors as well as of the penetrator speed and the shape of its nose on the deformations of the target.

2 Formulation of the problem

Since the axisymmetric deformations of the target appear to be independent of time to an observer situated on the penetrator nose and moving with it, we choose a cylindrical co-ordinate system attached to the nose tip with the positive z-axis pointing into the target material. With respect to these axes translating with a uniform velocity v_0e , being a unit vector along the penetrator axis

and in the direction of its motion, equations governing the thermomechanical deformations of the target are

div
$$v = 0$$
, (1)
div $\sigma = \rho (v \cdot \text{grad}) v$, (2)
 $- \operatorname{div} q + \operatorname{tr} (\sigma \mathbf{D}) = \rho (v \cdot \text{grad}) U$, (3)
 $\operatorname{tr} (\sigma \mathbf{D}) = (v \cdot \text{grad}) \varkappa$, (4)
 $\mathbf{D} = (\operatorname{grad} v + (\operatorname{grad} v)^T/2$. (5)

Equations (1) through (4) written in Eulerian description of motion express respectively the balance of mass, linear momentum, internal energy and the evolution of the work hardening parameter \varkappa . In Eq. (4) we have neglected the elastic deformations of the target and in Eq. (3) assumed that all of the plastic working, rather than a part of it (e.g. Farren and Taylor 1925) is dissipated in the form of heat. The operators grad and div signify the gradient and divergence operators on fields defined in the present configuration. Furthermore, σ is the Cauchy stress tensor, ϱ is the mass density of the target material, v is the velocity of the target particle relative to the penetrator, q is the heat flux per unit present area, **D** is the strain-rate tensor, and U the specific internal energy. Equations (1) through (4) are to be supplemented by constitutive relations and boundary conditions.

We assume the following constitutive relations for the target material.

$$q = -k \operatorname{grad} \theta, \tag{6}$$

$$U = c \theta, \tag{7}$$

$$\boldsymbol{\sigma} = -p\mathbf{1} + 2\mu(I,\theta,\psi)\mathbf{D}, \quad \text{if } \mathbf{D} \neq \mathbf{0},$$
(8)

tr (s²)
$$\leq \frac{2}{3} \sigma_0^2 (1 - a\theta)^2 \left(1 + \frac{\psi}{\psi_0}\right)^{2n}$$
, if **D** = **0**, (9)

$$\mathbf{s} = \boldsymbol{\sigma} + p\mathbf{1},\tag{10}$$

$$2\mu(I,\theta,\psi) = \frac{\sigma_0}{\sqrt{3}I}(1+bI)^m(1-a\theta)\left(1+\frac{\psi}{\psi_0}\right)^n,$$
(11)

$$\dot{\kappa}(\psi) = \sigma_0 \dot{\psi} (1 + \psi/\psi_0)^n,$$
(12)

$$I^2 = \frac{1}{2} \operatorname{tr} (\mathbf{D}^2).$$
(13)

Equation (6) is Fourier's law of heat conduction, k is the thermal conductivity, θ is the change in the temperature of a material particle from that in the underformed configuration, c is the specific heat which is assumed to be constant, p is the hydrostatic pressure not determined by the deformation history, and σ_0 is the yield stress in a simple tension or compression test. The material parameters b and m describe the strain-rate sensitivity of the material, the material parameter a describes its thermal softening, and ψ_0 and n characterize the strain hardening of the material. An integral form of Eq. (12) with \varkappa interpreted as the true stress and ψ the plastic strain represents the stress-strain curve in a quasistatic reference test. Equation (8) may be interpreted as a constitutive relation for a Non-Newtonian fluid whose viscosity μ depends upon the strain-rate, temperature and a material parameter ψ . Equation (8) implies that

$$\left(\frac{1}{2}\operatorname{tr} s^{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{3}}\sigma_{0}\left(1+b\,I\right)^{m}\left(1-a\,\theta\right)\left(1+\frac{\psi}{\psi_{0}}\right)^{n} \qquad (14)$$

