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Antiplane Shear Waves in Two Contacting Ferromagnetic Half Spaces

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Abstract The problem of reflection and refraction of antiplane shear (or magneto-elastic) waves at the interface between two ferromagnetic half-spaces with slipping contact (vacuum gap) is studied for waves propagating normal to the direction of the applied external magnetic field which is assumed to be parallel to the interface. We show the existence of new waves that are localized near the interface between the two ferromagnetic media and accompany the reflected and the transmitted waves. We call the new waves as *accompanying surface magneto-elastic* (ASME) *waves;* their amplitudes depend upon values of magnetoelastic parameters of the two media and the intensity of the applied magnetic field. We derive closed-form expressions for magnitudes (coefficients) of the reflected, the refracted (transmitted) and the ASME waves. We show that for a range of values of the applied magnetic field the coefficient of the reflected wave increases and that of the transmitted wave decreases with an increase in the magnitude of the applied magnetic field; these coefficients eventually approach 1 and 0, respectively. That is, the applied external magnetic field can totally eliminate the transmitted wave, and can control energies of the reflected, the refracted and the ASME waves.

Keywords Antiplane shear waves · Ferromagnetism · Contacting magneto-elastic half-spaces · Magnon-phonon interaction · Slipping contact

Mathematics Subject Classification (2000) 74J15 · 78A25 · 78A45

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1 Introduction

Multifunctional materials convert/transduce electrical, thermal, or magnetic energy into mechanical energy and vice versa. By suitably combining various active materials it is in principle possible to transduce between mechanical, electrical, thermal, and magnetic energies. Multifunctional materials having coupled electric, magnetic, and elastic parameters that result in simultaneous ferroelectricity (electro-elastic coupling), ferromagnetism (magneto-elastic coupling), and ferro-elasticity (magneto-electro-elastic coupling) are known as multiferroics. These multi-ferroic materials have been considered for applications in information storage, the emerging field of spintronics, sensors, MEMS, etc. [5, 14, 15]. Magnetic field sensors and magnetoelectric transducers based on multi-ferroic materials are self-energizing and require no external power to operate. Magneto-elastic interactions in magnetically ordered bodies are exploited in high-frequency magneto-strictive transducers, wave filters, sensors and actuators for smart structures, radio transmitters and seismic devices. Some of these devices work on the pulsing behavior of magnetoelastic waves and on the conversion of the fluctuation energy of magnetic spins into mechanical energy [5, 14, 15].

Many recent developments concerning spin waves have been directed towards understanding their behavior in limited magnetic samples. At the same time, there have been dramatic advances in experimental techniques, both for preparing high-quality magnetic thin films and superlattices as well as for studying the spin-wave excitations [1–20].

It is well known that due to the ordering induced by a magnetic field ferromagnetic materials can transmit pure magnetic excitations, known as spin waves [1, 8, 10], that can be regarded as oscillations in the magnetic moment density propagating through a magnetically ordered crystal. We can understand the origin of spin waves by considering deformations of a ferro-magnet at 0 K. In this case, all atomic magnetic moments are aligned and the configuration corresponds to the minimum energy of the ferromagnet. If we now perturb the magnetic moment of an atom it will spin about the local effective magnetic field. Due to interactions between spins of neighboring atoms, the spin will not remain localized at the original atom but will propagate through the crystal in the form of a wave called a spin wave.

We note that elastic waves can also propagate through a ferromagnetic material and these waves may interact with the spin waves. The interrelated spin and elastic waves propagating through a ferromagnetic material are called magneto-elastic or elastic-spin waves [1–4, 6, 7, 9, 11–13, 16–19].

The theory of magneto-elastic coupling in ferromagnets has been presented by Akhiezer et al. [1] and Kittel [8], and the coupling effect is known as the *magnetoacoustic effect* since it yields the excitation of magnons by phonons and vice versa. The pioneering works presented in [1, 8] were followed by many researchers who studied the propagation of coupled magneto-elastic waves, see e.g., [1–4, 6–13, 16–19]. In 1961 Damon and Eshbach [3] established the existence of a pure "magnetostatic" spin surface wave. One of their findings was that shear elastic surface waves do not exist in an elastic medium. However, the existence of shear magnetoelastic waves of Bleustein-Gulyaev type in ferromagnets has been shown by many authors; see e.g., [10, 11, 18]. The minimal conditions under which a surface mode of the Bleustein–Gulyaev type waves can be made to propagate in a ferromagnetic substrate are established in [11, 12].

An important property of such waves is their non-reciprocity, i.e., $\omega(+\alpha) \neq \omega(-\alpha)$, where ω is the wave frequency and α is the wave number. It has been demonstrated that magnetoelastic waves generated by mechanical or magnetic fields must be considered in devices where there is a radiated electromagnetic power generated from a vibrating magnetoelastic source. It appears that such waves play important roles in the design of hypersound generators, high-frequency magnetostrictive transducers, and wave filters, as well as in acoustic delay lines and wireless acoustic wave sensors where acoustic waves produce electromagnetic waves and vice versa [5, 10, 15].

The coupled magnetoelastic problems are studied using equations governing the mechanical displacements of a material point, equations describing electronic spin or equivalently equations of motion of a magnetic moment and Maxwell's quasistatic equations for a magnetic field that account for interactions between lattice and spin continua and a magnetic field. Here we focus on studying the interaction between the spin (magnon) and anti-plane elastic (phonon) waves in two homogeneous ferromagnetic half spaces in a magnetic field with the magnetic polarization perpendicular to the wave propagation direction. The slipping contact (vacuum gap) is assumed between the boundaries of the two ferromagnetic half-spaces. Equations governing the infinitesimal amplitudes of waves are linearized about one domain phase, and we study the reflection and the transmission of magneto-elastic waves at the interface between the two media. It is shown that the magnitudes of the reflected and the transmitted waves strongly depend on the wave frequency, the angle of the incident waves, the intensity of the applied magnetic field, and the mechanical as well as the physical parameters of the two half spaces. We should add that the propagation of gap waves in piezomagnetics, piezoelectrics and piezoelectromagnetics has been investigated in the literature; see, e.g., [6, 13, 20].

