

Original Article

Effective properties of a piezocomposite containing shape memory alloy and inert inclusions

B. Jiang, R. C. Batra

Department of Engineering Science and Mechanics, MC 0219, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

Received July 2, 2001 / Published online February 4, 2002 – © Springer-Verlag 2002
Communicated by Kolumban Hutter, Darmstadt

We use the energy equivalence principle and the Mori-Tanaka method of considering the interaction among inclusions to derive the effective thermo-electro-mechanical properties of a 4-phase composite consisting of an elastic matrix and shape memory alloy, piezoelectric and inert (nonpiezoelectric) inclusions. It is shown that the shapes and the volume fraction of inert (e.g. air) inclusions significantly influence the effective properties of the composite, and the addition of soft inert inclusions decreases the axial stress required to initiate the phase transformation in the SMA inclusions, and increases the electromechanical coupling constant. The 4-phase composite makes a very good sensor to measure low values of the applied axial stress since the axial strain induced by an axial stress of 80 MPa is 2%, and the sensitivity of the sensor is improved by the addition of soft spherical inert inclusions. With a suitable choice of the matrix material, its compliance can be adjusted and its shape made to conform to that of the host structure.

Key words: smart composite, effective moduli, Eshelby tensors, static deformations, ellipsoidal inclusions

1 Introduction

A “smart composite” comprised of piezoelectric ceramic (PZT*) and/or shape memory alloy (SMA) inclusions embedded in a polymer matrix can be designed to control its shape when subjected to different loads. However, the manufacturing process inevitably introduces in the composite nonpiezoelectric (NPZT) or inert inclusions such as voids. Soon after the appearance of barium titanate as a useful piezoelectric ceramic, researchers at the Naval Research Laboratory embedded it in a polymer matrix to make a flexible hydrophone material (Smith, 1989). Since then a variety of piezocomposite materials, usually called piezocomposites, have been manufactured by suitably combining a PZT with a polymer phase (e.g. see the web site <http://www.matsysinc.com/piezopg.html>). Piezocomposites have been used as electromechanical and medical ultrasonic imaging transducers. They are more suitable for damage monitoring since their impedance can be matched with that of the host structure and their mechanical properties can be tailored as needed. A cost effective piezocomposite with the desired electromechanical coupling constants and impedance can be manufactured by controlling the shape, the size and the volume fraction of the PZTs, and the materials of the two phases. Here we investigate effective electromechanical properties of a four phase composite comprised of PZT, SMA, and NPZT inclusions in a polymer matrix with the ultimate goal of ascertaining if the addition

Dedicated to Professor Ingo Müller on the occasion of his 65th birthday

* The abbreviation PZT is used for a generic piezoelectric ceramic rather than any specific material.

of suitable SMA and NPZT inclusions can be exploited to improve upon properties of the piezocomposite. We emphasize at the outset that not every known effect in a PZT and a SMA has been incorporated in the analysis. In order to keep the analysis tractable, only infinitesimal deformations are considered. This is reasonable because the ultimate tensile strain of the brittle PZT material is about 0.1 to 0.2%. Stress induced phase transformations in the SMA are considered because that is the primary actuation mechanism for an SMA. However, hysteresis and domain switching effects in the PZT and temperature induced phase transformations in the SMA have not been considered. Accordingly, when hysteresis and domain switching effects in the PZT are important, the effective moduli derived here are applicable only for isothermal monotonic mechanical or electric loading of the piezocomposite along the axis of polarization of the PZT inclusions. These material and geometric nonlinearities and coupled thermo-electro-mechanical effects can be considered in a numerical study which is not pursued here. Nevertheless expressions given here for the effective electroelastic moduli can be used to decide which combination of PZT, SMA and NPZT inclusions in a polymer matrix will be most suitable for achieving a desirable goal. Also, the goal of finding the optimum combination of PZT, SMA, and matrix materials and the NPZT inclusions for maximizing a given system property is not pursued here; however tools to achieve this goal are developed. The SMA inclusions are assumed to be electrically insulated from their surrounding medium so that they do not short circuit the electric potential applied to the bounding surfaces. Thus the composite can exhibit piezoelectric properties. We briefly review below some of the previous work on porous piezocomposites.

Ting (1990) observed that the hydrostatic piezoelectric coefficient of an 1-3 piezocomposite increases when cavities are added to the matrix. Lynn et al. (1981), Klicker et al. (1982) and Haun et al. (1983) found that making the matrix of an 1-3 piezocomposite porous enhances its sensitivity to the hydrostatic pressure. In the notation 1-3 the 1 refers to the one-dimensionally connected PZT phase, and the 3 to the three-dimensionally connected polymer or matrix phase. An 1-3 piezocomposite with a porous matrix is denoted as 1-3-0; the 0 referring to the polymer porosity which is not connected in the composite.

Hikita et al. (1983) have investigated the effect of pores on the electromechanical properties of a porous piezocomposite. Paul and Nelson (1996) have studied the effect of voids on the flexural vibrations of a composite hollow cylinder. Haun and Newnham (1986) have developed a model to predict the effective electroelastic constants of an 1-3-0 piezocomposite comprised of one large void containing PZT rods in the center of the composite. However, this geometric configuration is rather unrealistic. Avellaneda and Swart (1998) employed the effective medium approach to delineate the performance of a porous 1-3 piezocomposite and accounted for the effect of pores by modifying Young's modulus of the matrix. Their approach does not account for shapes of voids, the interaction among them, and the change in their shapes during the deformation process.

Here we develop constitutive relations for thermoelectroelastic deformations of a 4-phase composite made of PZT, NPZT and SMA inclusions embedded in an elastic matrix, and account for shapes of voids, interactions among inclusions, and phase transformations in the SMA. Note that cavities or air bubbles can be modeled as NPZT inclusions. The composite body is assumed to be at a uniform temperature, and expressions for effective thermal moduli such as thermal conductivity are not derived. In a previous paper (Jiang and Batra, 2001) we used the mean field theory and the Mori-Tanaka method to derive macroscopic constitutive relations of a 3-phase composite consisting of PZT and SMA inclusions embedded in an elastic matrix. Here we use the energy equivalence principle and deduce expressions for the effective moduli of a 4-phase composite wherein the fourth phase is the NPZT inclusions. These expressions are used to deduce effective moduli of the 4-phase composite made of an elastic matrix, PZT inclusions made of BaTiO_3 , SMA inclusions made of a NiTi alloy, and the NPZT inclusions are made of a material either harder or softer than the elastic matrix. No attempt has been made to select the best combination of materials from the ones commercially available to optimize a material modulus. It is shown that many of the thermoelectroelastic properties of the 4-phase composite are superior to those of the 3-phase composite when the NPZT inclusions of appropriate shapes and material are added to the 3-phase composite.

We note that the response of a SMA inclusion can be modeled as either pseudoelastic (Müller, 1998) or thermo-elasto-plastic (e.g. see Cherkaoui et al., 2000) and it undergoes a phase transformation with a change in stress and/or temperature. Thus the macroscopic response of the 4-phase composite studied herein will be thermo-electro-elasto-plastic.

2 Representative volume element

Consider a representative volume element (RVE) of randomly oriented SMA, PZT and NPZT inclusions embedded in an NPZT matrix. The RVE encompasses large enough volume so that its effective properties are invariant with respect to its rigid translations within the body. In rectangular Cartesian coordinates, linear constitutive relations for the constituents of the 4-phase composite are (e.g., see Lines and Glass, 1977)

$$\sigma_{ij} = C_{ijkl}^r (\varepsilon_{kl} - \bar{\varepsilon}_{kl}^r) - e_{kij}^r E_k - \lambda_{ij}^r \theta, \quad (1)$$

$$D_i = e_{ikl}^r \varepsilon_{kl} + k_{ij}^r E_j + p_i^r \theta, \quad (2)$$

$$s = \lambda_{ij}^r \varepsilon_{ij} + p_i^r E_i + \omega^r \theta, \quad (3)$$

where $r = M, P, N$ and S for the matrix, PZT, NPZT and SMA inclusions respectively. Note that

$$\bar{\varepsilon}_{kl}^r = 0 \text{ for } r \neq S, \quad (4a)$$

and

$$p_i^r = 0 \text{ and } e_{ikl}^r = 0 \text{ for } r \neq P. \quad (4b)$$

Here σ_{ij} is the stress tensor, ε_{ij} the strain tensor appropriate for infinitesimal deformations, $\bar{\varepsilon}_{ij}^S$ the transformation strain in the SMA, E_i the electric field, D_i the electric displacement, θ the change in the temperature from that in the reference configuration, and s the entropy density. Furthermore, $C_{ijkl} = C_{jikl} = C_{klij}$ are the elasticities, λ_{ij} the stress-temperature moduli, k_{ij} the dielectric constants, p_i the pyroelectric constants, $e_{ijk} = e_{ikj}$ the piezoelectric constants, and ω the change in entropy density caused by a unit change in the temperature. A repeated index implies summation over the range of the index. For $r = S$ the transformation induced stress represented by $C_{ijkl}^S \bar{\varepsilon}_{kl}^S$ essentially smears the austenite/martensite together into a homogeneous medium. A more rigorous treatment would have been to consider the martensite moduli and the transformation strain to compute this transformation induced stress. However, for practical purposes, the approach followed herein should be adequate.

We note that two phases, namely austenite and martensite, can coexist in the SMA, and their volume fractions depend upon the state of stress and the temperature. Here we assume that the temperature is kept constant and study only the stress induced phase transformations. The material properties of an SMA can be determined from those of the austenite and the martensite phases by using either the mean field theory or the energy equivalence method or the rule of mixtures. Lu and Weng (2000) regarded the martensite phase as an inclusion in the austenite phase of the SMA to estimate its effective moduli. However, we follow Boyd and Lagoudas (1994) and assume that the rule of mixtures approximates well the overall properties of an SMA; this simplifies the work significantly without introducing appreciable errors. Thus

$$C_{ijkl}^S = (1 - \xi) C_{ijkl}^a + \xi C_{ijkl}^m \quad (5)$$

and similar relations hold for α_{ij}^S , k_{ij}^S , p_i^S and ω^S , where α_{ij}^S is the coefficient of thermal expansion of the SMA. In (5) superscripts a and m signify quantities for the austenite and the martensite respectively, and the volume fraction ξ of the martensite phase is given by

$$\xi = \frac{\bar{\varepsilon}_e}{\bar{\varepsilon}_{\max}}, \quad \bar{\varepsilon}_e = \left(\frac{2}{3} \bar{\varepsilon}_{ij} \bar{\varepsilon}_{ij} \right)^{\frac{1}{2}}, \quad \bar{\varepsilon}_{ij} = \bar{\varepsilon}_{ij}^S, \quad (6)$$

where $\bar{\varepsilon}_{\max}$ is the maximum value of the equivalent transformation strain $\bar{\varepsilon}_e$, which is regarded as a material property (Song et al., 1999).

3 Free energy of the RVE

Taking the infinitesimal strains, the electric field and the temperature as independent variables, the electric Gibbs free energy density, g , for electro-thermo-mechanical deformations of a body is given by (e.g. see Lines and Glass, 1977)

$$g = \int_{(0,0,0)}^{(\varepsilon_{ij}, E_i, \theta)} (\sigma_{ij} d\varepsilon_{ij} - D_i dE_i - s d\theta). \quad (7)$$

Since the electric Gibbs free energy density is a state function, the integration in (7) is path independent. Substitution from constitutive relations (1)-(3) into (7) yields

$$\begin{aligned} g^r &= \frac{1}{2}(\sigma_{ij}\varepsilon_{ij} - D_i E_i - s\theta), \quad r = M, P, N, \\ g^S &= \frac{1}{2}(\sigma_{ij}\varepsilon_{ij} - D_i E_i - s\theta) - \frac{1}{2}C_{ijkl}^S \varepsilon_{ij} \bar{\varepsilon}_{kl}. \end{aligned} \quad (8)$$

In the derivation of (8)₂, the transformation strain $\bar{\varepsilon}_{ij}$ in the SMA has been taken to be a constant; in general, $\bar{\varepsilon}_{ij}$ will be a function of ε_{ij} . A possibility is to set $\bar{\varepsilon}_{ij}$ equal to the mean value of the transformation strain during the deformation process. The average electric Gibbs free energy density of the RVE of the composite is related to that of its constituents by

$$G = \frac{1}{v} \int_v g dv = \frac{1}{v} \sum_{r=M,P,S,N} \int_{v^r} g^r dv = \sum_{i=1}^4 G^i, \quad (9)$$

where v is the volume of the RVE, v^r is the volume of the r -th phase, and

$$\begin{aligned} G^1 &= \frac{1}{2v} \int_v \sigma_{ij} \varepsilon_{ij} dv, \quad G^2 = -\frac{1}{2v} \int_v D_i E_i dv, \\ G^3 &= -\frac{1}{2v} \int_v s \theta dv, \quad G^4 = -\frac{1}{2v} \int_{v^S} \bar{\varepsilon}_{ij} C_{ijkl}^S \varepsilon_{kl} dv. \end{aligned} \quad (10)$$

Let the essential boundary conditions prescribed on the bounding surfaces of the RVE correspond to the uniform strain ε_{ij}^0 , the uniform electric field E_i^0 , and the uniform temperature change θ^0 in the RVE. Then

$$\begin{aligned} \int_v \sigma_{ij} \varepsilon_{ij} dv &= \varepsilon_{ij}^0 \int_v \sigma_{ij} dv, \quad \int_v \varepsilon_{ij} dv = \varepsilon_{ij}^0 v, \\ \int_v D_i E_i dv &= E_i^0 \int_v D_i dv, \quad \int_v E_i dv = E_i^0 v. \end{aligned} \quad (11)$$