which can be viewed as a generalized Von-Mises yield criterion when the flow stress (given by the right-and side of (14)) at a material particle depends upon its strain-rate, strain and temperature. A constitutive relation similar to Eq. (8) has been used by Zienkiewicz et al. (1981) who took

$$2\mu = [\sigma_0 + (2I/\sqrt{3}\gamma)^{1/n}]/\sqrt{3}I, \qquad (15)$$

where γ and *n* are functions of θ . They asserted that it represents Perzyna's viscoplastic model. For a simple shearing deformation, Litonski (1959) proposed that

$$\tau = c \left(1 - a\theta\right) (1 + b\gamma)^m \dot{\gamma}^n \tag{16}$$

where τ and γ equal the shear stress and shear strain, and c is a material constant. Note that this relation implies that τ is zero whenever $\dot{\gamma} = 0$. Another stress-strain law proposed by Lindholm and Johnson (1984), based on fitting curves to experimental data obtained from torsion tests, is

$$\tau = (A + B\gamma^n)(1 + C\ln(\dot{\gamma}/\dot{\gamma}_0))\frac{\theta_m - \theta}{\theta_m - \theta_0}$$
(17)

where θ_m is the melting point of the material, θ_0 is a reference temperature, and A, B and C are material constants. Lin and Wagoner (1986) recently reported that the following curve

$$\sigma = 556 \left(\varepsilon - 0.014\right)^{0.219} \left(\varepsilon/0.02\right)^{0.018} \left(1 - 0.0012 \left(\theta - 298\right)\right) \text{MPa},$$
⁽¹⁸⁾

fitted well their experimental data derived from a uniaxial tension test on Armco I. F. steel. In Eq. (18), σ and ε are the axial stress and the axial strain respectively and θ is in °K. The linear dependence of the flow stress upon temperature has also been observed by Bell (1968).

The constitutive relation (8) with μ given by Eq. (11) is an attempt to generalize the one used by Wright and Batra (1986) for simple shearing deformations of nonpolar and dipolar materials. They used it to study shear strain localization phenomenon in metals and derived it by using arguments similar to those employed by Green, McInnis and Naghdi (1968). A curve fit to the torsion test data of Costin et al. (1979) for a 1018 cold rolled steel gives n = 0.09, $\psi_0 = 0.017$, $b = 10^4 \text{ sec}^{-1}$ and m = 0.025.

Before stating the boundary conditions we non-dimensionalize the variables as follows.

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/\sigma_0, \quad \bar{p} = p/\sigma_0, \quad \bar{\mathbf{s}} = \mathbf{s}/\sigma_0, \quad \bar{\boldsymbol{v}} = \boldsymbol{v}/v_0, \quad \bar{r} = r/r_0, \quad \bar{z} = z/r_0, \quad \bar{\theta} = \theta/\theta_0,$$

$$b = b \frac{v_0}{r_0}, \quad \bar{a} = a\theta_0, \quad \alpha = \varrho v_0^2/\sigma_0, \quad \beta = k/(\varrho c v_0 r_0), \quad \theta_0 = \sigma_0/(\varrho c).$$

$$(19)$$

Substituting from Eqs. (6) through (12) into the balance laws (1) through (4), rewriting these in terms of non-dimensional variables, and denoting the gradient and and divergence operators in non-dimensional coordinates by grad and div, we arrive at the following set of equations.

$$\operatorname{div} \boldsymbol{v} = \boldsymbol{0}, \tag{20}$$

$$\operatorname{div} \boldsymbol{\sigma} = \alpha \left(\boldsymbol{v} \cdot \operatorname{grad} \right) \boldsymbol{v}, \tag{21}$$

$$\frac{\operatorname{tr}(\boldsymbol{\sigma}\mathbf{D})}{\operatorname{tr}(\boldsymbol{\sigma}\mathbf{D})} = (\mathbf{r} \cdot \operatorname{grad})\boldsymbol{w}, \tag{22}$$

$$\frac{1}{(1+\psi/\psi_0)^n} = (v \cdot \operatorname{grad})\psi, \qquad (23)$$

where

$$\sigma = -p\mathbf{1} + \frac{1}{\sqrt{3}I}(1+bI)^m(1-a\theta)\left(1+\frac{\psi}{\psi_0}\right)^n\mathbf{D},$$
(24)

and we have dropped the superimposed bars.