2 Formulation of the Problem

A schematic sketch of the problem studied is exhibited in Fig. 1. We assume that two magnetoelastic half-spaces occupying the regions $x_2 > 0$ and $x_2 < 0$ are in a constant magnetic field $\vec{H}_0 = (0, 0, H_0)$. The saturation magnetization per unit mass in each medium is also directed along the x_3-axis, i.e., $\vec{\mu}_0 = (0, 0, \mu_0)$, where $\mu_0 = \text{const}$. The non-zero mechanical displacements occur only in the x_3-direction, i.e., $\vec{u} = (0, 0, u_3(x_1, x_2, t))$, and the mechanical displacement, the magnetic potential, and the magnetization are independent of the x_3-coordinate. The material of each half space has cubic symmetry with the cube axes parallel to the coordinate axes.

Equations of motion of a magnetoelastic body are [1–7, 9, 10, 17–19, 21]:

$$\frac{\partial^2 u_3}{\partial t^2} = c_0^2 \nabla_\perp^2 u_3 + \mu_0 f(\nabla_\perp \cdot \vec{\mu}), \tag{2.1}$$

$$\frac{\partial \mu_1}{\partial t} - \omega_m \hat{b} \mu_2 = \gamma_0 \mu_0 \frac{\partial}{\partial x_2}(\varphi + \bar{b}\mu_0 u_3), \tag{2.2}$$

$$\frac{\partial \mu_2}{\partial t} - \omega_m \hat{b} \mu_1 = -\gamma_0 \mu_0 \frac{\partial}{\partial x_1}(\varphi + \bar{b}\mu_0 u_3), \tag{2.3}$$

$$\nabla_\perp^2 \varphi = \rho_0 (\nabla_\perp \cdot \mu), \tag{2.4}$$

where

$$\nabla_\perp = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, 0\right), \qquad c_0^2 = G/\rho_0.$$

In (2.1)–(2.4), G is the shear modulus, ρ_0 the mass density of the medium, $\vec{\mu} = \vec{m}/\rho_0, \vec{m} = (m_1, m_2, 0)$ the vector of transverse magnetization, φ the magnetic potential, $\gamma_0 = 1.8 \times 10^7$ (oersted $\times$ s)$^{-1}$ the gyromagnetic ratio, $\omega_m = \gamma_0 M_0 = \gamma_0 \rho_0 \mu_0$, and $\mu_0 = M_0/\rho_0$ a saturation magnetization of unit mass. Furthermore, $\hat{b} = b + H_0/M_0$ and $\bar{b} = b + f$, where b

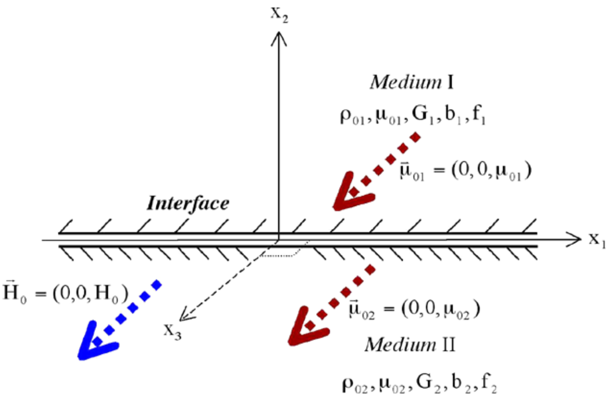


Fig. 1 Two magnetoelastic half-spaces in a magnetic field with a slipping contact at the interface

Table 1 Values of material parameters of YIG and Ga-doped YIG

Pure YIG (Yttrium Iron Garnet)	
$\mu_0 = 139.3$ G	$\rho_0 = 5.17$ g/cm^3
$G = 7.64 \times 10^{11}$ dyn/cm^2	$f = 4, b = 0$
Ga-doped YIG	
$\mu_0 = 27.9$ *G*	$\rho_0 = 5.17$ g/cm^3
$G = 7.64 \times 10^{11}$ dyn/cm^2	$f = 20, b = 0$

and f are magnetoelastic coupling coefficients, and H_0 is the constant external magnetic field applied to the system. The nonzero magnetoelastic stress components are:

$$t_{13} = G\frac{\partial u_3}{\partial x_1} + \rho_0\mu_0\bar{b}\mu_1 \quad \text{and} \quad t_{23} = G\frac{\partial u_3}{\partial x_2} + \rho_0\mu_0\bar{b}\mu_2. \tag{2.5}$$

Equations for media $x_2 > 0$ and $x_2 < 0$ are obtained from (2.1)–(2.4) by substituting values of parameters for the particular medium denoted, respectively, by adding subscripts "1" and "2" to the parameters.

For example, two half-spaces $x_2 > 0$ and $x_2 < 0$ can be made of pure YIG (Yttrium- Iron-Garnet) $Y_3Fe_5O_{12}$ and Ga-doped YIG materials with values of material parameters listed in Table 1.

The conditions at the interface between the two half-spaces with a slipping contact at the interface are

$$\varphi^{(1)} = \varphi^{(2)}, \tag{2.6}$$

$$\frac{\partial\varphi^{(1)}}{\partial x_2} - m_2^{(1)} = \frac{\partial\varphi^{(2)}}{\partial x_2} - m_2^{(2)}, \tag{2.7}$$

$$t_{23}^{(1)} = 0, \tag{2.8}$$

$$t_{23}^{(2)} = 0. \tag{2.9}$$

Equations (2.6) and (2.7) imply that components of the magnetic field vector parallel to the interface and components of the magnetic induction vector normal to the interface are continuous across the interface. It follows from (2.8) and (2.9) that the interface is traction free.

3 Solution of the Problem

We seek a solution of (2.1)–(2.4) of the following form

$$\begin{aligned} u_3 &= Ue^{i(px_2+qx_1+\omega t)}, \qquad \varphi = \Phi e^{i(px_2+qx_1+\omega t)}, \\ m_1 &= M_1 e^{i(px_2+qx_1+\omega t)}, \qquad m_2 = M_2 e^{i(px_2+qx_1+\omega t)}, \end{aligned} \tag{3.1}$$

where U, Φ, M_1, and M_2 are amplitudes of the corresponding physical fields; p and q are wave numbers in the x_2- and x_1-directions, respectively. Substituting (3.1) into (2.1)–(2.4) we get the following characteristic equation for the wave numbers p and q:

$$(p^2+q^2)\left(p^2+q^2-\frac{\omega^2}{c_0^2+\delta\omega_m\omega_H/(\omega^2-\omega_0^2)}\right)=0, \tag{3.2}$$

where $\omega_m=\gamma_0 M_0=\gamma_0\rho_0\mu_0$, $\omega_H=\gamma_0\rho_0\mu_0\hat{b}$, $\omega_0^2=\omega_H^2+\omega_m\omega_H$, and $\delta=M_0^2\bar{b}f/\rho_0$.