Substituting from constitutive relations (1)-(3) into (10) and using relations (11), we arrive at

$$\begin{aligned} G^1 &= \frac{1}{2} \varepsilon_{ij}^0 \left\{ C_{ijkl}^M \varepsilon_{kl}^0 - \lambda_{ij}^M \theta^0 + \sum_{r=P,S,N} f^r \langle \Delta C_{ijkl}^r \varepsilon_{kl} - \Delta \lambda_{ij}^r \theta \rangle_r - f^P \langle e_{kij}^P E_k \rangle_P - f^S \langle C_{ijkl}^S \bar{\varepsilon}_{kl} \rangle_S \right\}, \\ G^2 &= -\frac{1}{2} E_i^0 \left\{ k_{ij}^M E_j^0 + p_i^M \theta^0 + \sum_{r=P,S,N} f^r \langle \Delta k_{ij}^r E_j + \Delta p_i^r \theta \rangle_r + f^P \langle e_{ikl}^P \varepsilon_{kl} \rangle_P \right\}, \\ G^3 &= -\frac{1}{2} \theta^0 \left\{ \lambda_{ij}^M \varepsilon_{ij}^0 + p_i^M E_i^0 + \omega^M \theta^0 + \sum_{r=P,S,N} f^r \langle \Delta \lambda_{ij}^r \varepsilon_{ij} + \Delta p_i^r E_i + \Delta \omega^r \theta \rangle_r - f^S \langle \Delta \lambda_{ij}^S \bar{\varepsilon}_{ij} \rangle_S \right\}, \\ G^4 &= -\frac{1}{2} f^S \langle \bar{\varepsilon}_{ij} C_{ijkl}^S \varepsilon_{kl} \rangle_S, \end{aligned} \quad (12)$$

where f^r ($r = P, S, N$) is the volume fraction of the r -th phase,

$$\Delta C_{ijkl}^r = C_{ijkl}^r - C_{ijkl}^M, \quad (r = P, S, N), \quad (13)$$

and similar relations hold for $\Delta \lambda_{ij}^r$, Δk_{ij}^r , Δp_i^r and $\Delta \omega^r$. In (12),

$$\langle (\cdot) \rangle_r = \frac{1}{v^r} \int_{v^r} (\cdot) dv, \quad r = P, S, N. \quad (14)$$

In terms of the effective thermoelectroelastic moduli $\bar{C}_{ijkl}$, $\bar{e}_{ijk}$, $\bar{k}_{ij}$, $\bar{\lambda}_{ij}$, $\bar{p}_i$, $\bar{\omega}$ and fields, $\bar{\sigma}_{ij}^*$, $\bar{D}_i^*$, $\bar{s}^*$ and $\bar{G}^*$ of the RVE, the energy density of the RVE can be written as

$$\begin{aligned} \bar{G} = \frac{1}{2} \{ & \varepsilon_{ij}^0 \bar{C}_{ijkl} \varepsilon_{kl}^0 - 2E_i^0 \bar{e}_{ikl} \varepsilon_{kl}^0 - E_i^0 \bar{k}_{ij} E_j^0 - 2\theta^0 \bar{\lambda}_{ij} \varepsilon_{ij}^0 \\ & - 2\theta^0 \bar{p}_i E_i^0 - \theta^0 \bar{\omega} \theta^0 + 2\bar{\sigma}_{ij}^* \varepsilon_{ij}^0 - 2\bar{D}_i^* E_i^0 - 2\bar{s}^* \theta^0 - \bar{G}^* \}, \end{aligned} \quad (15)$$

where $\bar{G}^*$ is the initial value of the electric Gibb's free energy, and $\bar{\sigma}_{ij}^*$, $\bar{D}_i^*$ and $\bar{s}^*$ are, respectively, values of the residual stress, the residual electric displacement and the residual entropy density caused by the transformation strain. These are determined from the transformation strain $\bar{\varepsilon}_{ij}$, the electromechanical properties of the matrix and the inclusions, and the method used to account for the interaction among inclusions and the matrix.

The principle of energy equivalence states that

$$\sum_{i=1}^4 G^i = \bar{G}. \quad (16)$$

However, from (16), one cannot obtain effective properties of the composite because one does not know strains and electric fields within the inclusions. Therefore, we employ approximate methods to get strains and electric fields from fields applied on the bounding surfaces of the RVE and eigenfields within the inclusions.

4 Effective thermoelectroelastic properties of the composite

Recalling that boundary conditions applied on the boundaries of the RVE correspond to the uniform strain ε_{ij}^0 , the uniform electric field E_i^0 , and the uniform temperature change θ^0 , the average strain $\bar{\varepsilon}_{ij}^r$ and the average electric field $\hat{E}_i^r$ in the PZT ($r = P$), the SMA ($r = S$) and the NPZT ($r = N$) inclusions are given by

$$\begin{aligned} \bar{\varepsilon}_{ij}^r &= L_{ijkl}^r \varepsilon_{kl}^0 + N_{ijk}^r E_k^0 + R_{ij}^r \theta^0 + \varepsilon_{ij}^{**r}, \\ \hat{E}_i^r &= P_{ikl}^r \varepsilon_{kl}^0 + Q_{ik}^r E_k^0 + J_i^r \theta^0 + E_i^{**r}. \end{aligned} \quad (17)$$

Expressions for tensors L_{ijkl}^r , N_{ijk}^r , R_{ij}^r , P_{ikl}^r , Q_{ik}^r , J_i^r and for the interaction tensors, ε_{ij}^{**r} , and E_i^{**r} that account for the interaction among inclusions vary with the approximate method used to derive them. For example, if the dilute solution method is used to obtain $\bar{\varepsilon}_{ij}^r$ and $\hat{E}_i^r$, then tensors $\mathbf{L}$, $\mathbf{N}$ etc. equal the thermoelectroelastic Eshelby tensors for a single inclusion embedded in an infinite matrix, and ε_{ij}^{**r} and E_i^{**r} equal respectively $\bar{\varepsilon}_{ij}^r$ and 0 in the inclusion. However, if the interaction among inclusions is considered, then tensors $\mathbf{L}$, $\mathbf{N}$, ε_{ij}^{**r} and E_i^{**r} etc. can be obtained by the Mori-Tanaka method, the self-consistent estimates or other approximate methods. Expressions for tensors $\mathbf{L}$, $\mathbf{N}$ etc. and interaction fields ε_{ij}^{**r} and E_i^{**r} based on the Mori-Tanaka method are given in the Appendix.

Substituting from (17) into (12) and the results into (16), and considering the arbitrariness of the applied fields, we obtain following expressions for the effective thermoelectroelastic moduli of the composite:

$$\begin{aligned}
\bar{C}_{ijkl} &= C_{ijkl}^M + \text{sym} \left\{ \sum_{r=P,S,N} f^r \langle \Delta C_{ijmn}^r L_{mnkl}^r \rangle_r - f^P \langle e_{mij}^P P_{mkl}^P \rangle_P \right\}, \\
\bar{e}_{kij} &= \frac{1}{2} \left\{ f^P \langle e_{kmn}^P L_{mnij}^P + e_{mij}^P Q_{mk}^P \rangle_P + \sum_{r=P,S,N} f^r \langle \Delta k_{km}^r P_{mij}^r - \Delta C_{ijmn}^r N_{mnk}^r \rangle_r \right\}, \\
\bar{k}_{ij} &= k_{ij}^M + \text{sym} \left\{ \sum_{r=P,S,N} f^r \langle \Delta k_{im}^r Q_{mj}^r \rangle_r + f^P \langle e_{imn}^P N_{mnj}^P \rangle_P \right\}, \\
\bar{\lambda}_{ij} &= \lambda_{ij}^M + \frac{1}{2} \left\{ f^P \langle e_{mij}^P J_m^P \rangle_P + \sum_{r=P,S,N} f^r \langle \Delta \lambda_{mn}^r (L_{mnij}^r + I_{mnij}^r) + \Delta p_m^r P_{mij}^r - \Delta C_{ijmn}^r R_{mn}^r \rangle_r \right\}, \\
\bar{p}_i &= p_i^M + \frac{1}{2} \left\{ f^P \langle e_{imn}^P R_{mn}^P \rangle_P + \sum_{r=P,S,N} f^r \langle \Delta k_{ij}^r J_j^r + \Delta \lambda_{mn}^r N_{mni}^r + \Delta p_m^r (Q_{mi}^r + \delta_{mi}) \rangle_r \right\}, \\
\bar{\omega} &= \omega^M + \sum_{r=P,S,N} f^r \langle \Delta \omega^r + \Delta \lambda_{ij}^r R_{ij}^r + \Delta p_i^r J_i^r \rangle_r, \\
\bar{\sigma}_{ij}^* &= \frac{1}{2} \left\{ \sum_{r=P,S,N} f^r \langle \Delta C_{ijmn}^r \varepsilon_{mn}^{**r} \rangle_r - f^P \langle e_{mij}^P E_m^{**P} \rangle_P - f^S \langle \bar{\varepsilon}_{mn} C_{mnpq}^S (I_{pqij}^S + L_{pqij}^S) \rangle_S \right\}, \\
\bar{D}_i^* &= \frac{1}{2} \left\{ \sum_{r=P,S,N} f^r \langle \Delta k_{ij}^r E_j^{**r} \rangle_r + f^P \langle e_{imn}^P \varepsilon_{mn}^{**P} \rangle_P + f^S \langle \bar{\varepsilon}_{mn} C_{mnpq}^S N_{pqi}^S \rangle_S \right\}, \\
\bar{s}^* &= \frac{1}{2} \left\{ \sum_{r=P,S,N} f^r \langle \Delta \lambda_{ij}^r \varepsilon_{ij}^{**r} + \Delta p_i^r E_i^{**r} \rangle_r - f^S \langle \bar{\varepsilon}_{ij} (\lambda_{ij}^S - C_{ijmn}^S R_{mn}^S) \rangle_S \right\}, \\
\bar{G}^* &= f^S \langle \bar{\varepsilon}_{ij} C_{ijmn}^S \varepsilon_{mn}^{**S} \rangle_S.
\end{aligned} \tag{18}$$

Here “sym” stands for the symmetric part of a tensor, i.e., $\text{sym}(A_{ijkl}) = \frac{1}{2}(A_{ijkl} + A_{klij})$, and $\text{sym}(B_{ij}) = \frac{1}{2}(B_{ij} + B_{ji})$, and I_{ijkl} is the fourth-order unit tensor, viz., $I_{ijkl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2$ where δ_{ij} is the Kronecker delta.

In the space of strain, electric field and temperature, constitutive relations of the composite obtained from

$$\sigma_{ij}^0 = \frac{\partial G}{\partial \varepsilon_{ij}^0}, \quad D_i^0 = -\frac{\partial G}{\partial E_i^0}, \quad s^0 = -\frac{\partial G}{\partial \theta^0}, \tag{19}$$

are

$$\begin{aligned}
\sigma_{ij}^0 &= \bar{C}_{ijkl} \varepsilon_{kl}^0 - \bar{e}_{kij} E_k^0 - \bar{\lambda}_{ij} \theta^0 + \bar{\sigma}_{ij}^*, \\
D_i^0 &= \bar{e}_{ikl} \varepsilon_{kl}^0 + \bar{k}_{ij} E_j^0 + \bar{p}_i \theta^0 + \bar{D}_i^*, \\
s^0 &= \bar{\lambda}_{ij} \varepsilon_{ij}^0 + \bar{p}_i E_i^0 + \bar{\omega} \theta^0 + \bar{s}^*.
\end{aligned} \tag{20}$$

Note that effective elasticities $\bar{\mathbf{C}}$ and effective dielectric constants $\bar{\mathbf{k}}$ exhibit symmetries $\bar{C}_{ijkl} = \bar{C}_{klij}$ and $\bar{k}_{ij} = \bar{k}_{ji}$ even when the shapes of the PZT, SMA and NPZT inclusions are different. However, when effective properties are derived by the mean field theory, effective elasticities $\bar{\mathbf{C}}$ and effective dielectric constants $\bar{\mathbf{k}}$ may not exhibit such symmetries; e.g. see Qiu and Weng (1990).