We assume smooth contact at the target/penetrator interface. Thus the boundary conditions on this surface are

| $t\cdot(\sigma n)=0,$ | (25.1) |
|---|--------|
| $v \cdot n = 0$, | (25.2) |
| $\boldsymbol{q}\cdot\boldsymbol{n}=h(\theta-\theta_a),$ | (25.3) |

where h is the heat transfer coefficient between the penetrator and the target, θ_a is an average

temperature of the penetrator, and n and t are, respectively, a unit normal and a unit tangent vector to the surface. At points far away from the penetrator

$$|v+e| \to 0, \quad \theta \to 0, \quad \psi \to 0 \quad \text{as} \quad (r^2+z^2)^{i} \to \infty, \quad z > -\infty,$$
(26.1)

$$|\sigma n| \to 0, \quad |q \cdot n| \to 0, \quad \psi \to 0 \quad \text{as} \quad z \to -\infty, \quad r \ge r_0.$$
 (26.2)

The boundary condition (26.1) states that target particles at a large distance from the penetrator appear to be moving at a uniform velocity with respect to it and experience no change in their temperature. Equation (26.2) implies that far to the rear the traction and heat flux fields vanish.

Note that the governing Eqs. (20)-(23) are coupled and are nonlinear in v, θ and ψ . Their solution, if there is one, may not be unique and will depend, in general, upon the rates at which quantities in (26) decay to zero. Since we are unable to solve these equations analytically and prove any uniqueness theorem, we will seek a numerical solution of these equations which we hope will be physically meaningful.

3 Finite element formulation of the problem

The numerical solution of the problem necessitates the consideration of a finite region. Since the target deformations are axisymmetric, only the target region R shown in Fig. 1 is studied. The adequacy of the finite domain considered will be verified by solving the problem for two separate regions, one larger and containing the other, and ensuring that the two sets of computed values of various field quantities are close to each other. The boundary conditions (26) are replaced by the following.

$$\sigma_{zz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{on the surface } AB,$$
 (27.1)

$$\sigma_{rz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial r} = 0 \quad \text{on the axis of symmetry } DE,$$
 (27.2)

$$v_r = 0$$
, $v_r = -1.0$, $\theta = 0$, $\psi = 0$ on the boundary surface *EFA*. (27.3)

Referring the reader to Becker et al. (1981) and Zienkiewicz et al. (1981) for details, we simply note that a weak formulation of the problem defined on the region R by Eqs. (20)–(24) and boundary conditions (25) and (27) is that equations

$$\int_{B} \lambda (\operatorname{div} v) \, \mathrm{d}v = 0,$$

(28.1)



Fig. 1. The finite region studied

$$\int_{R} p(\operatorname{div}\varphi) \, \mathrm{d}v - \int_{R} \mu(I,\theta,\psi) (\mathbf{D}: (\operatorname{grad}\varphi + (\operatorname{grad}\varphi)^T)) \, \mathrm{d}v = \alpha \int_{R} ((v \cdot \operatorname{grad})v) \cdot \varphi \, \mathrm{d}v, \tag{28.2}$$

$$\int_{R} ((v \cdot \operatorname{grad})\theta) \eta \, \mathrm{d}v + \beta \int_{R} \operatorname{grad} \theta \cdot \operatorname{grad} \eta \, \mathrm{d}v + h \int_{\partial_{1}R} \theta \eta \, \mathrm{d}A = \int_{R} \eta \, (\sigma: \mathbf{D}) \, \mathrm{d}v - \int_{\partial_{1}R} h \theta_{a} \eta \, \mathrm{d}A, \qquad (28.3)$$