Equation (3.2) has following solutions:

$$p_{1,2}=\pm p_0=\sqrt{\omega^2/s^2-q^2}, \qquad p_{3,4}=\pm i|q|, \tag{3.3}$$

where $s=c_0\sqrt{1+\delta\omega_m\omega_H/(\omega^2-\omega_0^2)c_0^2}$ is the speed of propagation of the magnetoelastic shear wave. Solutions $p_1=p_0$ and $p_1=-p_0$ correspond to the incident and the reflected waves, respectively, and solutions $p_3=i|q|$ and $p_4=-i|q|$ correspond to the new waves stimulated by the magnetostriction effect. The waves corresponding to the wave numbers $p_3=i|q|$ and $p_4=-i|q|$ exponentially decay as $x_2\to\mp\infty$ because $e^{i(px_2+qx_1+\omega t)}=e^{\pm|p|x_2}e^{i(qx_1+\omega t)}\to 0$ as $x_2\to\mp\infty$. These waves are localized near the interface between the two ferromagnetic media and accompany the reflected and the transmitted waves, and exist because of the incident waves on the interface between the two magnetoelastic media. We call these waves as *accompanying surface magneto-elastic* (ASME) waves.

We write solutions of (2.1)–(2.4) as (see Fig. 2 for the geometrical representation):

$$\begin{cases} u_3^{(1)}=U_I e^{i(-p_1x_2+q_1x_1+\omega t)}+U_R e^{i(p_1x_2+q_1x_1+\omega t)}, \\ \varphi^{(1)}=\frac{a_{11}}{1-a_{11}}u_3^{(1)}+\Phi_1 e^{i(-q_1x_2+q_1x_1+\omega t)}; \end{cases} \tag{3.4}$$

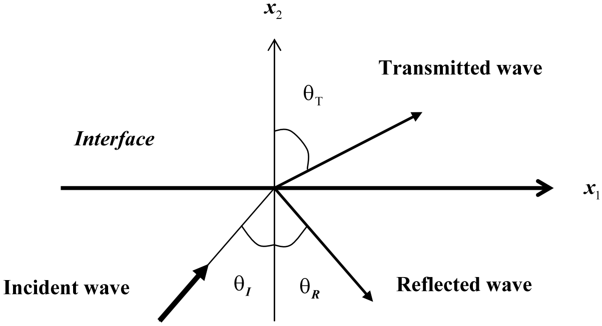


Fig. 2 Reflection and transmission of an incident wave on the interface between two ferromagnetic semi-spaces

$$\begin{cases} m_1^{(1)} = -a_{11}\frac{\partial(\varphi^{(1)}+u_3^{(1)})}{\partial x_1} + ia_{21}\frac{\partial(\varphi^{(1)}+u_3^{(1)})}{\partial x_2}, \\ m_2^{(1)} = ia_{21}\frac{\partial(\varphi^{(1)}+u_3^{(1)})}{\partial x_1} + a_{11}\frac{\partial(\varphi^{(1)}+u_3^{(1)})}{\partial x_2}; \end{cases} \tag{3.5}$$

in medium "1" with $x_2 > 0$, and

$$\begin{cases} u_3^{(2)} = U_T e^{i(-p_2 x_2 + q_2 x_1 + \omega t)}, \\ \varphi^{(2)} = \frac{a_{12}}{1-a_{12}} u_3^{(2)} + \Phi_2 e^{i(-q_2 x_2 + q_2 x_1 + \omega t)}; \end{cases} \tag{3.6}$$

$$\begin{cases} m_1^{(2)} = -a_{12}\frac{\partial(\varphi^{(2)}+u_3^{(2)})}{\partial x_1} + ia_{22}\frac{\partial(\varphi^{(2)}+u_3^{(2)})}{\partial x_2}, \\ m_2^{(2)} = ia_{22}\frac{\partial(\varphi^{(2)}+u_3^{(2)})}{\partial x_1} + a_{12}\frac{\partial(\varphi^{(2)}+u_3^{(2)})}{\partial x_2}; \end{cases} \tag{3.7}$$

in medium "2" with $x_2 < 0$. In (3.4)–(3.7) U_I, U_R, U_T equal amplitudes of the incident, the reflected and the transmitted waves, respectively, and

$$a_{1j} = \frac{\omega_{mj}\omega_{Hj}}{\omega^2 - \omega_{Hj}^2}, \qquad a_{2j} = \frac{\omega_{mj}\omega}{\omega^2 - \omega_{Hj}^2}, \qquad \omega_{mj} = \gamma_0 \rho_{0j}\mu_{0j}, \ \omega_{Hj} = \gamma_0\rho_{0j}\mu_{0j}\hat{b}_j, \ j = (1,2).$$

The wave numbers p_j, q_j $(j = 1, 2)$ are related to angles of the incident and the transmitted waves through $p_1 = p_{01}\sin(\theta_I)$, $q_1 = p_{01}\cos(\theta_I)$ and $p_2 = p_{02}\sin(\theta_T)$, $q_2 = p_{02}\cos(\theta_T)$ where for $j = 1, 2$, $p_{0j} = \omega/c_j$,

$$\begin{aligned} &c_j = c_j(\omega, \mu_{0j}) = c_{0j}\left[1 + \frac{\delta_j}{c_{0j}^2}\frac{\omega_{mj}\omega_{Hj}}{\omega^2 - \omega_{0j}^2}\right]^{1/2}, \qquad \omega_{0j}^2 = \omega_{Hj}^2 + \omega_{Hj}\omega_{mj}, \\ &\delta_j = M_{0j}^2 \bar{b}_j f_j/\rho_{0j}, \qquad c_{0j} = \sqrt{G_j/\rho_{0j}}. \end{aligned} \tag{3.8}$$

Equation (3.8) implies that the speed of the transverse magnetoelastic volume wave strongly depends on the magnetization of the material and the wave frequency.