If the independent variables are stress, electric field and temperature, the thermodynamic function is the Gibbs free energy F which can be obtained from the electric Gibbs free energy G by the Legendre transformation

$$F = -(G - \sigma_{ij}^0 \varepsilon_{ij}^0), \tag{21}$$

where σ_{ij}^0 is given by (20)₁. Therefore, we have

$$\begin{aligned}
F &= \frac{1}{2} \left\{ \sigma_{ij}^0 \tilde{M}_{ijkl} \sigma_{kl}^0 + 2E_i^0 \tilde{d}_{ikl} \sigma_{kl}^0 + E_i^0 \tilde{k}_{ij} E_j^0 + 2\theta^0 \tilde{\alpha}_{ij} \sigma_{ij}^0 \right. \\
&\quad \left. + 2\theta^0 \tilde{p}_i E_i^0 + \theta^0 \tilde{\omega} \theta^0 + 2\tilde{\varepsilon}_{ij}^* \sigma_{ij}^0 + 2\tilde{D}_i^* E_i^0 + 2\tilde{s}^* \theta^0 + \tilde{G}^* \right\},
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\tilde{M}_{ijkl} &= \bar{C}_{ijkl}^{-1}, & \tilde{d}_{ikl} &= \bar{e}_{imn} \tilde{M}_{mnkl}, \\
\tilde{k}_{ij} &= \bar{k}_{ij} + \tilde{d}_{imn} \bar{e}_{jmn}, & \tilde{\alpha}_{ij} &= \tilde{M}_{ijkl} \bar{\lambda}_{kl}, \\
\tilde{p}_i &= \bar{p}_i + \bar{e}_{imn} \tilde{\alpha}_{mn}, & \tilde{\varepsilon}_{ij}^* &= -\tilde{M}_{ijkl} \bar{\sigma}_{kl}^*, \\
\tilde{D}_i^* &= \bar{D}_i^* + \bar{e}_{imn} \tilde{\varepsilon}_{mn}^*, & \tilde{\omega} &= \bar{\omega} + \tilde{\alpha}_{ij} \bar{\lambda}_{ij}, \\
\tilde{s}^* &= \bar{s}^* + \bar{\lambda}_{ij} \tilde{\varepsilon}_{ij}^*, & \tilde{G}^* &= \bar{G}^* - \bar{\sigma}_{ij}^* \tilde{\varepsilon}_{ij}^*.
\end{aligned} \tag{23}$$

Thus, constitutive relations of the composite in the space of stress, electric field and temperature obtained from

$$\varepsilon_{ij}^0 = \frac{\partial F}{\partial \sigma_{ij}^0}, \quad D_i^0 = \frac{\partial F}{\partial E_i^0}, \quad s^0 = \frac{\partial F}{\partial \theta^0}, \tag{24}$$

are

$$\begin{aligned}
\varepsilon_{ij}^0 &= \tilde{M}_{ijkl} \sigma_{kl}^0 + \tilde{d}_{pij} E_p^0 + \tilde{\alpha}_{ij} \theta^0 + \tilde{\varepsilon}_{ij}^*, \\
D_i^0 &= \tilde{d}_{ikl} \sigma_{kl}^0 + \tilde{k}_{ip} E_p^0 + \tilde{p}_i \theta^0 + \tilde{D}_i^*, \\
s^0 &= \tilde{\alpha}_{ij} \sigma_{ij}^0 + \tilde{p}_i E_i^0 + \tilde{\omega} \theta^0 + \tilde{s}^*.
\end{aligned} \tag{25}$$

5 Phase transformations in the SMA

The evolution of the volume fraction, ξ , of the martensite depends upon the state of stress and temperature in the SMA. The stress state in the SMA is given by

$$\sigma_{ij}^S = U_{ijkl}^S \sigma_{kl}^0 + V_{ijk}^S E_k^0 + W_{ij}^S \theta^0 + \sigma_{ij}^{**S}, \tag{26}$$

which is analogous to (17). Here

$$\begin{aligned}
U_{ijkl}^S &= H_{ijmn}^{5S} L_{mnkl}^\sigma, \quad V_{ijk}^S = H_{ijmn}^{5S} N_{mnk}^\sigma, \\
W_{ij}^S &= F_{ij}^{3S} + H_{ijmn}^{5S} R_{mn}^\sigma, \quad \sigma_{ij}^{**S} = S_{ijkl}^{5S} \bar{\varepsilon}_{kl} - f^S U_{ijkl}^S \langle S_{klmn}^{5S} \bar{\varepsilon}_{mn} \rangle_S,
\end{aligned} \tag{27}$$

and expressions for tensors H_{ijkl}^{5S} , S_{ijkl}^{5S} , L_{ijkl}^σ , N_{ijk}^σ and R_{ij}^σ are given in the Appendix. When all of the SMA inclusions are likewise oriented, we have

$$\sigma_{ij}^{**S} = Y_{ijkl}^S \bar{\varepsilon}_{kl}, \tag{28}$$

where

$$Y_{ijkl}^S = (I_{ijmn} - f^S U_{ijmn}^S) S_{mnkl}^{5S}. \tag{29}$$

Let $\dot{\Sigma}_{ij}^0$ equal the change of stress in the SMA inclusions due to the variation of the applied fields σ_{ij}^0 , E_i^0 and θ^0 , and $\dot{\Sigma}_{ij}^\xi$ the change of stress in the SMA due to the evolution of the martensite phase. That is

$$\begin{aligned}
\dot{\Sigma}_{ij}^0 &= U_{ijkl}^S \dot{\sigma}_{kl}^0 + V_{ijk}^S \dot{E}_k^0 + W_{ij}^S \dot{\theta}^0, \\
\dot{\Sigma}_{ij}^\xi &= \left(\frac{\partial U_{ijkl}^S}{\partial \xi} \sigma_{kl}^0 + \frac{\partial V_{ijk}^S}{\partial \xi} E_k^0 + \frac{\partial W_{ij}^S}{\partial \xi} \theta^0 + \frac{\partial Y_{ijkl}^S}{\partial \xi} \bar{\varepsilon}_{kl} \right) \dot{\xi} + Y_{ijkl}^S \dot{\bar{\varepsilon}}_{kl},
\end{aligned} \tag{30}$$

where a superimposed dot indicates the change or the increment. Using (6)₁ and (6)₂, one can express $\dot{\xi}$ in terms of $\dot{\bar{\varepsilon}}_{ij}$. Following Boyd and Lagoudas (1994), Song et al. (1999) and Cherkaoui et al. (2000), we assume that

$$\bar{\varepsilon}_{ij} = \begin{cases} \dot{A}s_{ij}^S, & \text{for the forward transformation,} \\ \dot{A}^{re}\bar{\varepsilon}_{ij} & \text{for the reverse transformation,} \end{cases} \quad (31)$$

where $\dot{A}$ and $\dot{A}^{re}$ are proportionality factors to be determined from the consistency condition, and s_{ij}^S is the deviatoric stress tensor in the SMA inclusion. Note that dimensions of $\dot{A}$ and $\dot{A}^{re}$ are different. Finding the incremental change in quantities from (26) and using (6), (30) and (31), we arrive at

$$\dot{A} = - \frac{s_{ij}^S \dot{\Sigma}_{ij}^0 - \frac{2}{3} s_e^S \frac{\partial s_e^{fS}}{\partial \theta} \dot{\theta}^0}{s_{ij}^S \left[\frac{2}{3\bar{\varepsilon}_{\max}\bar{\varepsilon}_e} \dot{\Sigma}_{ij}^\xi (\bar{\varepsilon}_{kl} s_{kl}^S) + Y_{ijkl}^S s_{kl}^S \right]}, \quad (32)$$

for the forward (austenite $\rightarrow$ martensite) transformation, and

$$\dot{A}^{re} = - \frac{s_{ij}^S \dot{\Sigma}_{ij}^0 - \frac{2}{3} s_e^S \frac{\partial s_e^{rS}}{\partial \theta} \dot{\theta}^0}{s_{ij}^S \left[\frac{\bar{\varepsilon}_e}{\bar{\varepsilon}_{\max}} \dot{\Sigma}_{ij}^\xi + Y_{ijkl}^S \bar{\varepsilon}_{kl} \right]} \quad (33)$$

for the reverse (martensite $\rightarrow$ austenite) transformation. Here

$$s_e^S = \left(\frac{3}{2} s_{ij}^S s_{ij}^S \right)^{\frac{1}{2}} \quad (34)$$

is the equivalent or the effective or the von Mises stress. The forward transformation initiates when $s_e^S = s_e^{fS}(\theta)$ and the reverse transformation begins when $s_e^S = s_e^{rS}(\theta)$. Note that $s_e^{fS}(\theta)$ and $s_e^{rS}(\theta)$ are temperature dependent material parameters. However, in the present work, the temperature is assumed to be uniform throughout the body and is held constant. Also, $\dot{\theta}^0 = 0$.

The transformation relation (31) smears out finer details of the phase transformation in the SMA. At the microscopic level, both the austenite and the martensite are orthotropic materials, and the transformation strain depends upon lengths of Burger's vectors and rotations of lattice vectors. Lu and Weng (2000) consider some of these effects in developing constitutive relations of a composite consisting of an elastomer matrix and SMA inclusions.

6 Comparison with results from other models

To the authors's knowledge, there is no experimental data available in the open literature on the effective moduli of a 4-phase composite comprised of a polymer matrix, PZT, SMA and NPZT inclusions. Therefore, we can compare results with either a 2-phase or a 3-phase composite. In order to establish the validity of our results, we note that present values of the effective thermoelectroelastic moduli of a 3-phase composite with PZT and SMA inclusions of the same shape embedded in an elastic matrix (i.e. $f^N = 0$) reduce to those of Jiang and Batra (2001) who derived them by using the mean field theory. The formulae of the effective properties are different in the two approaches, but the numerical values come out to be same. However, when the PZT and the SMA inclusions are of different shapes, the energy equivalence method used here and the mean field theory approach adopted in Jiang and Batra (2001) may not give same values of the effective electroelastic moduli. For example, the energy equivalence method preserves the symmetries $\bar{C}_{ijkl} = \bar{C}_{klij}$ and $\bar{k}_{ij} = \bar{k}_{ji}$ but the mean field theory does not. In the absence of SMA and NPZT inclusions, i.e., $f^N = f^S = 0$, values of the effective moduli are identical with those of Jiang et al. (1999). Jiang et al.'s results compared well with experimental findings of Chan and Unsworth (1989).

Setting $f^S = 0$, and taking the PZT inclusions as cylindrical with their axis of polarization and hence of transverse isotropy along the x_3 -axis and the NPZT inclusions as spherical voids, we compare in Figs. 1–3 present values of the hydrostatic charge coefficient, $d_h = (\tilde{d}_{3ii})$, the hydrostatic figure of merit, $d_h g_h (= d_h / \tilde{k}_{33})$, and the hydrostatic electromechanical coupling factor $k_h (= d_h^2 / (\tilde{k}_{33} m))$, $m = (2(\tilde{M}_{1111} + \tilde{M}_{1122}) + 4\tilde{M}_{1133} + \tilde{M}_{3333})$

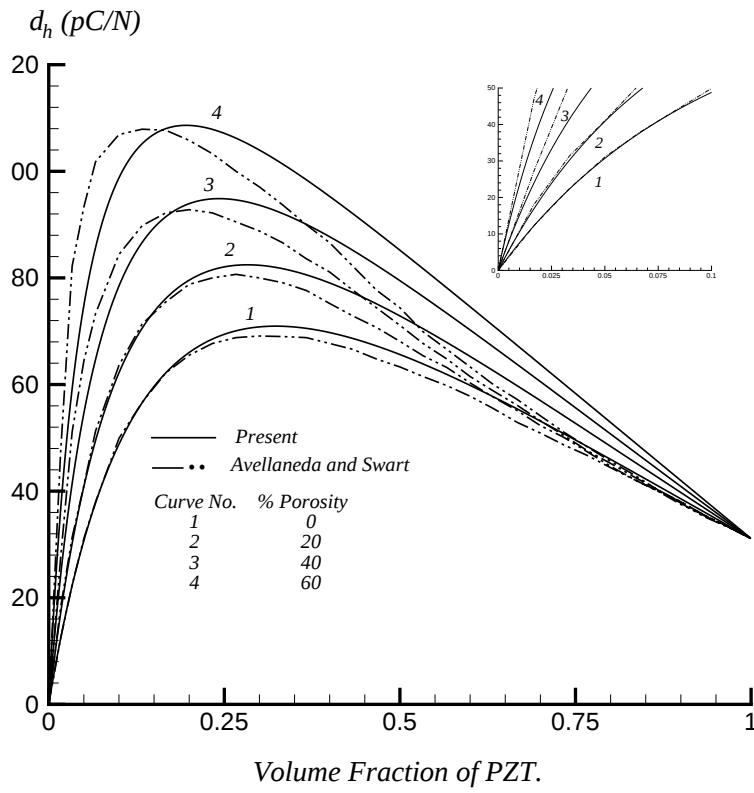


Fig. 1. For different values of the porosity, the variation of the hydrostatic charge coefficient, d_h , with the volume fraction of piezoceramic particles. Results are compared with those of Avellaneda and Swart

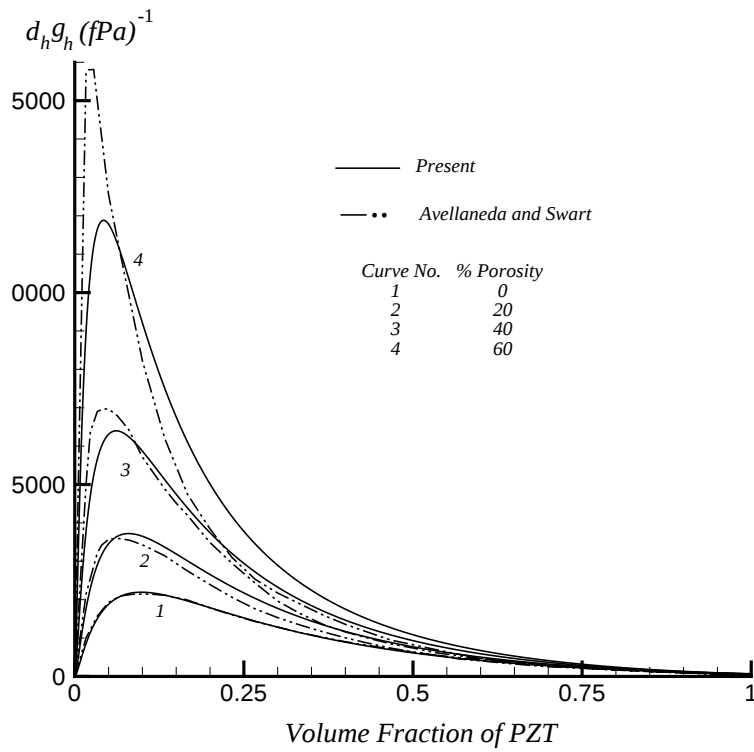


Fig. 2. For different values of the porosity, the variation of the figure of merit, $d_h g_h$, with the volume fraction of piezoceramic particles. Results are compared with those of Avellaneda and Swart