$$\int_{R} ((v \cdot \operatorname{grad}) \psi) \xi \, \mathrm{d}v = \int_{R} \frac{(\sigma : \mathbf{D})}{\left(1 + \frac{\psi}{\psi_0}\right)^n} \, \xi \, \mathrm{d}v,$$
(28.4)

hold for arbitrary smooth functions λ , φ , η and ξ defined on R such that $\varphi_r = 0$ on AB, $\varphi = 0$ on EFA, $\varphi \cdot n = 0$ on the target/penetrator interface BCD, and $\eta = \xi = 0$ on EFA. In these equations, $A: B = tr(AB^T)$ for linear transformations A and B, and $\partial_1 R$ denotes the target/penetrator interface BCD. Since these equations are nonlinear in v, θ and ψ , the following iterative technique has been employed. At the *i*th iteration, equations

$$\int_{R} \lambda \operatorname{div} v^{i} \mathrm{d}v = 0, \tag{29.1}$$

$$\int_{R} p^{i}(\operatorname{div}\varphi) \,\mathrm{d}v - \int \mu(I^{i-1}, \theta^{i-1}, \psi^{i-1}) (\mathbf{D}^{i}: (\operatorname{grad}\varphi + (\operatorname{grad}\varphi)^{T})) \,\mathrm{d}v = \alpha \int_{R} ((v^{i-1} \cdot \operatorname{grad}) v^{i}) \cdot \varphi \,\mathrm{d}v, \quad (29.2)$$

 $\int_{R} ((v^{i-1} \cdot \operatorname{grad})\theta^{i})\eta \,\mathrm{d}v + \beta \int \operatorname{grad} \theta^{i} \cdot \operatorname{grad} \eta \,\mathrm{d}v + h \int_{\partial_{1}R} \theta^{i}\eta \,\mathrm{d}A = \int_{R} \eta (\sigma^{i-1}:\mathbf{D}^{i-1}) \,\mathrm{d}v - \int_{\partial_{1}R} h \theta_{a}\eta \,\mathrm{d}A, (29.3)$

$$\int_{R} \left(\left(\boldsymbol{v}^{i-1} \cdot \operatorname{grad} \right) \boldsymbol{\psi}^{i} \right) \boldsymbol{\xi} \, \mathrm{d} \boldsymbol{v} = \int_{R} \frac{\boldsymbol{\sigma}^{i-1} \cdot \mathbf{D}^{i-1}}{\left(1 + \frac{\boldsymbol{\psi}^{i-1}}{\boldsymbol{\psi}_{0}} \right)^{n}} \, \boldsymbol{\xi} \, \mathrm{d} \boldsymbol{v} \,, \tag{29.4}$$

are solved for v^i , θ^i , ψ^i and p^i . The iterative process is stopped when, at each nodal point,

$$\|v^{i} - v^{i-1}\| + |\theta^{i} - \theta^{i-1}| + |\psi^{i} - \psi^{i-1}| \le \varepsilon [\|v^{i-1}\| + |\theta^{i-1}| + |\psi^{i-1}|]$$
(30)

where $||v||^2 = v_r^2 + v_z^2$, and ε is a preassigned small number. Values of p^i are not included in Eq. (30) since p appears linearly in Eq. (29.2).

4 Computation and discussion of results

The finite element code developed earlier [Batra and Wright (1986)] to solve the problem when the target material is modeled as rigid/perfectly plastic and the penetrator nose is hemispherical has been modified to solve the present problem. It employes six-noded triangular elements with v_r , v_z , ψ and θ approximated by quadratic functions over an element and p by a linear function defined in terms of its values at the vertices of the triangular element. The validity of the code was established by first modifying Eqs. (29) to include arbitrary source terms on their right-hand sides, and then solving simple problems for incompressible Navier-Stokes-Fourier fluids. The source terms were adjusted so that the governing equations were satisfied by the presumed analytic expressions for v_r , v_{θ} , p, θ and ψ . Results for a sample problem that does not include thermal effects are given in Batra and Wright (1986).

