

THE INTERACTION AMONG ADIABATIC SHEAR BANDS IN SIMPLE AND DIPOLAR MATERIALS

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Abstract—The effect of average strain-rate on the interaction among shear bands in nonpolar and dipolar thermoviscoplastic materials has been investigated. The uniform temperature and stress fields in a block undergoing simple shearing deformations are perturbed when the body just starts deforming plastically and the governing nonlinear coupled partial differential equations are solved numerically. The temperature perturbations introduced are symmetric about the centerline of the block, and have two bumps of unequal heights on each side of the centerline. For nonpolar materials one band on either side of the centerline eventually emerges, the location of the center of the band and its width depend upon the overall strain-rate. For dipolar materials only one band situated at the center of the block forms irrespective of the applied strain-rate. The initiation of the band is significantly delayed in dipolar materials, especially at the higher overall strain-rate studied.

INTRODUCTION

Since the time Zener and Hollomon [1] observed shear bands in a steel plate punched by a standard die and postulated that a negative slope of the stress-strain curve implies an intrinsic instability of the material, there have been many analytical (e.g. Recht [2], Staker [3], Clifton [4], Burns [5], Wright [6], Anand *et al.* [7], Bai [8], Coleman and Hodgdon [9]), experimental (e.g. Moss [10], Costin *et al.* [11], Marchand and Duffy [12]), and numerical (e.g. Clifton *et al.* [13], Merzer [14], Wu and Freund [15], Wright and Batra [16, 17], Wright and Walter [18], Batra [19, 20], Batra and Kim [21–24]) studies aimed at understanding factors that enhance or inhibit the shear strain localization. Since the strain-rate gradients in the region where the deformation localizes are extremely high Wright and Batra [17] and Batra [19] considered strain gradient as an independent variable. Further motivation for the consideration of dipolar effects is provided by the work of Dillon and Kratochvil [25] who have suggested this to be one way of accounting for the interaction among dislocations, and the recent work of Batra and Kim [24] that suggests that the experimental observations of Marchand and Duffy [12] are in close agreement with the predictions from the dipolar theory. Batra [19] has also studied the interaction among two shear bands and showed that the two bands that would grow independently in nonpolar materials coalesced in dipolar materials even when the material characteristic length was 1/20 of the distance between the bands at the time of their initiation.

In this paper we further investigate the interaction among shear bands and especially the effect of the applied strain-rate on the coalescence, initiation and growth of bands in simple and dipolar materials. Backman and Finnegan [26] observed that adiabatic shear bands initiated at flaws, pits, scratches and inhomogeneities in the material. As has been done earlier by Clifton *et al.* [13], Wright and Batra [16, 17] and Batra [19, 20] we model these inhomogeneities by introducing a temperature perturbation; the amplitude and width of the perturbation represent, respectively, the strength and size of the inhomogeneity. Herein we assume that the material has two flaws of different strengths located symmetrically about the centerline of the specimen. We model these by a temperature perturbation introduced at the instant when the material just starts deforming plastically. The temperature perturbation has two bumps of heights 0.067 and 0.1 centered at 0.02H and 0.06H; 2H being the thickness of the specimen undergoing simple shearing deformations. The effects of locating the strong bump near the center or away from it and deforming the specimen at nominal strain-rates of 500 and 50,000 s⁻¹ are studied.

FORMULATION OF THE PROBLEM

We envisage a homogeneous, isotropic and semi-infinite viscoplastic body bounded by the planes $Y = \pm H$ undergoing simple shearing deformations given by

$$x = X + u(Y, t), \quad y = Y, \quad z = Z, \quad \theta = \theta(Y, t). \quad (1)$$

We use a fixed rectangular Cartesian set of axes and denote by (x, y, z) the current coordinates of a material point that occupied the place (X, Y, Z) in a stress-free reference configuration. The functions u and θ give, respectively, the x -displacement of a material point and its temperature change from that in the reference configuration. In the absence of body forces and external sources of energy, equations governing the thermomechanical deformations of the block are (e.g. see Green *et al.* [27], Wright and Batra [17])

$$\rho \dot{v} = (s - \sigma_{,y})_{,y}, \quad (2)$$

$$\rho \dot{e} = -q_{,y} + sv_{,y} + \sigma v_{,yy}. \quad (3)$$

Equation (2) expresses the balance of linear momentum and equation (3) the balance of internal energy. In these equations ρ is the mass density which stays constant since the simple shearing is an isochoric deformation and the effect of temperature change on the mass density is being neglected, $v = \dot{u}$ is the x -velocity of a material particle, a superimposed dot indicates material time differentiation, a comma followed by y stands for partial differentiation with respect to y , s is the shear stress in the x -direction on the plane $y = \text{const.}$, σ is the dipolar stress on this plane, e is the specific internal energy and q is the heat flux. We assume that the shear strain and the shear strain gradient have additive decompositions

$$\gamma = u_{,y} = \gamma_e + \gamma_p, \quad d = \gamma_{,y} = u_{,yy} = d_e + d_p, \quad (4)$$

and that a scalar loading or yield function f exists such that

$$f(s, \sigma, \theta, \dot{\gamma}_p, \dot{d}_p) = \kappa, \quad (5)$$

and

$$\dot{\gamma}_p = \Lambda \alpha, \quad \dot{d}_p = \Lambda \beta, \quad (6)$$

where α and β are constitutive functions that depend upon s, σ, θ , and κ . In equation (5) κ is a measure of the work hardening of the material. We assume that f satisfies

$$\frac{\partial f}{\partial \Lambda}(s, \sigma, \theta, \Lambda \alpha, \Lambda \beta) < 0 \quad (7)$$

for all admissible values of s, σ and θ . The criterion for elastic and plastic loading is

$$f(s, \sigma, \theta, 0, 0) \leq 0, \quad \text{elastic}, \quad (8.1)$$

$$f(s, \sigma, \theta, 0, 0) > 0, \quad \text{plastic}. \quad (8.2)$$

In the later case, because of (7), equation (5) will have a unique solution $\Lambda > 0$. The reader is referred to Green *et al.* [27] and Wright and Batra [17] for a discussion of the preceding equations.