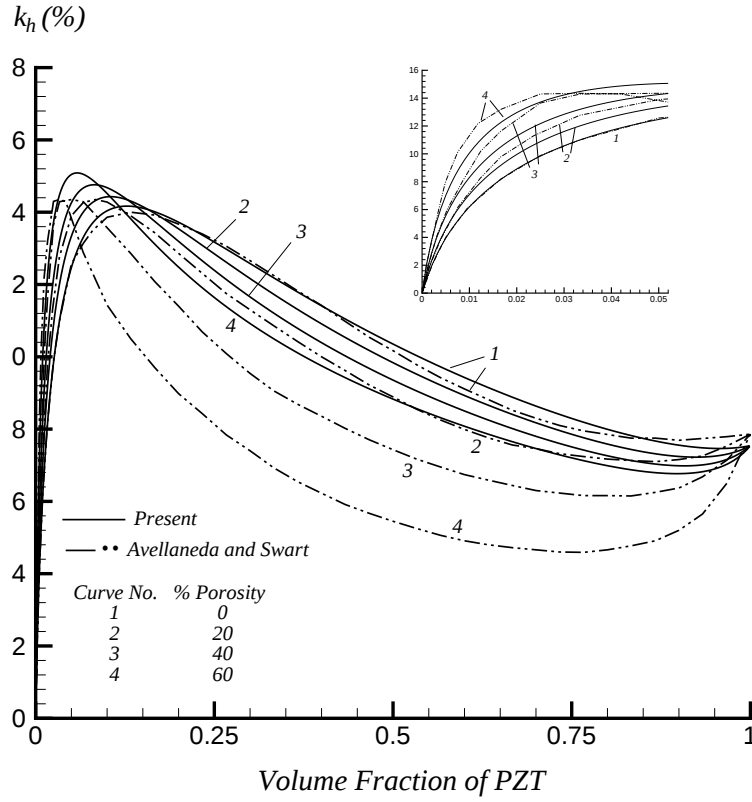


Fig. 3. For different values of the porosity, the variation of the hydrostatic electromechanical coupling factor, k_h , with the volume fraction of piezoceramic particles. Results are compared with those of Avellaneda and Swart

with those obtained by Avellaneda and Swart (1998). We have taken Young's modulus of spherical voids equal to zero, Poisson's ratio equal to that of the stycast matrix which is modeled as an isotropic material and the relative dielectric constant equal to 1. We presume that voids are filled with air, and their volume fraction, f^N , is determined from

$$f^N = \phi(1 - f^P)/(1 - \rho^{\text{air}}/\rho^{\text{stycast}}) \simeq \phi(1 - f^P), \quad (35)$$

where ϕ is the porosity. Equation (35)₂ follows from (35)₁ since $\rho^{\text{air}} \ll \rho^{\text{stycast}}$. Values of material moduli used in the calculations are listed in Table 1. Results plotted in Figs. 1 through 3 reveal that values of d_h , $d_h g_h$ and k_h computed with the present method are close to those obtained by Avellaneda and Swart (1998) for small volume fractions of the PZT inclusions. Whereas the two sets of results agree qualitatively for large values of f^P and f^N , they differ quantitatively. Possible reasons for these differences are the neglect in Avellaneda and Swart's work of the interaction among the PZT5A inclusions, and of the effect of their shapes and orientations. Avellaneda and Swart set the dielectric constant of the porous matrix equal to that of the stycast material and account for the presence of pores by modifying Young's modulus of the stycast matrix.

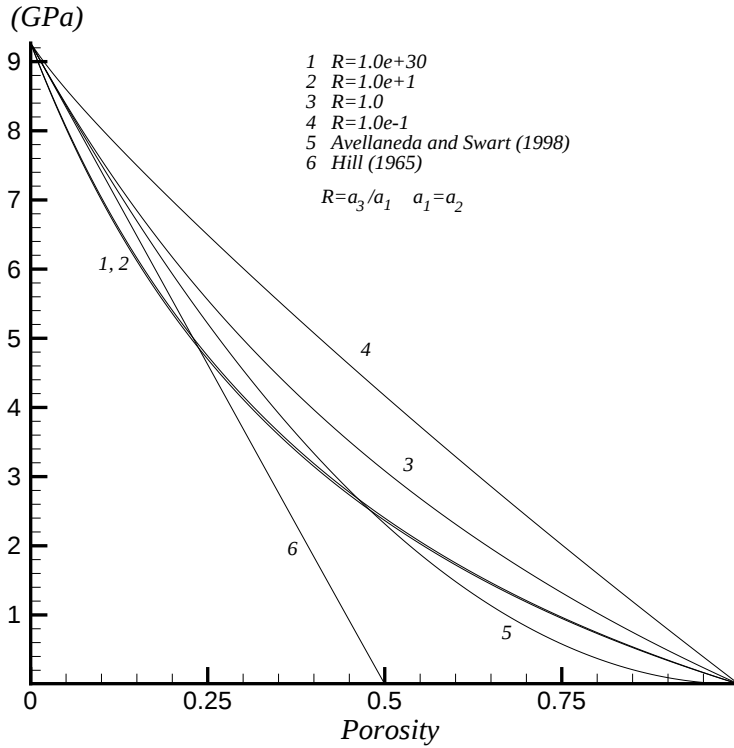
For the same volume fraction of the PZT inclusions, values of the hydrostatic charge coefficient and of the hydrostatic figure of merit increase with an increase in the porosity of the composite. The same trend holds for the electromechanical coupling factor k_h only for small values of the volume fraction of the PZT inclusions. For $f^P > 0.075$, an increase in the porosity of the composite decreases the value of the electromechanical coupling factor. We note that the dielectric constant of air is less than that of the stycast, and the interactions among densely distributed PZT5A inclusions play a significant role in influencing the electroelastic moduli of a piezocomposite. Also, as shown below, these moduli are very sensitive to the shapes and the orientations of the inclusions.

Figure 4 depicts the variation of Young's modulus, E , in the x_3 -direction with the porosity for an isotropic linear elastic matrix containing ellipsoidal voids of aspect ratio R ; similar results hold for the bulk modulus. The principal axes of lengths a_1 , a_2 and a_3 of the ellipsoid are aligned along the x_1 , x_2 and x_3 axes respectively,

Table 1. Values of nonvanishing material moduli used to compare results with those of Avellaneda and Swart

	PZT 5A	Stycast
C_{1111} (GPa)	120.0	12.34
C_{1122} (GPa)	75.18	5.19
C_{1133} (GPa)	75.09	5.19
C_{3333} (GPa)	111.0	12.34
C_{1313} (GPa)	21.1	3.575
e_{131} (C/m ²)	12.3	0
e_{311} (C/m ²)	−5.4	0
e_{333} (C/m ²)	15.8	0
k_{11}/k_0	916.0	4.0
k_{33}/k_0	830.0	4.0

$$k_0 = 8.85 \times 10^{-12} \text{ C/Vm}^2$$

**Fig. 4.** Variation of Young's modulus with the porosity for different shapes of ellipsoidal voids; a_1 , a_2 and a_3 are principal axes of the ellipsoid along x_1 , x_2 and x_3 axes respectively. Results are compared with those obtained by the Mori-Tanaka method, the self consistent technique, and those assumed by Avellaneda and Swart who took it from Gibson and Ashby's book

and $R = a_3/a_1$, $a_1 = a_2$. Large values of the aspect ratio R imply that the pores are cylindrical, and small values are for penny shaped pores. For penny shaped pores, E decreases nearly affinely with the increase in the porosity. The rule of mixtures gives an affine decrease of E with an increase in the value of ϕ ; e.g. see Passman and Batra (1984). Passman and Batra, like Cowin and Nunziato (1983) allow for the deformations of voids and thus the porosity changes as the body is deformed. However, they do not account for the shapes of the pores. For the same value of the porosity, cylindrical pores of height/diameter ≥ 10 change Young's modulus by the same amount, and the decrease in the value of E is more for cylindrical pores than that for the spherical and the penny shaped pores. The parabolic dependence of E upon the porosity ϕ employed by Avellaneda and Swart, who took it from Gibson and Ashby's (1988) book, agrees with that obtained by presuming spherical pores only if $\phi \leq 0.1$.

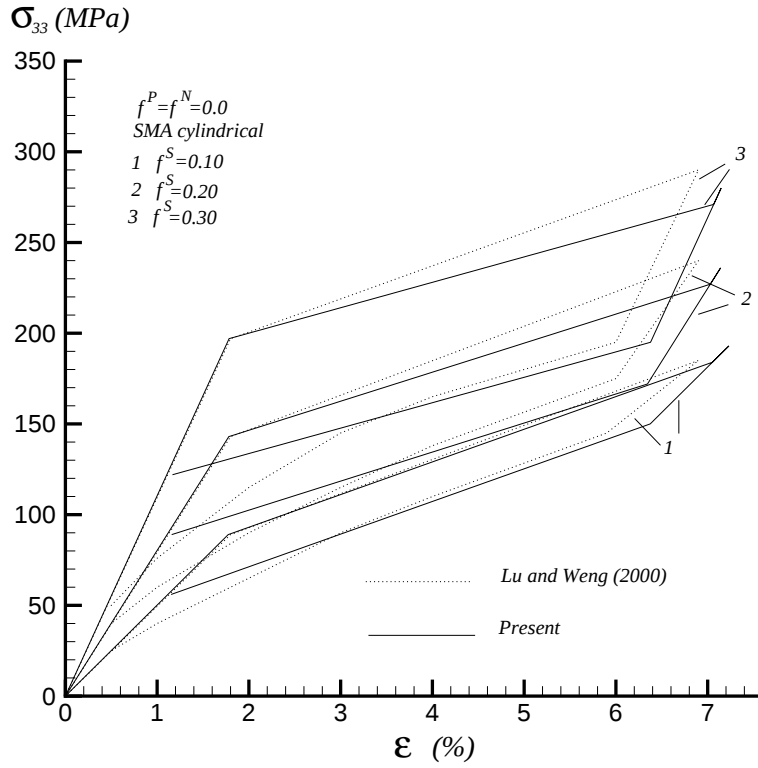


Fig. 5. Axial stress vs. axial strain for a composite comprised of an elastic matrix and SMA inclusions

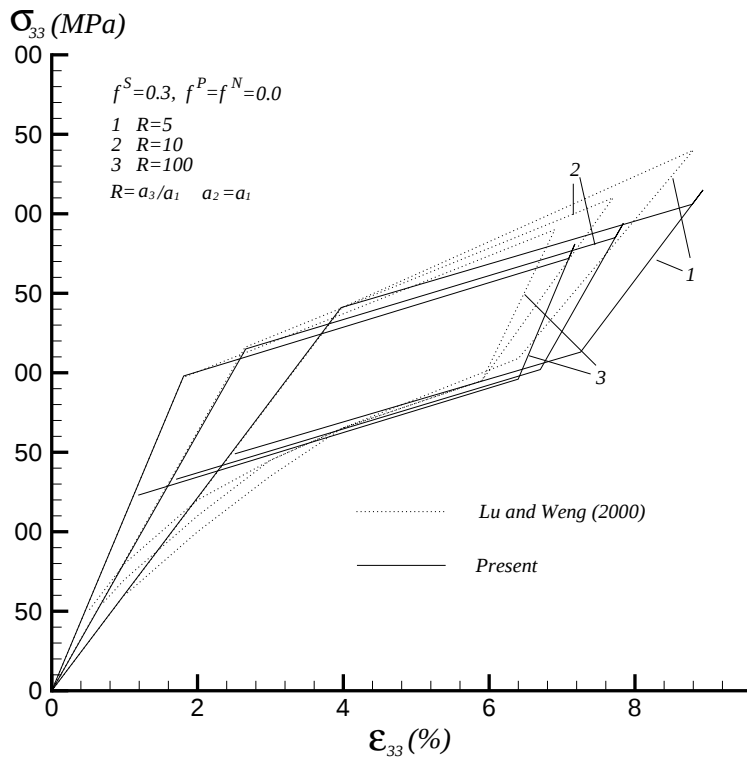


Fig. 6. Effect of the shape of the prolate SMA inclusions on the axial stress vs. the axial strain for a composite made of an elastic matrix and SMA inclusions

Figures 5 and 6 compare the presently computed effective axial stress vs. the effective axial strain behavior of a composite made of an elastomer matrix and SMA inclusions with those of Lu and Weng (2000). Material properties of the elastomer and the SMA are given in Lu and Weng's paper. They employed a two-level micromechanical theory, one at the inclusion (austenite-martensite) level, and the other at the macroscopic level consisting of the transforming inclusions and the inactive matrix. Here we have glossed over the first one by using the rule of mixtures (5) to derive properties of the SMA from those of its constituents. Figure 5 exhibits results for three values of the volume fractions of cylindrical SMA inclusions and Fig. 6 for three shapes ($R = 5, 10, 100$) of the 30% prolate SMA inclusions. Results from the two approaches agree qualitatively but differ quantitatively because of the different phase transformation criteria and the difference in deriving properties of the SMA inclusions from its two phases.