A major difference between the problem studied herein and those studied earlier by Batra and Wright (1986) and Batra (1987) is that Eq. (28.4) does not have any diffusive term in it. This necessitates the use of either an ultrafine mesh or a fine mesh with an artificial diffusive term included in Eq. (28.4). Brooks and Hughes (1982) have discussed in detail the justification for including such a term and have given equivalent ways of achieving the same objective. We added a term $\delta \int \operatorname{grad} \psi \cdot \operatorname{grad} \xi \, dv$ to the left-hand side of Eq. (28.4) and computed results for $\delta = 10^{-6}$ and 10^{-7} .

The two sets of values of θ , p, v_r and v_z differed by less than one percent at each node. The results presented below are for $\delta = 10^{-6}$. We next ascertained the adequacy of the region considered by increasing *DE* in Fig. 1 from $3.25r_0$ to $4.5r_0$. Again the difference in the values of θ , p, v_r , v_z and ψ at points in the vicinity of the penetrator nose was negligible.

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 $\langle h\rangle_{A}$

We note that experimental data for the range of deformations expected to occur in the penetration problem under study is not available in the open literature. Therefore, values of material parameters b, m, a, ψ_0 and n in Eq. (14) found by fitting a curve to the experimental data in torsion of Costin et al. (1979) were assumed to be valid under the more general state of stress studied here. This should enable us to undertake the parametric study for a reasonable range of values of various material parameters. The values of various parameters used to compute numerical results are:

 $\begin{array}{ll} n=0.09, & \psi_0=0.017, & b=10^4\,{\rm sec}^{-1}, & m=0.025, & a=0.000555/^{\circ}{\rm C}, & k=48\,{\rm W/m\,^{\circ}C}, \\ c=473\,{\rm J/kg\,^{\circ}C}, & \varrho=7800\,{\rm kg/m^3}, & \sigma_0=180\times10^6\,{\rm Pa}, & h=20\,{\rm W/m^2\,^{\circ}C}, & r_0=2.54\,{\rm mm}, \\ \varepsilon=0.02, & \theta_a=0. \end{array}$

However, the results presented below are in terms of non-dimensional quantities and the variables that are assigned values different from those given above are so indicated in the figures along with their new values.

In Fig. 2 is plotted the pressure distribution on the penetrator nose for a relatively blunt nose $(r_n/r_0 = 0.2)$, a hemispherical nose $(r_n/r_0 = 1)$ and an ellipsoidal nose $(r_n/r_0 = 2.0)$. As expected the normal pressure on the blunt nose stays essentially uniform over most of its surface and drops off sharply near its extremities. Note the change in the curvature of the pressure curve in going from hemispherical to an ellipsoidal nose. The non-dimensional axial resisting force decreased from 17.091 for the blunt nose to 8.902 for the hemispherical nose and further to 5.085 for the ellipsoidal nose. The axial resisting force F is given by

$$F = 2 \int_{0}^{\pi/2} (n \cdot \sigma n) \sin \theta \cos \varphi \left[\sin^2 \theta + \left(\frac{r_n}{r_0} \right)^2 \cos^2 \theta \right]^{\frac{1}{2}} d\theta,$$

where angles θ and φ are defined in Fig. 1. The corresponding axial force in physical units is given by $F(\pi r_0^2 \sigma_0)$. The normal pressure on the hemispherical and the ellipsoidal nose surface for the



Fig. 2. Distribution of the normal pressure on the penetrator nose for three different nose shapes. — Blunt nose $(r_n/r_0 = 0.2);$ ---- hemispherical nose $(r_n/r_0 = 1.0);$ ----- ellipsoidal nose $(r_n/r_0 = 2.0)$