At the interface between the two half spaces, Snell's law gives

$$q_1 = q_2 \quad \text{or} \quad \frac{\cos(\theta_I)}{c_1} = \frac{\cos(\theta_T)}{c_2}. \tag{3.9}$$

We thus get the direction, θ_T, of the transmitted wave if the angle θ_I of the incident wave is known. Equation (3.9) implies the following:

(1) for $c_1 > c_2$, angle $\theta_T = \cos^{-1}(\frac{c_2}{c_1}\cos(\theta_I))$;
(2) for $c_1 < c_2$, angle $\theta_T = \cos^{-1}(c_2\cos(\theta_I)/c_1)$ is well defined only for $\theta_I < \theta_* = \cos^{-1}(c_1/c_2)$.

4 Coefficients of Reflected, Transmitted and ASME Waves

We define

$$K_R = \frac{U_R}{U_I}, \qquad K_T = \frac{U_T}{U_I}, \qquad K_1 = \frac{\Phi_1}{U_I}, \qquad K_2 = \frac{\Phi_2}{U_I}, \tag{4.1}$$

as coefficients of the reflected, the transmitted and the ASME waves, respectively. Boundary conditions (2.6)–(2.9) lead to the following equation for the determination of K_R, K_T, K_1 and K_2:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \begin{pmatrix} K_R \\ K_T \\ K_1 \\ K_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix}, \tag{4.2}$$

where

$$A_{11} = F_1, \quad A_{12} = 0, \quad A_{13} = -\gamma_1(a_{11} + a_{21}), \quad A_{14} = 0, \quad A_1 = -F_1,$$

$$A_{21} = \frac{a_{21}}{1 - a_{11}}, \quad A_{22} = -\frac{a_{22}}{1 - a_{12}}, \quad A_{23} = a_{11} + a_{21} - 1,$$

$$A_{24} = -1 + a_{12} - a_{22}, \quad A_2 = -\frac{a_{21}}{1 - a_{11}},$$

$$A_{31} = 0, \quad A_{32} = F_2, \quad A_{33} = 0, \quad A_{34} = -\gamma_2(a_{12} - a_{22}), \quad A_3 = 0,$$

$$A_{41} = \frac{a_{11}}{1 - a_{11}}, \quad A_{42} = -\frac{a_{12}}{1 - a_{12}}, \quad A_{43} = 1, \quad A_{44} = -1, \quad A_4 = -\frac{a_{11}}{1 - a_{11}},$$

$$\frac{c_{14}^{(2)}}{c_{44}^{(1)}} = c,$$

$$F_1 = i \tan\theta_I \left(1 + \frac{\gamma_1 a_{11}}{1 - a_{11}}\right) - \gamma_1 \frac{a_{21}}{1 - a_{11}}, \quad F_2 = i \tan\theta_T \left(1 + \frac{\gamma_2 a_{22}}{1 - a_{12}}\right) + \gamma_2 \frac{a_{22}}{1 - a_{12}}.$$

5 Results for Special Cases

We now discuss some cases for which expressions for coefficients K_R, K_T, K_1, K_2 have a simple analytical form.

5.1 Medium "1" is Ferromagnetic and Medium "2" is Vacuum

For this case $a_{12} = 0$, $a_{22} = 0$, $\gamma_2 = 0$ and the solution of (4.2) is:

$$K_R = \frac{(-2 + a_{11} + a_{21})[1 + a_{11}(\gamma_1 - 1)]\tan\theta_I - i[a_{11}^2 + (-2 + a_{11})a_{21}]\gamma_1}{(-2 + a_{11} + a_{21})[1 + a_{11}(\gamma_1 - 1)]\tan\theta_I + i[a_{11}^2 + (-2 + a_{11})a_{21}]\gamma_1},$$

$$K_T = 0,$$

$$K_1 = \frac{2(a_{11} - a_{12})[1 + a_{11}(\gamma_1 - 1)]\tan\theta_I}{(a_{11} - 1)\{(-2 + a_{11} + a_{21})\tan\theta_I[1 + a_{11}(-1 + \gamma_1)] + i[a_{11}^2 + (a_{11} - 2)a_{21}]\gamma_1\}},$$

$$K_2 = \frac{2(a_{11} + a_{21})[1 + a_{11}(\gamma_1 - 1)]\tan\theta_I}{(-2 + a_{11} + a_{21})\tan\theta_I[1 + a_{11}(\gamma_1 - 1)] + i[a_{11}^2 + (a_{11} - 2)a_{21}]\gamma_1}.$$

Magnitudes of the coefficients are:

$$|K_R| = 1, \tag{5.1}$$

$$K_T = 0, \tag{5.2}$$

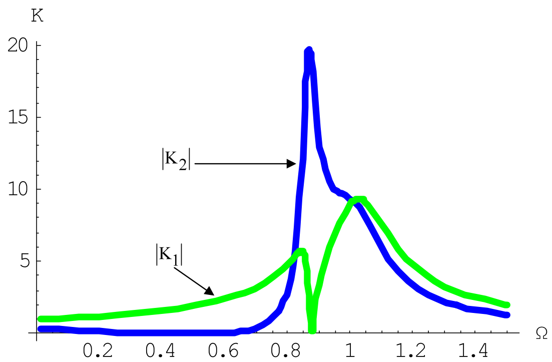


Fig. 3 Dependence of the magnitude of ASME wave coefficients $|K_1|$ and $|K_2|$ on the dimensionless frequency for $H = 0.5$, $\theta_I = 0.001$; the material of the half-space 1 is YIG and that of 2 is vacuum