Having satisfactorily compared the presently computed values of the effective moduli of the two-phase and the three-phase composites with those obtained from other models and/or experimental results, we conclude that the derived expressions for the effective moduli are correct.

7 Results and discussion

We now give results for the 4-phase composite made of an elastic matrix, PZT inclusions made of BaTiO_3 , SMA inclusions made of a NiTi alloy, and NPZT inclusions. Values of nonvanishing moduli of these materials are listed in Table 2, and all of the material moduli of the NPZT inclusions are set equal to β times the corresponding material moduli of the matrix. The SMA, the matrix and the NPZT materials are modeled as isotropic and the PZT as transversely isotropic with the x_3 -axis as the axis of polarization and hence of transverse isotropy. The surfaces of the SMA inclusions are assumed to be electrically insulated from the surrounding media so that they do not short circuit the electric field applied to the boundaries of the RVE. We investigate cases for which the PZT and the SMA inclusions are either cylindrical or spherical and the NPZT inclusions are either cylindrical, spherical or penny shaped. The temperature of the composite in the stress free reference configuration is taken to be uniform and equal to 10°C ; it is assumed to stay constant throughout the deformation process. Results have been computed with $f^P = f^S = 0.15$, f^N is taken as either 0 or 0.15, and $\beta = 10^{-3}$ and 10^2 which respectively model soft and hard particles.

7.1 Axial stress-axial strain relations

Figures 7 through 10 display effects of the shapes of the soft and the hard NPZT inclusions on the axial stress vs. the axial strain relations of the 4-phase composite loaded axially in the x_3 -direction. These results have been computed from (25)₁ by setting $\mathbf{E}^0 = \mathbf{0}$, $\theta^0 = 0$ and $\sigma_{ij}^0 = \sigma_{33}^0 \delta_{i3} \delta_{j3}$. It is clear that the energy dissipated during cyclic axial load, as indicated by the area of the hysteresis loop, is more for the spherical PZT and SMA inclusions than that for the cylindrical PZT and SMA inclusions. Also, the applied axial stress at the initiation of the phase transformation in the SMA inclusions is a little lower for the cylindrical PZT and SMA inclusions than that for the spherical PZT and SMA inclusions. Note that the average axial strain in the cylindrical SMA and PZT inclusions and the matrix is the same. However, in general, the lateral strains in them will be different. For the spherical SMA and PZT inclusions, even the axial strains in the three phases need not be equal to each other.

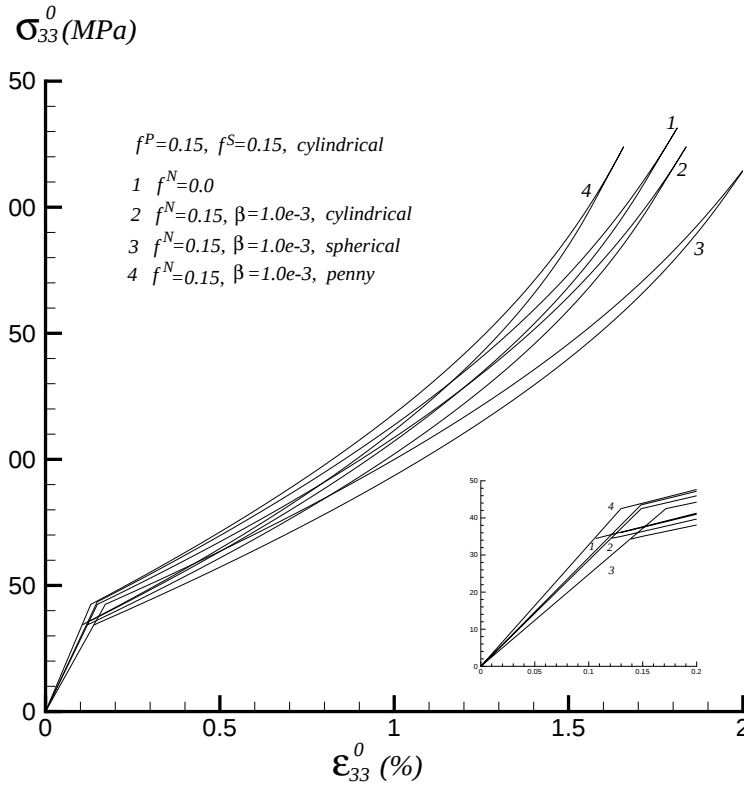
Irrespective of the shapes of the NPZT inclusions, hard NPZT particles which sustain more of the applied tractions than the remaining constituents of the composite increase the applied axial stress required to initiate the forward transformation in the SMA inclusions; this may not be very desirable for the hybrid composite. However, as should be evident from the results plotted in Figs. 7 and 9, the soft NPZT particles of the three shapes considered herein decrease the applied axial stress required to ensue the forward phase transformation in the SMA inclusions; the exception being the penny shaped NPZT particles with the cylindrical PZT and SMA inclusions in which case the axial stress required at the initiation of the transformation is increased a little. When no phase transformation occurs in the SMA inclusions, the axial macroscopic strain induced by the applied axial stress is more for soft cylindrical and spherical NPZT particles than that for soft penny shaped particles. When the phase transformation is in progress, for the same value of the axial stress soft

Table 2. Values of nonvanishing material moduli of the constituents of the 4-phase composite

	Polymer ^(a)	BaTiO ₃ ^(a)	Austenite ^(b)	Martensite ^(b)
C_{1111} (GPa)	8.0	150.0	143.57	56.36
C_{1122} (GPa)	4.4	66.0	95.71	37.57
C_{1133} (GPa)	4.4	66.0	95.71	37.57
C_{3333} (GPa)	8.0	146.0	143.57	56.36
C_{1313} (GPa)	1.8	44.0	23.93	9.39
e_{311} (C/m ²)	0	-4.35	0	0
e_{333} (C/m ²)	0	17.5	0	0
e_{131} (C/m ²)	0	11.4	0	0
$k_{11}/k_0^{(c)}$	4.0	1115.0	0	0
k_{33}/k_0	4.0	1260.0	0	0
λ_{11} (MPa/°C)	1.008	1.974	3.685	0.8679
λ_{33} (MPa/°C)	1.008	1.471	3.685	0.8679
$p_3(10^{-3}\text{C}/\text{m}^2\text{ }^\circ\text{C})$	0	1.877	0	0
At 10°C, $s_e^{fS} = 100$ MPa, $s_e^{rS} = 80$ MPa, $\bar{\varepsilon}_{\max} = 0.048$				
$\frac{\partial s_e^{fS}}{\partial \theta} = \frac{\partial s_e^{rS}}{\partial \theta} = 8$ MPa/°C (Ref. (b))				

(a) Dunn, M.D., 1993;

(b) Song, G.Q., Sun, Q.P. and Cherakoui, C., 1999;

(c) $k_0 = 8.85 \times 10^{-12}$ C/Vm²**Fig. 7.** Effect of the shapes of the soft NPZT particles on the axial stress vs. the axial strain curves of the 4-phase composite comprised of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

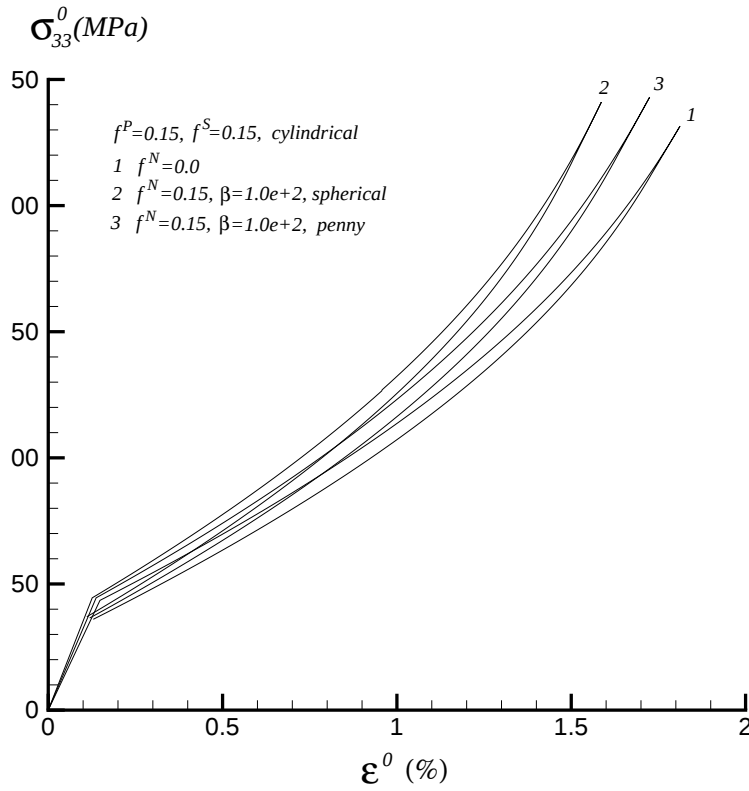


Fig. 8. Effect of the shapes of the hard NPZT particles on the axial stress vs. the axial strain curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

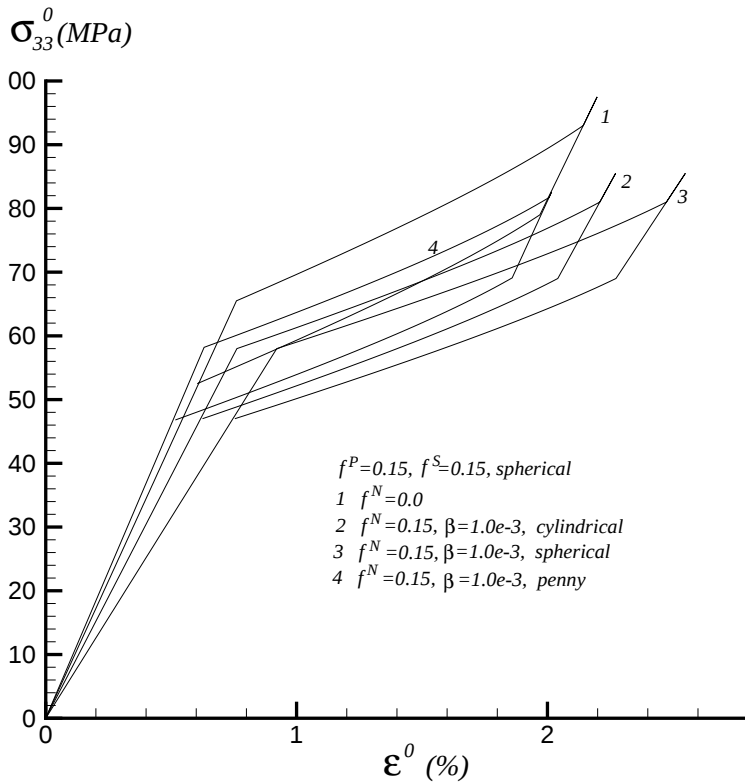


Fig. 9. Effect of shapes of the soft NPZT particles on the axial stress vs. the axial strain curves of the 4-phase composite made of an isotropic elastic matrix and spherical PZT and SMA inclusions

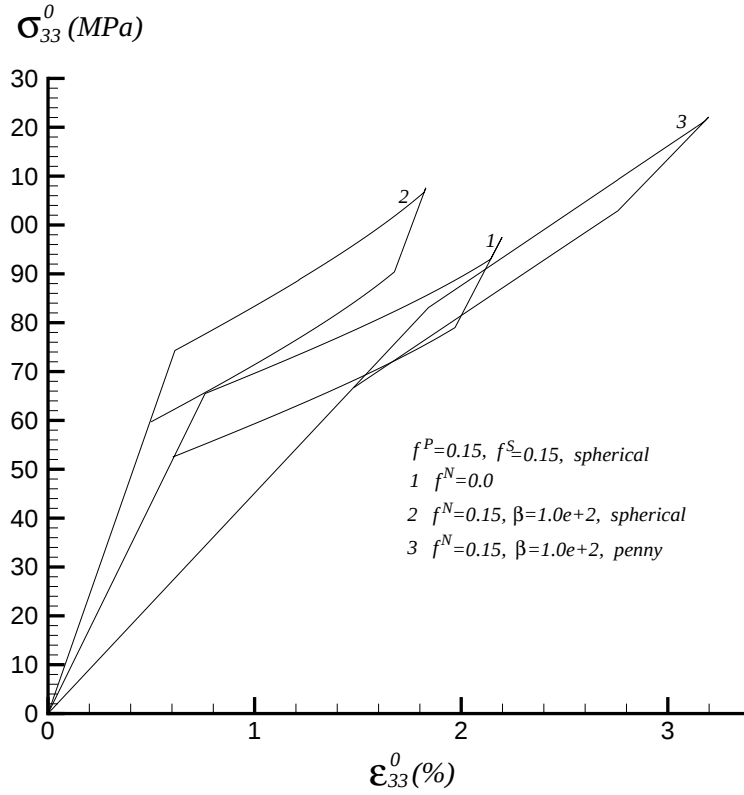


Fig. 10. Effect of shapes of the hard NPZT particles on the axial stress vs. the axial strain curves of the 4-phase composite comprised of an isotropic elastic matrix and spherical PZT and SMA inclusions

cylindrical, spherical and the penny shaped NPZT inclusions increase the macroscopic axial strain if the SMA and the PZT inclusions are spherical (cf. Fig. 9). However, as shown in Fig. 7, for cylindrical SMA and PZT inclusions and the same value of the applied axial stress, soft cylindrical and spherical NPZT particles increase the macroscopic axial strain but the penny shaped ones decrease it. Thus the shapes of all three inclusions and the interaction among them strongly influence the elastic properties of the 4-phase composite. An examination of the results plotted in Figs. 7 through 10 suggests that soft spherical NPZT particles change most the properties of the composite and this effect is more noticeable for spherical PZT and SMA inclusions than for cylindrical PZT and SMA inclusions. Prior to the initiation of the phase transformation in the SMA inclusions, the NPZT particles have very little effect on the elastic moduli of the 4-phase composite containing cylindrical PZT and SMA inclusions but the effect is significant when the PZT and the SMA inclusions are spherical.