angle θ greater than nearly 75° is not plotted because of the difficulties encountered in computing it accurately. The mesh in this region was not fine enough to yield reliable values. Figure 3 depicts the variation of the strain-rate invariant I and temperature change θ on the nose surface. Whereas the maximum value 4.21 of I occurs at the penetrator nose tip for the ellipsoidal nose, it assumed very high values at the extremities for the blunt nose. For these two nose shapes significant values of I occur near the nose tip and the nose periphery respectively. For the hemispherical nose shape I decreases almost linearly from its maximum value of 2.16 at the nose tip to 0.4 at its periphery ($\theta = 90^{\circ}$). The dimensional values of I equal 1.1 (10⁵) times the non-dimensional values. The values of temperature at the nose tip do not depend that much on the nose shape. However the temperature decreases with θ for the ellipsoidal and the hemispherical nose, it increases with θ for the blunt nose. Because of the high-strain rates near the vicinity of the periphery of the blunt nose, there is a lot of heat generated in this narrow region. Since material particles near the periphery of the nose are moving relatively slowly, not much of the heat produced is convected or transported away. In Fig. 4 is plotted the variation of the strain rate I and temperature change θ on the axial line. For the blunt nose, the deformation has spread to a larger distance as compared to that for the ellipsoidal nose. Accordingly the temperature drops off slowly for the blunt nose than it does for the other two cases. The actual temperatures in °C are obtained by multiplying their nondimensional values by 48.9. Thus temperatures as high as 605 °C occur at and near the nose tip. the The maximum value of strain-rate I on the axial line appears to occur at a point slightly away from the nose tip. This initial rise is probably only an artifact and the curves should be extended smoothly to the nose-tip.









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Figures 5 and 6 illustrate the effect of speed of the penetrator on various solution parameters at or in the vicinity of its ellipsoidal nose with $r_n/r_0 = 2.0$. As shown in Fig. 5, the normal pressure near the penetrator nose tip increases with the speed but decreases near its periphery. Near $\vartheta = 45^{\circ}$, the speed has no effect on the normal pressure. Such a behavior was also observed for a hemispherical nose and a rigid/perfectly plastic target material by Batra and Wright (1986). The dependence of the non-dimensional axial force F upon the speed (through non-dimensional variable α) is given by

 $F = 5.021 + 0.0732 \alpha$, ellipsoidal nose $(r_n/r_0 = 2.0)$

 $F = 8.71 + 0.2145 \alpha$, hemispherical nose.

For rigid/perfectly plastic materials, Batra and Wright (1986) obtained $F = 3.903 + 0.0773 \alpha$ for a hemispherical nosed penetrator, and Batra (1987) computed $F = 2.58 + 0.019 \alpha$ for a penetrator with an ellipsoidal nose having $r_n/r_0 = 2.0$. Thus the consideration of strain and strain-rate hardening and thermal softening effects more than doubles the axial resisting force. In every case studied so far, F depends upon α weakly. This weak dependence of F upon α seems to explain why the choice of constant target resistance in the simple theory of Tate (1967, 69) gives such good qualitative results. On most of the nose surface, the temperature decreases with α . This is shown in Fig. 5. Figure 6 depicts that most of the target deformations are concentrated near the penetrator nose. The peak value of I on the axial line appears to occur not at the nose tip but slightly away from it. The plots of strain-rate invariant I and the temperature change in the target region, shown in Fig. 7, confirm that significant target deformations occur in the vicinity of the target/penetrator interface.



Fig. 6. Variation of the hydrostatic pressure and the strain-rate mesure I on the axial line for different values of α . Explanations see Fig. 5



Fig. 7. Temperature and strain-rate distribution in the target region for $\alpha = 3.0$.