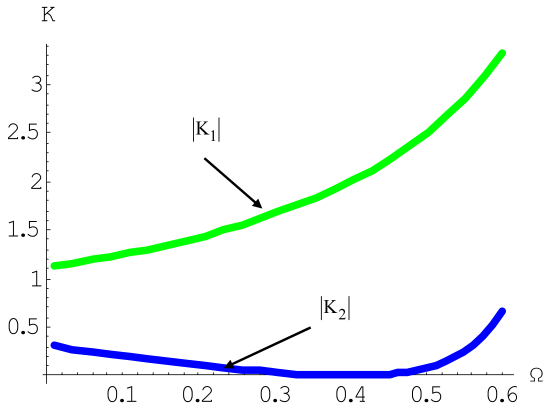


Fig. 4 Dependence of the magnitude of the ASME wave on the dimensionless frequency Ω for $H = 0.4$, $\theta_I = 0.1$; the material of the half-space 1 is YIG, and that of half-space 2 is vacuum

$$|K_1| = \frac{2\tan\theta_I |(a_{11}-1)(a_{11}-a_{21})[1+a_{11}(\gamma_1-1)]|}{(a_{11}-1)^2\sqrt{\tan^2\theta_I(a_{21}+a_{11}-2)^2[1+a_{11}(-1+\gamma_1)^2]^2+[a_{11}^2+a_{21}(a_{11}-2)]^2\gamma_1^2}}, \tag{5.3}$$

$$|K_2| = \frac{2\tan\theta_I |(a_{11}+a_{21})[1+a_{11}(\gamma_1-1)]|}{(a_{21}+a_{11}-2)^2\tan^2\theta_I[1+a_{11}(\gamma_1-1)]^2+[a_{11}^2+(a_{11}-2)a_{21}]^2\gamma_1^2}. \tag{5.4}$$

Using (5.1)–(5.4) and taking the material "1" to be YIG, we show in Figs. 3 and 4 the dependence of $|K_1|$ and $|K_2|$ on the dimensionless frequency $\Omega = \omega/\omega_{m1}$, the normalized external magnetic field $H = H_0/M_{01}$ and the angle θ_I of the incident wave. The dependence of $|K_1|$ on H and Ω for $\theta_I = 0.001$ is shown in Fig. 3; we get analogous results for $|K_2|$. From results exhibited in Fig. 3 we see that the magnitude of the ASME wave strongly depends on the frequency of the incident wave and on the magnitude of the applied magnetic field. However, the angle θ_I of the incident wave has a negligible influence on coefficients $|K_1|$ and $|K_2|$. By appropriately choosing the magnetic field we can concentrate the magnetoelastic energy near the interface between the magnetoelastic media and the vacuum. Numerical results reveal that for $\Omega \approx 1$ (resonant frequencies) the ASME wave coefficients can have very large values (see Figs. 3, 4, 5 and 6).

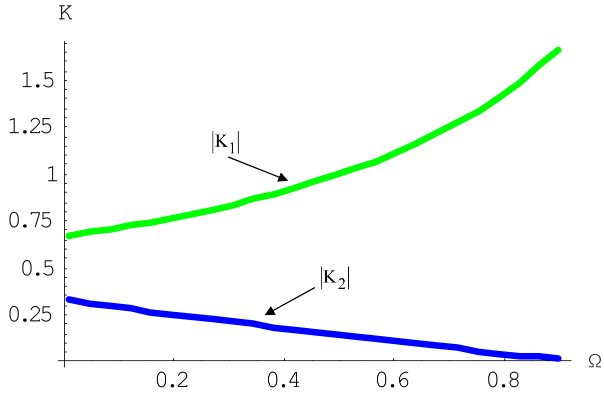


Fig. 5 Dependence of the magnitude of the ASME wave on the dimensionless frequency Ω for $H = 1$, $\theta_I = 0.1$; the material of the half-space 1 is YIG and that of half-space 2 is vacuum

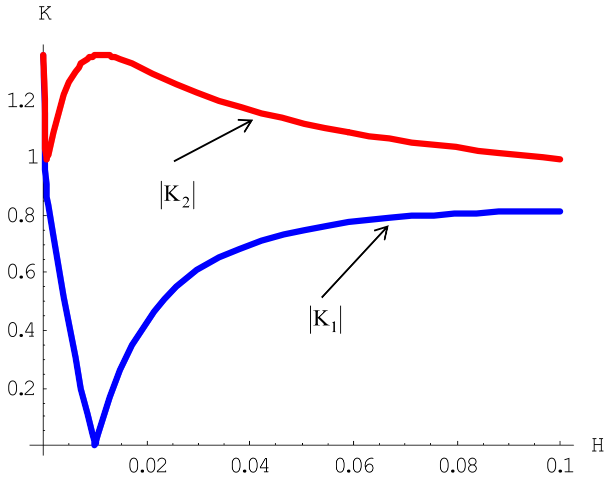


Fig. 6 Dependence of the magnitude of the ASME wave on the dimensionless magnetic field for $\Omega = 0.01$, $\theta_I = 0.001$; the material of the half-spaces 1 and 2 are YIG and vacuum, respectively

5.2 Two Half-Spaces Composed of the Same Ferromagnetic Material

Because values of material parameters of two semi-spaces are the same, therefore, $a_{11} = a_{22}, a_{21} = a_{22}$, $\gamma_1 = \gamma_2$. Snell's law (3.9) gives $\theta_T = \theta_I$. Omitting details we get

$$|K_R| = \sqrt{\frac{R_1}{R_2}}, \tag{5.5}$$

$$|K_T| = \frac{2|(a_{11} - a_{21})|(a_{11} + a_{21})\gamma_1 \tan\theta_I[1 + a_{11}(\gamma_1 - 1)^2]}{\sqrt{R_2}}, \tag{5.6}$$

$$|K_1| = \frac{2|(a_{11} - 1)(a_{11} - a_{21}) \tan\theta_I[1 + a_{11}(\gamma_1 - 1)^2]|\sqrt{a_{21}^2\gamma_1^2 + \tan^2\theta_I(a_{11} - a_{21}\gamma_1 - 1)^2}}{(a_{11} - 1)^2\sqrt{R_2}}, \tag{5.7}$$

$$|K_2| = \frac{2|(a_{11} - 1)(a_{11} + a_{21}) \tan\theta_I[1 + a_{11}(\gamma_1 - 1)^2]|\sqrt{a_{21}^2\gamma_1^2 + \tan^2\theta_I(a_{11} - a_{21}\gamma_1 - 1)^2}}{(a_{11} - 1)^2\sqrt{R_2}}, \tag{5.8}$$