Generally, the ultimate tensile strain of a PZT is about 0.1% to 0.2%. For the composite containing cylindrical PZT and SMA inclusions, the axial strain in the PZT inclusions is nearly the same as the macroscopic axial strain in the composite. However, for the composite with spherical PZT and SMA inclusions, the maximum principal tensile strain induced in the PZT inclusions is below 0.2% even at the completion of the phase transformation in the SMA inclusions when the macroscopic axial strain is about 2%. The addition to it of soft spherical NPZT inclusions increases this macroscopic strain. Thus the 4-phase composite can be used as a sensor to effectively measure low values of the applied axial stress. Since elasticities of the PZT material are higher than those of the matrix and the SMA, it is conceivable that a 3-phase composite comprised of the polymer matrix, soft NPZT inclusions and the SMA inclusions will be a better sensor than the 4-phase composite that also has PZT inclusions. An advantage of the composite sensor over that made of pure SMA is that the former can be made compliant and its impedance adjusted to match that of the host structure.

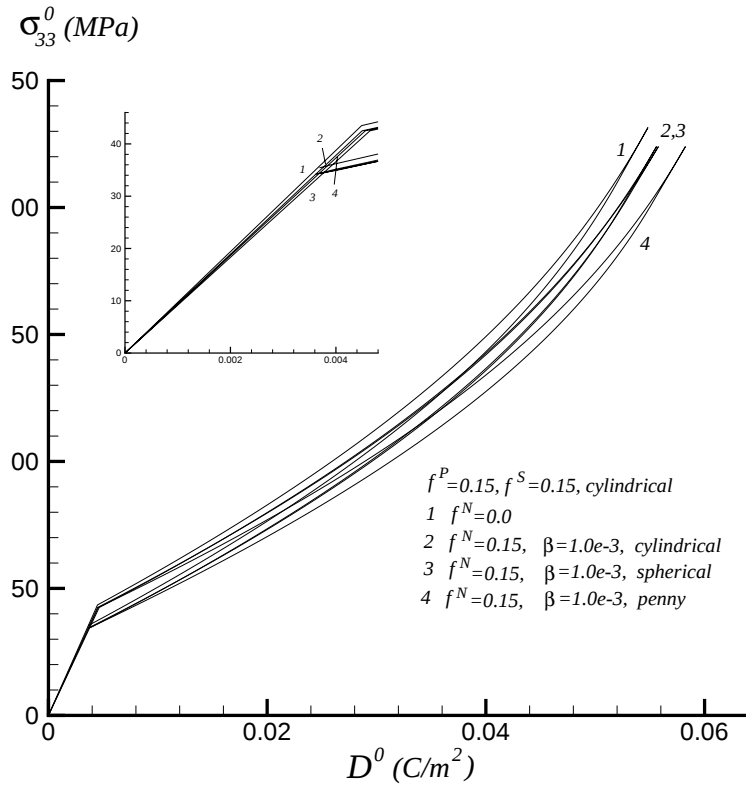


Fig. 11. Effect of shapes of the soft NPZT particles on the axial stress vs. the axial electric displacement curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

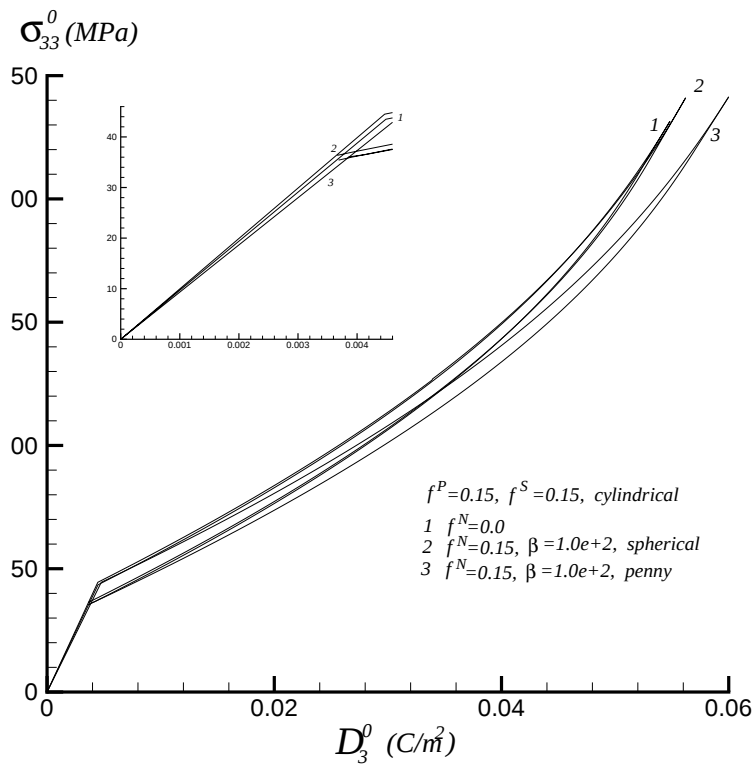


Fig. 12. Effect of shapes of the hard NPZT particles on the axial stress vs. the axial electric displacement curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

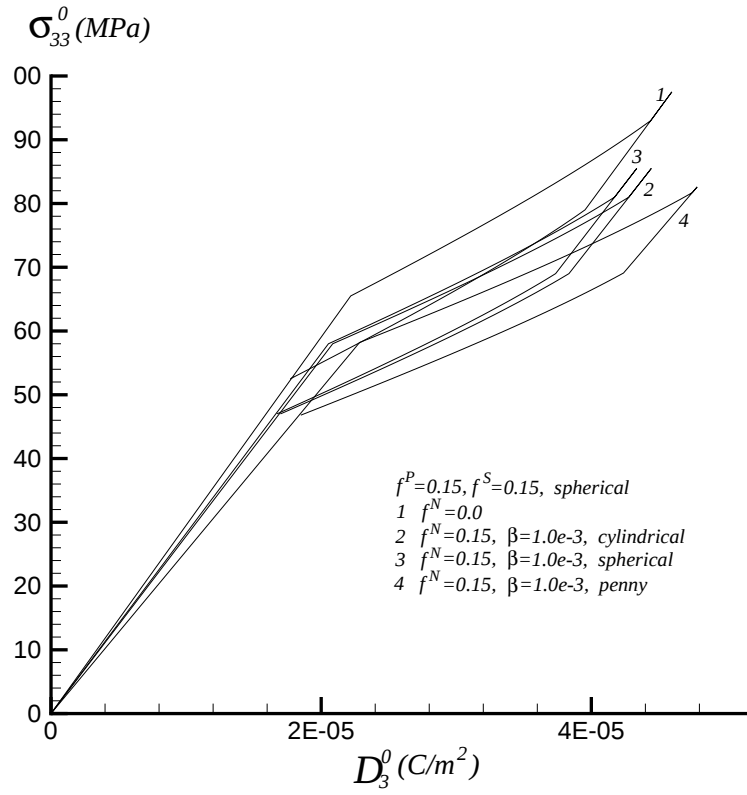


Fig. 13. Effect of shapes of the soft NPZT inclusions on the axial stress vs. the axial electric displacement curves of the 4-phase composite made of an isotropic elastic matrix and spherical PZT and SMA inclusions

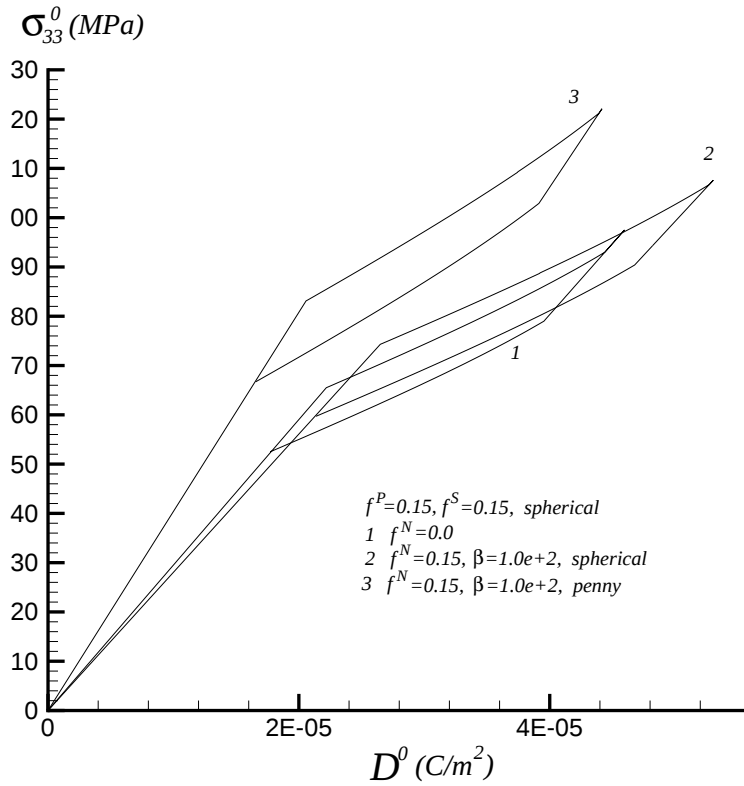


Fig. 14. Effect of shapes of the hard NPZT inclusions on the axial stress vs. the axial electric displacement curves of the 4-phase composite made of an isotropic elastic matrix and spherical PZT and SMA inclusions

7.2 Axial stress-axial electric displacement relations

We have plotted in Figs. 11 through 14 the axial electric displacement, D_3^0 , induced by the applied axial stress, σ_{33}^0 . These values are computed from (25)₂ by setting $\mathbf{E}^0 = \mathbf{0}$, $\theta^0 = 0$ and $\sigma_{ij}^0 = \sigma_{33}^0 \delta_{i3} \delta_{j3}$. For cylindrical PZT and SMA inclusions the addition of either soft or hard cylindrical, spherical and penny shaped NPZT inclusions does not noticeably affect the electroelastic modulus $\tilde{d}_{333}$ which equals the reciprocal of the slope of the σ_{33}^0 vs. D_3^0 curve. However, when the PZT and the SMA inclusions are spherical (cf. Fig. 13), the addition of soft NPZT inclusions noticeably alters the value of $\tilde{d}_{333}$, and the change in its value depends upon the shape of the NPZT inclusions. Out of the cylindrical, spherical and penny shaped soft NPZT inclusions, the penny shaped ones increase most the value of $\tilde{d}_{333}$. However, as shown in Fig. 14, hard penny shaped NPZT particles significantly decrease the value of $\tilde{d}_{333}$. Thus one way to improve upon the electromechanical coupling constant $\tilde{d}_{333}$ of the 3-phase composite made of an elastic matrix and spherical SMA and PZT inclusions is to add soft penny shaped NPZT particles to it.

7.3 Axial electric field-axial strain relations

As was the case for the 3-phase hybrid composite comprised of the spherical PZT and SMA inclusions (Jiang and Batra, 2001), a very high electric field is required to induce phase transformation in the SMA particles even when NPZT inclusions are added to the 3-phase composite. Recall that the SMA inclusions are assumed to be electrically insulated from the surrounding medium and the heat produced by the electric field has not been considered. Therefore, we study the 4-phase composite only with cylindrical SMA and PZT inclusions. Results computed from (25)₁ by setting $\sigma^0 = \mathbf{0}$, $E_i^0 = E_3^0 \delta_{i3}$ and $\theta^0 = 0$ and plotted in Fig. 15 reveal that the addition of soft spherical NPZT particles increases the axial electric field required to induce the forward phase transformation in the SMA inclusions; however, the addition of penny shape NPZT particles decreases it. During the initial stage of the transformation, soft penny shape NPZT particles significantly increase the macroscopic strain of the composite. However at the completion of the phase transformation in the SMA, the macroscopic axial strain in the composite is less than that for no NPZT inclusions and with either spherical or cylindrical PZT and SMA inclusions. From the plots of Fig. 16 we conclude that the addition of hard NPZT particles decreases the applied electric field required to induce the initial forward phase transformation in the SMA particles. During the early stages of the phase transformation, either spherical or penny shape NPZT particles increase the electromechanical coupling properties of the 4-phase composite. The macroscopic axial strain in the composite at the completion of the phase transformation is more for penny shape NPZT particles than that for spherical NPZT particles.

Most of the commercially available PZTs can withstand an electric field of about 10 MV/m. Thus cylindrical PZT inclusions do not improve the effective piezoelectric constants of the 4-phase composite even when rigid NPZT inclusions are added to the composite. However, when the piezoelectric constants, e_{ijk} , of the PZT are doubled, the applied electric field required to induce the phase transformation in the SMA inclusion is essentially halved.

7.4 Axial electric field-axial electric displacement relations

From the results obtained from (25)₂ and plotted in Figs. 17 and 18, we see that the addition of hard spherical or penny shape NPZT particles increases the dielectric constant of the 4-phase composite, because hard particles have large dielectric constant. However, the addition of even soft cylindrical or penny shape NPZT particles (cf. Fig. 17) whose dielectric constants are smaller than those of the matrix also enhances the dielectric constant of the 4-phase composite.