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How different material parameters influence the deformations of the target is demonstrated by results presented in Figs. 8 through 11. Figure 8 shows that strain-rate hardening increases the normal pressure more than the work-hardening does. The inclusion of thermal softening affects inclusion of work-hardening and strain-rate hardening decreases the value of the strain-rate invariant I but increases the temperature. This is due to the fact that these hardening effects increase the material's flow stress and cause more plastic working which is converted into heat. The thermal softening has a noticeable effect on the temperature distribution at the penetrator nose. From the plot of the strain-hardening parameter ψ on the penetrator nose and on the axial line in Fig. 10,



Fig. 10. Effect of different material parameters on the strain-hardening parameter ψ . Explanations see Fig. 8

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Fig. 11. Effect of strain-hardening, strain-rate hardening and thermal softening on relative z-velocity, temperature change and the hydrostatic pressure at points on the axial line. Explanations see Fig. 8

one may conclude that the thermal softening reduces its value significantly, especially at points near the nose surface. On the axial line as well as on the nose surface, strain-rate hardening in turn increases strain-hardening. We should add that a steady-state penetration problem is being studied and thus it is tacitly assumed that the increased energy required for deforming the target is available whenever necessary. Fig. 11 depicts the variation of the hydrostatic pressure, temperature and relative z-velocity on the axial line. The hydrostatic pressure, the temperature and the absolute z-velocity of a target particle on the axial line increase with the inclusion of hardening effects but change very little by the consideration also of thermal softening. In order to investigate further the effect of thermal softening, we arbitrarily doubled the value of the thermal softening parameter a. This increased the value of the strain-rate invariant I but changed very little the value of other quantities on the penetrator nose.

On the axial line uniaxial strain conditions prevail approximately. Thus the magnitude of the deviatoric stress s_{zz} at a point should equal 2/3 the effective flow stress σ_e defined as

$$\sigma_e = \sigma_0 (1+bI)^m (1-a\theta) (1+\psi/\psi_0)^n.$$

Of all the points on the axial line, the nose tip is the most critical one since the strain-hardening parameter ψ assumes very high values there. In Table 1 below are compared the values of $(-s_{zz})$ and $2/3\sigma_e$ at the nose tip. Whereas the error is negligible when the target material is rigid/perfectly plastic, it is rather high for the other three cases. A possible reason for the high error is that values of ψ at the center are sensitive to the value of the artificial viscosity δ eventhough other field variables show negligible dependence upon the precise value of δ within a certain range. To support this reasoning, we list in Table 2 values of the same variables but with the effect of strain-hardening neglected. Note that these are for a higher value of the speed of the penetrator.

Finally we remark that results presented here are valid only for the constitutive model used herein.

| Table 1. Values of s., and $2/3\sigma$, at the nose-tip for different | ent material models ($\alpha = 1$) |
|--|--------------------------------------|
|--|--------------------------------------|

| Model | $-\sigma_{u}$ | - <i>p</i> | -\$ ₁₁ | 2/3σ, | % differ | ence |
|--|-----------------------|----------------------|---------------------|----------------------|---------------------|---------|
| Perfectly plastic Strain-hardening + Rate dependence | 6.34 7.05 11.07 | 5.64 6.36 9.88 | 0.7 0.69 1.19 | 0.67 1.12 2.01 | 4.5 40.2 40.7 | Sec. 20 |
| + Thermal softening | 11.10 | 10.25 | 0.85 | 1.42 | 40.1 | 1000 |

Table 2. Values of s_{rr} and $2/3\sigma_r$ at the nose-tip for different material models ($\alpha = 5$)

| Model | -σ, | | | 2/3σ. | % difference |
|---------------------|------|--------------------|------|-------|--------------|
| Perfectly plastic | 8.9 | 7 | 0.62 | 0.67 | 7.46 |
| + Thermal softening | 12.3 | 6 11.26 8 11.39 | 0.89 | 0.96 | 7.29 |



Fig. 12. Variation of strain-hardening parameter ψ in the target region. ($\alpha = 3.0, r_n/r_0 = 2.0$)

Conclusion

The computed results show that during the steady state portion of the penetration process, the penetrator nose shape has a significant effect on the deformations of the target. Whereas the strainrates are higher for the sharper ellipsoidal nose, deformations spread to a larger distance away from the nose surface for the blunt nose. The speed of the penetrator has a weak effect on the axial resisting force experienced by the penetrator even though the hydrostatic pressure does increase with the speed. The inclusion of thermal softening effects increases the strain-rate in the target material but does not alter the pressure distribution on the penetrator nose.

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