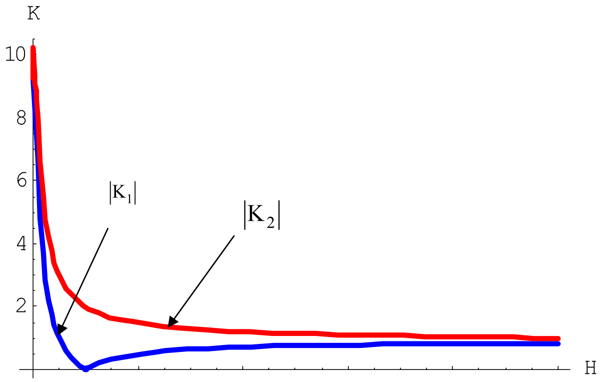


Fig. 7 For the two half-spaces made of YIG, the dependence of the magnitudes of the ASME wave coefficients on the dimensionless magnetic field for $\Omega = 0.01$ and $\theta_I = 0.01$

where

$$\begin{aligned}
R_1 &= \tan^2\theta_I \gamma_1^2\{-4(a_{11}-1)a_{21} + (a_{11}+a_{21})[(a_{11}-a_{21})^2 + 2a_{21}]\gamma_1\}^2 \\
&\quad + [-2a_{21}^2\gamma_1^2 - 2\tan^2\theta_I(1-a_{11}+a_{11}\gamma_1)(-1+a_{11}\gamma_1)^2], \\
R_2 &= (a_{11}-a_{21})^2\tan^2\theta_I\gamma_1^2\{-2(a_{11}-1)(a_{11}+a_{21}) + [(2a_{21}+(a_{11}+a_{21})^2]\gamma_1\}^2 \\
&\quad + \{2a_{21}^2\gamma_1^2 - 2\tan^2\theta_I[1-a_{11}+a_{11}\gamma_1](-1+a_{11}-a_{21}\gamma_1)\}^2.
\end{aligned}$$

The dependence of $|K_R|$, $|K_T|$, $|K_1|$, $|K_2|$ on the dimensionless frequency $\Omega = \omega/\omega_{m1}$ and on the dimensionless magnetic field $H = H_0/M_{01}$ is shown in Figs. 7–13 when the material of the two half-spaces is YIG. These results imply the following:

- Values of coefficients $|K_1|$ and $|K_2|$ of the ASME waves strongly depend on the magnetic field as well as on the frequency and the angle of the incident wave.
- For a given applied magnetic field, the frequency and the angle of the incident wave strongly influence values of coefficients $|K_R|$ and $|K_T|$ of the reflected and the transmitted waves (see Figs. 9–13).
- There are regions of the applied magnetic field in which the coefficient $|K_R|$ of the reflected wave increases to one, and the coefficient $|K_T|$ of the transmitted wave approaches zero (see Figs. 9–13). In other words, the applied magnetic field can totally eliminate the transmitted and the reflected waves.

5.3 Two Different Ferromagnetic Half-Spaces with a Gap Between Them

We assume that the material of half-space 1 is YIG and that of half-space 2 is Ga-doped YIG; values of parameters for the two materials are listed in Table 1.

The variation with H of coefficients $|K_T|$ and $|K_R|$ of the transmitted and the reflected waves is shown in Figs. 14 and 15. From results exhibited in Fig. 15 we see that there is a range of values of H for which $|K_T| = 0$ and $|K_R| = 0$ (see Fig. 15, region $0.2 < H < 0.4$). The magnetic field H, the wave frequency Ω and the incident wave angle θ_I strongly influence values of $|K_R|$ and $|K_T|$. Results plotted in Figs. 14 and 15 enable us to conclude the following.

(a) There is a range of the magnetic field for which both $|K_R|$ and $|K_T|$ vanish.
(b) For values of the magnetic field H for which $|K_R|$ decreases with an increase in the value of H the coefficient $|K_T|$ increases and vice versa.

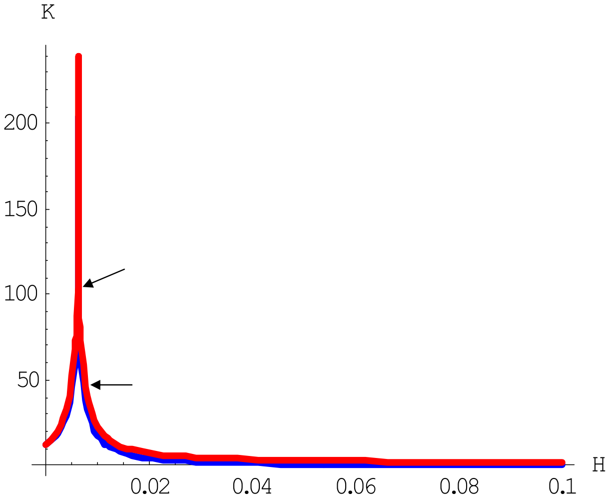


Fig. 8 (Color online) For the two half-spaces made of YIG, the dependence of the magnitudes of the ASME wave coefficients on the dimensionless magnetic field for $\Omega = 0.08$ and $\theta_I = 0.01$ (*blue curve* for $|K_1|$, *red curve* for $|K_2|$)

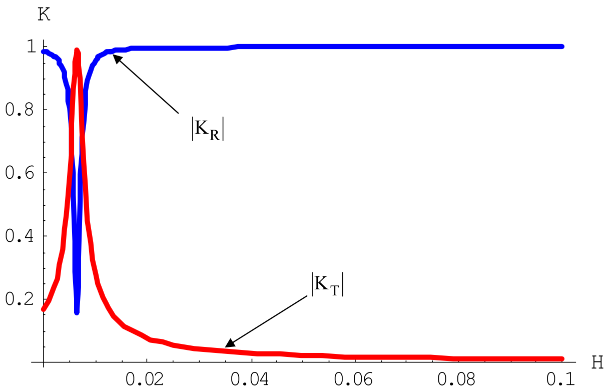


Fig. 9 For the two half-spaces made of YIG, the dependence of the magnitudes of the coefficients of the ASME reflected and transmitted waves for $\Omega = 0.08$ and $\theta_I = 0.01$