7.5 Numerical values of effective moduli

In Tables 3 through 6 we have listed the computed values of the nonvanishing effective electromechanical moduli of the 4-phase composite with the x_3 -axis aligned along the polarization axis of the PZT inclusions.

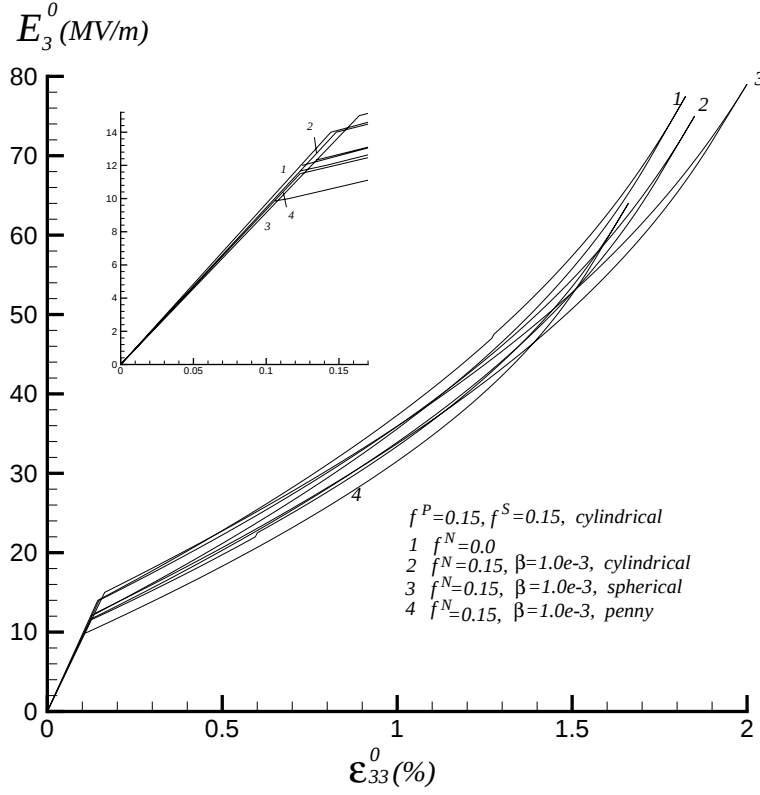


Fig. 15. Effect of shapes of the soft NPZT particles on the axial electric field vs. the axial strain curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

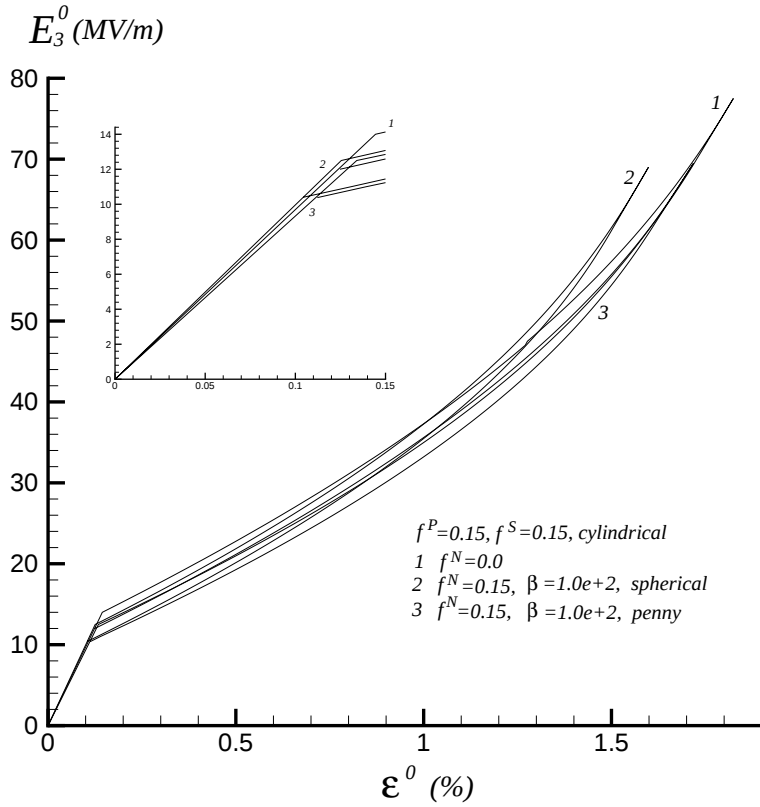


Fig. 16. Effect of shapes of the hard NPZT particles on the axial electric field vs. the axial strain curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

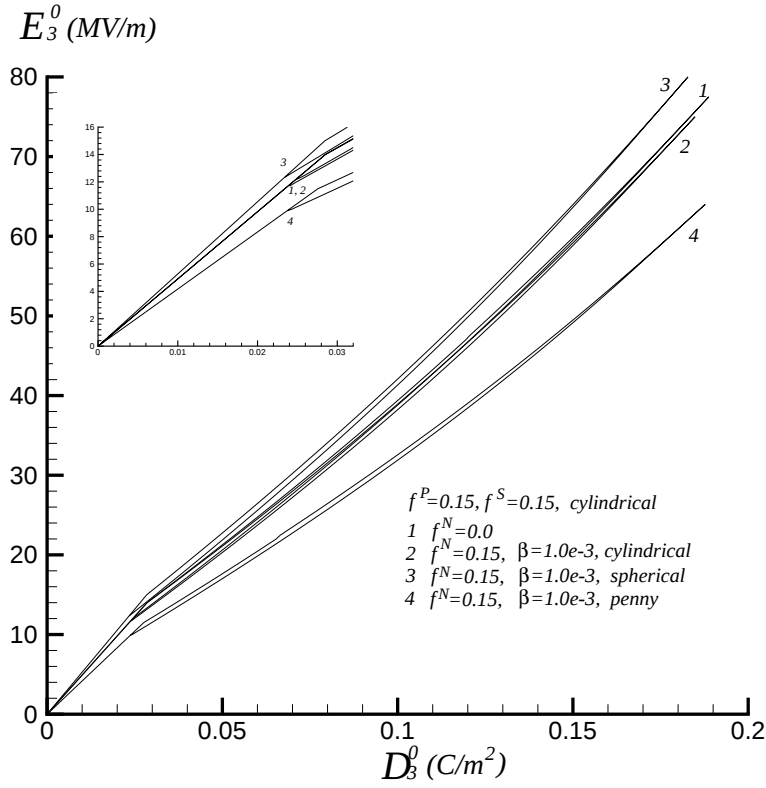


Fig. 17. Effect of shapes of the soft NPZT particles on the axial electric field vs. the axial electric displacement curves of the 4-phase composite made of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

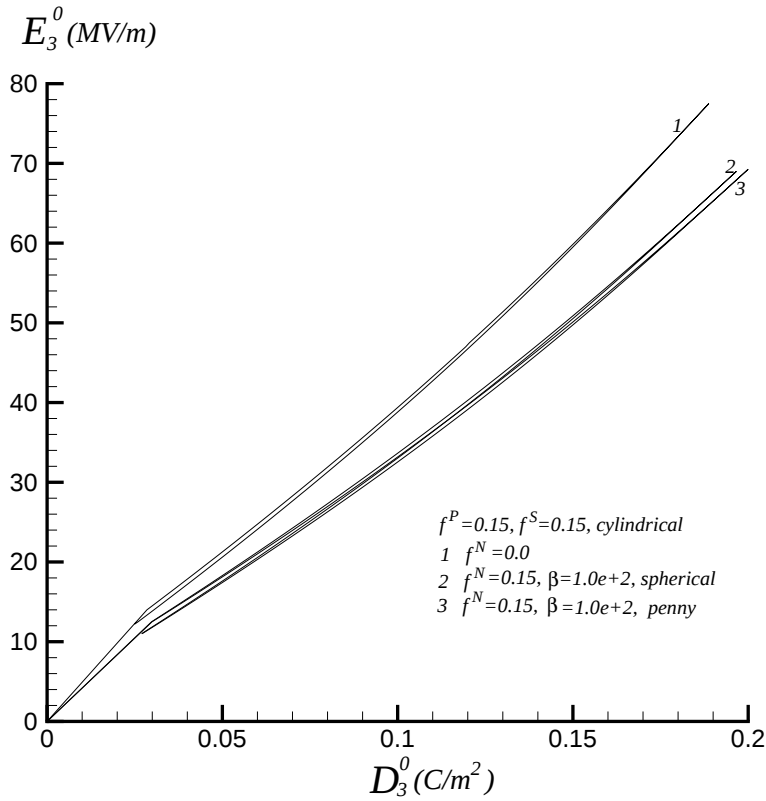


Fig. 18. Effect of shapes of the hard NPZT particles on the axial electric field vs. the axial electric displacement curves of the 4-phase composite comprised of an isotropic elastic matrix and cylindrical PZT and SMA inclusions

PZT inclusions are assumed to be cylindrical and SMA inclusions either cylindrical or spherical with $f^P = f^S = 0.15$. The soft NPZT inclusions are modeled as cylindrical, spherical and penny shape with $\beta = 10^{-3}$. Values given in Tables 3 and 5 are for the austenite SMA inclusions, and those in Tables 4 and 6 for the martensite SMA inclusions. The material parameters for the constituents used in finding the effective moduli are listed in Table 2. Knowing the volume fractions of the austenite and the martensite phases of the SMA inclusions, approximate values of the effective moduli of the four phase composite can be estimated from those given in these Tables by linear interpolation.

Table 3. Values of nonvanishing effective moduli for the 4-phase composite made of a polymer matrix, and cylindrical PZT and SMA inclusions ($f^P = f^S = 0.15$) and soft NPZT inclusions ($\beta = 10^{-3}$) of different shapes. The SMA material is modeled as austenite

	$f^N = 0.15$			$f^N = 0.30$		
	Cylindrical	Spherical	Penny	Cylindrical	Spherical	Penny
$\bar{C}_{1111}$ (GPa)	6.58	7.68	10.84	4.08	5.16	9.50
$\bar{C}_{1122}$ (GPa)	2.77	3.38	5.78	1.46	1.96	5.19
$\bar{C}_{1133}$ (GPa)	3.31	2.90	7.07	1.96	1.20	7.92
$\bar{C}_{3333}$ (GPa)	30.93	26.30	38.76	29.25	21.58	45.76
$\bar{C}_{1212}$ (GPa)	1.90	2.15	2.53	1.31	1.60	2.16
$\bar{C}_{1313}$ (GPa)	2.29	2.35	3.47	1.70	1.77	4.06
$\bar{e}_{311}$ (C/m ²)	-3.85×10^{-2}	-0.136	1.11×10^{-2}	-2.72×10^{-2}	-0.157	0.124
$\bar{e}_{333}$ (C/m ²)	2.99	2.64	3.52	3.01	2.38	4.30
$\bar{e}_{131}$ (C/m ²)	7.38×10^{-4}	7.90×10^{-4}	1.14×10^{-3}	5.58×10^{-4}	6.27×10^{-4}	1.34×10^{-3}
$\bar{k}_{11}/k_0$	2.95	3.16	3.39	2.15	2.43	2.80
$\bar{k}_{33}/k_0$	194.10	180.45	228.45	193.50	168.15	276.77
$\bar{\lambda}_{11}$ (MPa/°C)	0.496	0.576	0.945	0.264	0.329	0.831
$\bar{\lambda}_{33}$ (MPa/°C)	0.663	0.469	1.27	0.455	0.129	1.525
$\bar{p}_1$ (C/m ² °C)	0.0	0.0	0.0	0.0	0.0	0.0
$\bar{p}_3(10^{-4} \text{ C/m}^2 \text{ °C})$	2.73	2.866	2.77	2.71	2.90	2.768

Table 4. Values of nonvanishing effective moduli for the 4-phase composite made of a polymer matrix, and cylindrical PZT and SMA inclusions ($f^P = f^S = 0.15$) and soft NPZT inclusions ($\beta = 10^{-3}$) of different shapes. The SMA material is modeled as martensite

	$f^N = 0.15$			$f^N = 0.30$		
	Cylindrical	Spherical	Penny	Cylindrical	Spherical	Penny
$\bar{C}_{1111}$ (GPa)	6.43	7.49	10.51	3.99	5.03	9.20
$\bar{C}_{1122}$ (GPa)	2.77	3.37	5.69	1.47	1.97	5.09
$\bar{C}_{1133}$ (GPa)	3.26	3.02	6.74	1.93	1.42	7.32
$\bar{C}_{3333}$ (GPa)	24.79	21.20	31.36	23.12	17.18	36.61
$\bar{C}_{1212}$ (GPa)	1.83	2.06	2.41	1.26	1.53	2.05
$\bar{C}_{1313}$ (GPa)	2.17	2.22	3.24	1.61	1.68	3.71
$\bar{e}_{311}$ (C/m ²)	-3.80×10^{-2}	-0.134	1.09×10^{-2}	-2.69×10^{-2}	-0.156	0.121
$\bar{e}_{333}$ (C/m ²)	2.998	2.64	3.52	3.01	2.39	4.30
$\bar{e}_{131}$ (C/m ²)	7.16×10^{-4}	7.66×10^{-4}	1.09×10^{-3}	5.44×10^{-4}	6.10×10^{-4}	1.26×10^{-3}
$\bar{k}_{11}/k_0$	2.95	3.16	3.39	2.15	2.43	2.80
$\bar{k}_{33}/k_0$	194.10	180.45	228.45	193.50	168.15	276.77
$\bar{\lambda}_{11}$ (MPa/°C)	0.478	0.556	0.902	0.254	0.319	0.787
$\bar{\lambda}_{33}$ (MPa/°C)	0.566	0.425	1.098	0.362	0.121	1.265
$\bar{p}_1(10^{-4} \text{ C/m}^2 \text{ °C})$	0.0	0.0	0.0	0.0	0.0	0.0
$\bar{p}_3(10^{-4} \text{ C/m}^2 \text{ °C})$	2.73	2.86	2.77	2.71	2.89	2.77