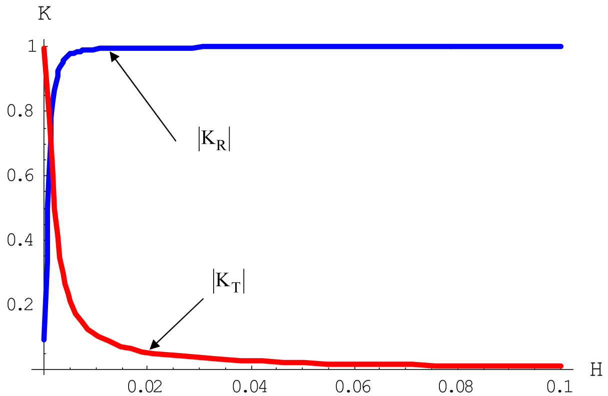


Fig. 10 For the two half-spaces made of YIG, the dependence of the magnitudes of the coefficients of the ASME reflected and transmitted waves for $\Omega = 0.01$ and $\theta_I = 0.01$

(c) With an increase in the value of the frequency Ω the range of the magnetic field H for which $|K_R| = 0$ and $|K_T| = 0$ increases because of the appearance of new intervals.

Fig. 11 For the two half-spaces made of YIG, the dependence of the magnitudes of the coefficients of the ASME reflected and transmitted waves for $\Omega = 0.01$ and $\theta_I = 0.001$

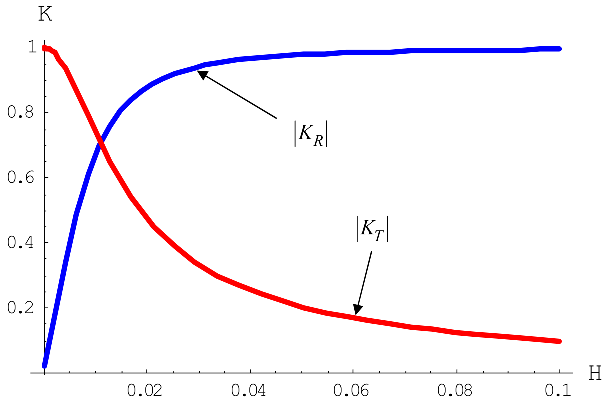


Fig. 12 For the two half-spaces made of YIG, the dependence of the magnitudes of the coefficients of the ASME reflected and transmitted waves for $\Omega = 0.2$ and $\theta_I = 0.001$

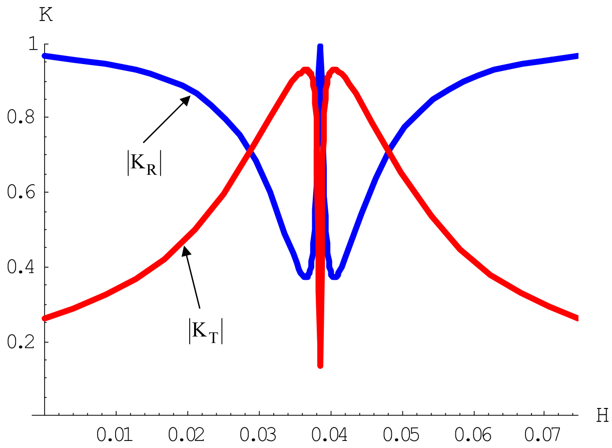
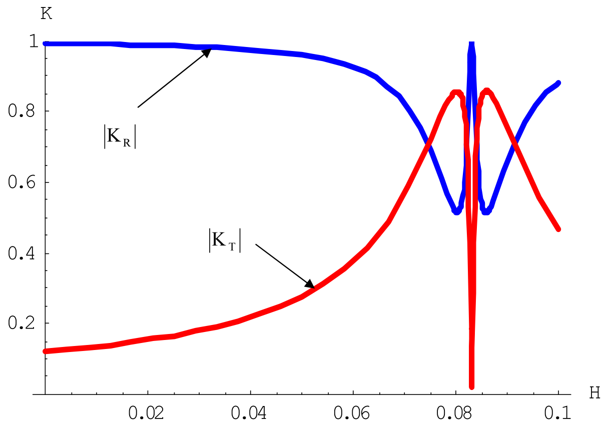


Fig. 13 For the two half-spaces made of YIG, the dependence of the magnitudes of the coefficients of the ASME reflected and transmitted waves for $\Omega = 0.3$ and $\theta_I = 0.001$



(d) The angle θ_I of the incident wave strongly influences the variations of $|K_R|$ and $|K_T|$. However, for $\theta_I > 0.8$ there is almost no influence of θ_I on $|K_R|$ and $|K_T|$.

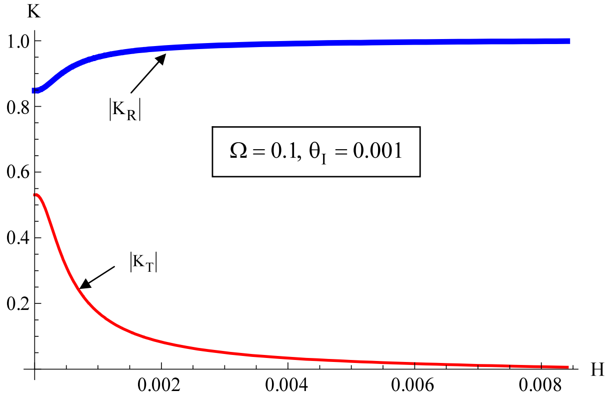


Fig. 14 Dependence of $|K_R|$ and $|K_T|$ on non-dimensional magnetic field H

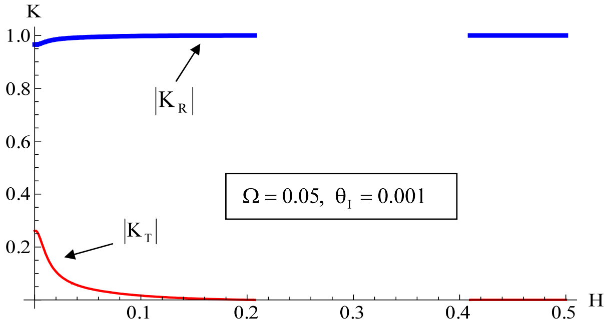


Fig. 15 Dependence of $|K_R|$ and $|K_T|$ on non-dimensional magnetic field H

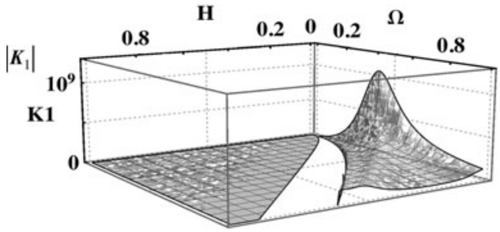


Fig. 16 3D representation of the dependence of $|K_1|$ on the non-dimensional magnetic field H and the non-dimensional frequency Ω

For $\theta_I = 0.001$ we have depicted in Fig. 16 the dependence of $|K_1|$ on the non-dimensional frequency Ω and the non-dimensional magnetic field H. The analogous results obtained for $|K_2|$ are not shown here. It is clear that there is a strong influence of Ω and H on values of $|K_1|$ and $|K_2|$. These results imply that

(a) there are regions of Ω and H for which values of $|K_1|$ and $|K_2|$ monotonically increase;
(b) there are intervals of the magnetic field where values of $|K_1|$ and $|K_2|$ approach infinity.

6 Conclusions

We have studied the propagation of antiplane shear waves in two ferromagnetic half spaces with emphasis on the reflection and transmission of waves at the interface between them where a slipping contact is assumed. The direction of propagation of waves is perpendicular to the direction of the spontaneous magnetization in each medium, which is assumed to be parallel to the interface.

We show the existence of new waves that are localized at the interface between two ferromagnetic media and accompany the reflected and the transmitted waves; we call these waves *accompanying surface magneto-elastic* (ASME) *waves*. Expressions for the magnitudes (coefficients) of the reflected, the refracted (transmitted) and the ASME waves and the computed results show that these coefficients strongly depend on the applied magnetic field, the magnitude of the incident wave, the angle of the incident wave, and values of material properties of the two media. It is shown that there are regions of the magnetic field and the frequency of the incident wave for which the coefficient of the reflected wave approaches one (i.e., the magnitude of the incident wave) and that of the transmitted wave approaches zero. In other words, the magnetic field can totally eliminate the transmitted and the reflected waves and can serve as a control parameter for exchanging energies between the reflected and the refracted waves. Thus the magnetic field can control the magnitude of the ASME wave.

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References

1. Akhiezer, A.I., Baryakhtar, V.G., Peletminskiy, S.: The coupled magnetoelastic waves in ferromagnets and the ferroacustical resonance. J. Exp. Theor. Phys. **35**, 228–240 (1958)
2. Camley, R.E., Maradudin, A.A.: Pure shear elastic surface wave guided by the interface between two semi-infinite magnetoelastic media. Appl. Phys. Lett. **38**(8), 610–612 (1981)
3. Damon, R.W., Eshbach, J.R.: Magnetoelastic modes of a ferromagnetic slab. J. Phys. Chem. Solids **19**, 308–320 (1961)
4. Danoyan, Z., Piliposian, G., Hasanyan, D.J.: Reflection of spin and spin-elastic waves at the interface of a ferromagnetic half-space. Waves Random Complex Media **19**(4), 567–584 (2009)
5. Fiebig, M.: Revival of the magnetoelectric effect. J. Phys. D, Appl. Phys. **38**, R123–R152 (2005)
6. Hasanyan, D.J., Marzocca, P., Harutyunyan, S.: Gap waves in ferro-magneto-elastic materials. In: CANSMART 2004, 7th Cansmart Meeting International Workshop on Smart Materials and Structures, Montreal, Quebec, Canada, October 21–22, 2004, pp. 201–209 (2004)
7. Hasanyan, D.J., Sahakyan, S.L.: Propagation of spin waves in a periodic layered space. Selected Papers from Symposium on Electro-Magneto-Mechanics, USNCTAM 14, pp. 67–76 (2003)
8. Kittel, C.: Interaction of spin waves and ultrasonic waves in ferromagnetic crystals. Phys. Rev. B **110**, 836–841 (1958)
9. Levchenko, V.V.: Propagation of magnetoelastic shear waves through a regularly laminated medium with metallized interfaces. Int. Appl. Mech. **40**(1), 97–102 (2004)
10. Maugin, G.A.: Continuum Mechanics of Electromagnetic Solids. North Holland-Elsevier, Amsterdam (1988)
11. Maugin, G.A.: Wave motion in magnetizable deformable solids: review article. Int. J. Eng. Sci. **19**, 321–388 (1981)
12. Maugin, G.A., Hakmi, A.: Magnetoelastic surface waves in elastic ferromagnets, I: orthogonal setting of the bias field. J. Acoust. Soc. Am. **77**(3), 1010–1026 (1985)
13. Nowacki, J.P., Alshits, V.I., Lyubimov, V.N.: Magnetoelastic waves in a bicrystalline gap structure under a bias magnetic field. Int. J. Appl. Electromagn. Mech. **4**, 223–232 (2000)
14. Roduner, E.: Nanoscopic Materials. Size-Dependent Phenomena. RSC Publishing (2006)
15. Schmid, H.: In: Fiebig, M., et al. (ed.) Magnetoelectric Interaction Phenomena in Crystals. Kluwer, Dordrecht (2004)
16. Shul'ga, N.A., Levchenko, V.V., Ratushnyak, T.V.: Propagation of magnetoelastic shear waves across layers in a periodically layered medium. Int. Appl. Mech. **42**(6), 655–660 (2006)

17. Tiersten, H.F.: Thickness vibration of saturated magnetoelastic plates. J. Appl. Phys. **36**(7), 2550–2559 (1965)
18. Van de Vaart, V.: Magnetoelastic love-wave propagation in metal-coated layered substrates. J. Appl. Phys. **42**(3), 5305–5312 (1971)
19. Wadhawan, V.K.: Introduction to Ferroic Materials. Gordon and Breach Science, New York (2000)
20. Yang, J.S.: Acoustic gap waves in piezoelectromagnetic materials. Math. Mech. Solids **11**, 451–458 (2006)
21. Ericksen, J.L.: Magnetizable and polarizable elastic materials. Math. Mech. Solids **13**, 38–54 (2008)