Table 5. Values of effective moduli for the 4-phase composite made of a polymeric matrix, cylindrical PZT, spherical SMA and ellipsoidal NPZT inclusions. The SMA material is modeled as austenite and $f^P = f^S = 0.15$, $\beta = 10^{-3}$

	$f^N = 0.15$			$f^N = 0.30$		
	Cylindrical	Spherical	Penny	Cylindrical	Spherical	Penny
$\bar{C}_{1111}$ (GPa)	6.83	7.98	11.21	4.27	5.41	9.90
$\bar{C}_{1122}$ (GPa)	2.79	3.42	5.86	1.47	1.99	5.29
$\bar{C}_{1133}$ (GPa)	3.21	2.99	6.90	1.87	1.39	7.72
$\bar{C}_{3333}$ (GPa)	25.04	21.03	32.42	23.27	16.76	39.05
$\bar{C}_{1212}$ (GPa)	2.02	2.28	2.67	1.40	1.71	2.31
$\bar{C}_{1313}$ (GPa)	2.33	2.39	3.53	1.73	1.81	4.13
$\bar{e}_{311}$ (C/m ²)	-3.18×10^{-2}	-0.141	3.88×10^{-2}	-2.25×10^{-2}	-0.167	0.184
$\bar{e}_{333}$ (C/m ²)	3.12	2.71	3.70	3.13	2.43	4.58
$\bar{e}_{131}$ (C/m ²)	7.77×10^{-4}	8.36×10^{-4}	1.21×10^{-3}	5.83×10^{-4}	6.60×10^{-4}	1.42×10^{-3}
$\bar{k}_{11}/k_0$	3.16	3.40	3.67	2.28	2.60	3.02
$\bar{k}_{33}/k_0$	180.56	168.68	209.96	180.00	157.84	250.09
$\bar{\lambda}_{11}$ (MPa/°C)	0.493	0.578	0.946	0.258	0.331	0.836
$\bar{\lambda}_{33}$ (MPa/°C)	0.603	0.457	1.173	0.393	0.149	1.385
$\bar{p}_1$ (C/m ² °C)	0	0	0	0	0	0
$\bar{p}_3$ (C/m ² °C)	2.62×10^{-4}	2.80×10^{-4}	2.57×10^{-4}	2.61×10^{-4}	2.854×10^{-4}	2.42×10^{-4}

Table 6. Values of effective moduli for the 4-phase composite made of a polymeric matrix, cylindrical PZT, spherical SMA and ellipsoidal NPZT inclusions. The SMA material is modeled as martensite and $f^P = f^S = 0.15$, $\beta = 10^{-3}$

	$f^N = 0.15$			$f^N = 0.30$		
	Cylindrical	Spherical	Penny	Cylindrical	Spherical	Penny
$\bar{C}_{1111}$ (GPa)	6.60	7.69	10.75	4.13	5.21	9.48
$\bar{C}_{1122}$ (GPa)	2.79	3.41	5.77	1.49	2.01	5.20
$\bar{C}_{1133}$ (GPa)	3.22	3.02	6.80	1.90	1.44	7.53
$\bar{C}_{3333}$ (GPa)	24.26	20.48	31.28	22.50	16.35	37.35
$\bar{C}_{1212}$ (GPa)	1.90	2.14	2.49	1.32	1.60	2.14
$\bar{C}_{1313}$ (GPa)	2.19	2.25	3.27	1.63	1.70	3.74
$\bar{e}_{311}$ (C/m ²)	-2.57×10^{-2}	-0.130	4.64×10^{-2}	-1.82×10^{-2}	-0.158	0.184
$\bar{e}_{333}$ (C/m ²)	3.07	2.68	3.63	3.08	2.41	4.48
$\bar{e}_{131}$ (C/m ²)	7.52×10^{-4}	8.08×10^{-4}	1.15×10^{-3}	5.67×10^{-4}	6.41×10^{-4}	1.33×10^{-3}
$\bar{k}_{11}/k_0$	3.16	3.40	3.67	2.28	2.60	3.02
$\bar{k}_{33}/k_0$	180.57	168.69	209.97	180.01	157.84	250.08
$\bar{\lambda}_{11}$ (MPa/°C)	0.475	0.555	0.90	0.248	0.317	0.792
$\bar{\lambda}_{33}$ (MPa/°C)	0.564	0.428	1.11	0.358	0.130	1.290
$\bar{p}_1$ (C/m ² °C)	0	0	0	0	0	0
$\bar{p}_3$ (C/m ² °C)	2.64×10^{-4}	2.81×10^{-4}	2.61×10^{-4}	2.63×10^{-4}	2.86×10^{-4}	2.49×10^{-4}

8 Conclusions

Based on the principle of energy equivalence and the Mori-Tanaka method of accounting for the interaction among inclusions, we have derived macroscopic constitutive relations for a 4-phase composite comprised of piezoelectric, nonpiezoelectric and shape memory alloy inclusions embedded in a non-electromechanically coupled elastic matrix. The computed results show that the electromechanical properties of the composite are significantly affected by the material moduli and the shapes of the nonpiezoelectric inclusions. In general, the addition of soft nonpiezoelectric inclusions of appropriate shape will enhance the electromechanical properties of the composite. The four-phase composite comprised of the spherical piezoelectric and shape memory alloy inclusions and either cylindrical or spherical soft nonpiezoelectric inclusions will make a very good sensor since an axial stress of approximately 80 MPa will produce an axial strain of about 2%. The addition of

15% spherical voids enhances the induced axial strain by nearly 20%. The piezoelectric constants of the four phase composite are not affected much by the addition of either soft or hard nonpiezoelectric inclusions. The impedance of the four phase composite sensor can be adjusted to match with that of the host structure.

Acknowledgements. This work was partially supported by the NSF grant CMS9713453 to Virginia Polytechnic Institute & State University.

Appendix

The Mori-Tanaka approximation for thermoelectroelastic concentration tensors

The Mori-Tanaka method approximately accounts for the interaction among inclusions embedded in a matrix. According to this method, expressions for the average strain and the average electric field in the PZT, the SMA and the NPZT inclusions are given by (17) in which

$$\begin{aligned} L_{ijkl}^r &= H_{ijmn}^{1r} L_{mnkl}^M + H_{ijm}^{2r} P_{mkl}^M, \quad N_{ijk}^r = H_{ijmn}^{1r} N_{mnk}^M + H_{ijm}^{2r} Q_{mk}^M, \\ P_{ijk}^r &= H_{imn}^{3r} L_{mnjk}^M + H_{im}^{4r} P_{mjk}^M, \quad Q_{ij}^r = H_{imn}^{3r} N_{mnj}^M + H_{im}^{4r} Q_{mj}^M, \\ R_{ij}^r &= F_{ij}^{1r} + H_{ijmn}^{1r} R_{mn}^M + H_{ijm}^{2r} J_m^M, \quad J_i^r = F_i^{2r} + H_{imn}^{3r} R_{mn}^M + H_{im}^{4r} J_m^M, \\ \varepsilon_{ij}^{**r} &= S_{ijmn}^{1r} \varepsilon_{mn}^{*r} + H_{ijmn}^{1r} \varepsilon_{mn}^{**M} + H_{ijm}^{2r} E_m^{**M}, \quad E_i^{**r} = H_{imn}^{3r} \varepsilon_{mn}^{**M} + H_{im}^{4r} E_m^{**M}. \end{aligned}$$

Tensors L_{ijkl}^M , N_{ijk}^M , R_{ij}^M , P_{ijk}^M , Q_{ij}^M , J_i^M , ε_{ij}^{**M} and E_i^{**M} are given by

$$\begin{aligned} L_{ijkl}^M &= [\mathcal{H}_{ijkl}^1 - \mathcal{H}_{ijm}^2 (\mathcal{H}_{mn}^4)^{-1} \mathcal{H}_{nkl}^3]^{-1}, \quad N_{ijk}^M = -L_{ijmn}^M \mathcal{H}_{mnp}^2 (\mathcal{H}_{pk}^4)^{-1}, \\ P_{ijk}^M &= -(\mathcal{H}_{im}^4)^{-1} \mathcal{H}_{mpq}^3 L_{pqjk}^M, \quad Q_{ij}^M = (\mathcal{H}_{im}^4)^{-1} (\delta_{mj} - \mathcal{H}_{mkl}^3 N_{klj}^M), \\ R_{ij}^M &= -(L_{ijkl}^M \mathcal{F}_{kl}^1 + N_{ijk}^M \mathcal{F}_k^2), \quad J_i^M = -(P_{imn}^M \mathcal{F}_{mn}^1 + Q_{im}^M \mathcal{F}_m^2), \\ \varepsilon_{ij}^{**M} &= -f^S L_{ijkl}^M \langle S_{klmn}^{1S} \bar{\varepsilon}_{mn} \rangle_S, \quad E_i^{**M} = -f^S P_{ikl}^M \langle S_{klmn}^{1S} \bar{\varepsilon}_{mn} \rangle_S, \\ \mathcal{H}_{ijkl}^1 &= I_{ijkl} - \sum_{r=P,S,N} f^r [I_{ijkl} - \langle H_{ijkl}^{1r} \rangle_r], \quad \mathcal{H}_{ijk}^2 = f^P \langle H_{ijk}^{2P} \rangle_P, \\ \mathcal{H}_{ijk}^3 &= f^P \langle H_{ijk}^{3P} \rangle_P, \quad \mathcal{H}_{ij}^4 = \delta_{ij} - \sum_{r=P,S,N} f^r [\delta_{ij} - \langle H_{ij}^{4r} \rangle_r], \\ \mathcal{F}_{ij}^1 &= \sum_{r=P,S,N} f^r \langle F_{ij}^{1r} \rangle_r, \quad \mathcal{F}_i^2 = \sum_{r=P,S,N} f^r \langle F_i^{2r} \rangle_r. \end{aligned}$$

Tensors $H_{ijkl}^{1r}, \dots, F_i^{2r}$ can be obtained from (A7) of Jiang and Batra (2001) wherein the added superscript r is not included. In order to get values of these tensors for $r = M, P, N$ and S , values of the corresponding material moduli should be substituted in the right hand sides of (A7) of Jiang and Batra's (2001) paper.

In the space of stress and electric field, we have

$$\begin{aligned} \bar{\sigma}_{ij}^\sigma &= L_{ijkl}^\sigma \sigma_{kl}^0 + N_{ijk}^\sigma E_k^0 + R_{ij}^\sigma \theta^0 + \sigma_{ij}^{**\sigma}, \\ \bar{E}_i^\sigma &= P_{ikl}^\sigma \sigma_{kl}^0 + Q_{ij}^\sigma E_j^0 + J_i^\sigma \theta^0 + E_i^{**\sigma}, \end{aligned}$$

where

$$\begin{aligned}
L_{ijkl}^\sigma &= [\mathcal{H}_{ijkl}^5 - \mathcal{H}_{ijm}^6 (\mathcal{H}_{mn}^4)^{-1} \mathcal{H}_{nkl}^7]^{-1}, \quad N_{ijk}^\sigma = -L_{ijmn}^\sigma \mathcal{H}_{mnp}^6 (\mathcal{H}_{pk}^4)^{-1}, \\
P_{ijk}^\sigma &= -(\mathcal{H}_{im}^4)^{-1} \mathcal{H}_{mpq}^7 L_{pqk}^\sigma, \quad Q_{ij}^\sigma = (\mathcal{H}_{im}^4)^{-1} (\delta_{mj} - \mathcal{H}_{mpq}^7 N_{pqj}^\sigma), \\
R_{ij}^\sigma &= -(L_{ijmn}^\sigma \mathcal{F}_{mn}^3 + N_{ijp}^\sigma \mathcal{F}_p^2), \quad J_i^\sigma = -(P_{imn}^\sigma \mathcal{F}_{mn}^3 + Q_{im}^\sigma \mathcal{F}_m^2), \\
\sigma_{ij}^{**\sigma} &= -f^S L_{ijkl}^\sigma \langle S_{klmn}^{5S} \bar{\varepsilon}_{mn} \rangle_S, \quad E_i^{**\sigma} = -f^S P_{ikl}^\sigma \langle S_{klmn}^{5S} \varepsilon_{mn}^{tr} \rangle_S, \\
\mathcal{H}_{ijkl}^5 &= I_{ijkl} - \sum_{r=P,S,N} f^r [I_{ijkl} - \langle H_{ijkl}^{5r} \rangle_r], \\
\mathcal{H}_{ijk}^6 &= f^P \langle H_{ijk}^{6P} \rangle_P, \quad \mathcal{H}_{ijk}^7 = f^P \langle H_{ijk}^{7P} \rangle_P, \quad \mathcal{F}_{ij}^3 = \sum_{r=P,S,N} f^r \langle F_{ij}^{3r} \rangle_r,
\end{aligned}$$

and tensors $H_{ijkl}^{1r}, \dots, F_i^{2r}$ can be evaluated from (A7) of Jiang and Batra's (2001) paper as outlined above.

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