We choose the following constitutive functions.

$$\rho e = \frac{1}{2} \mu (\gamma_e^2 + l^2 d_e^2) + \rho c_v \theta, \quad (9.1)$$

$$q = -k \theta_{,y}, \quad (9.2)$$

$$\alpha = s, \quad \beta = \sigma / l^2, \quad (9.3, 9.4)$$

$$\kappa = \kappa_0 \left(1 + \frac{\psi}{\psi_0} \right)^n, \quad (9.5)$$

$$\kappa \dot{\psi} = s \dot{\gamma}_p + \sigma \dot{d}_p, \quad (9.6)$$

$$f = (s^2 + \sigma^2 / l^2)^{1/2} / (1 - \nu \theta) (1 + b(\dot{\gamma}_p^2 + l^2 \dot{d}_p^2)^{1/2})^m. \quad (9.7)$$

Here μ is the constant shear modulus, l is a material characteristic length, c_v is the constant specific heat, k is the constant thermal conductivity, ψ is the plastic strain in a slow isothermal reference test for which equation (9.5) gives the stress-strain curve, the parameter ν describes the thermal softening of the material, and material parameters n and m characterize, respectively, the strain-hardening and strain-rate hardening of the material. The constitutive assumptions (9) and the standard thermodynamic arguments (e.g. see Green *et al.* [27]) give

$$s = \mu \gamma_e, \quad \sigma = \mu l^2 d_e. \quad (10)$$

Before summarizing the field equations for v and θ , we introduce nondimensional variables, indicated by a superimposed bar in equation (11) below.

$$\begin{aligned} y &= H\bar{y}, & u &= H\bar{u}, & l &= H\bar{l}, & s &= \kappa_0 \bar{s}, & \theta &= \frac{\kappa_0}{\rho c_v} \bar{\theta} = \theta_0 \bar{\theta}, & \bar{\sigma} &= \frac{\sigma}{l \kappa_0}, \\ t &= \bar{t}/\dot{\gamma}_0, & \gamma &= \bar{\gamma}, & d &= H\bar{d}, & \kappa &= \kappa_0 \bar{\kappa}, & \psi &= \bar{\psi}, & \bar{v} H \dot{\gamma}_0 &= v, \\ \bar{v} &= \nu \theta_0, & \bar{b} &= b \dot{\gamma}_0, & \bar{\rho} &= \rho H^2 \dot{\gamma}_0^2 / \kappa_0, & \bar{k} &= k / (\rho c_v \dot{\gamma}_0 H^2). \end{aligned} \quad (11)$$

Here $\dot{\gamma}_0 = v(H, t)/H$ is the average applied strain rate between the boundaries $Y = \pm H$.

Henceforth we use only the nondimensional variables except for the applied strain-rate and drop the superimposed bars. Thus the governing equations may be summarized as

$$\dot{v} = \frac{1}{\rho} (s - l \sigma_{,y})_{,y}, \quad (12.1)$$

$$\dot{\theta} = k \theta_{,yy} + \Lambda (s^2 + \sigma^2), \quad (12.2)$$

$$\dot{s} = \mu (v_{,y} - \Lambda s), \quad (12.3)$$

$$\dot{\sigma} = l \mu \left(v_{,yy} - \frac{\Lambda}{l} \sigma \right), \quad (12.4)$$

$$\dot{\psi} = \Lambda (s^2 + \sigma^2) / \left(1 + \frac{\psi}{\psi_0} \right)^n, \quad (12.5)$$

$$\Lambda = \max \left[0, \left\{ \left(\frac{(s^2 + \sigma^2)^{1/2}}{\left(1 + \frac{\psi}{\psi_0} \right)^n (1 - \nu \theta)} \right)^{1/m} - 1 \right\} / \{ b (s^2 + \sigma^2)^{1/2} \} \right]. \quad (12.6)$$

We presume that the specimen is put in a hard and perfectly insulated loading device, i.e. the boundary conditions are

$$v(\pm 1, t) = \pm 1, \quad q(\pm 1, t) = 0, \quad \sigma(\pm 1, t) = 0. \quad (13)$$

For the initial conditions we take

$$v(y, 0) = y, \quad \sigma(y, 0) = 0, \quad \psi(y, 0) = 0, \quad (14.1)$$

$$\theta(y, 0) = \bar{\theta}(y), \quad \dot{\gamma}_p(y, 0) = 1, \quad (14.2)$$

$$s(y, 0) = (1 + b)^m (1 - \nu \bar{\theta}), \quad (14.3)$$

and seek solutions of equations (12)–(14) such that

$$\begin{aligned} v(-y, t) &= -v(y, t), & \sigma(-y, t) &= -\sigma(y, t), & \theta(-y, t) &= \theta(y, t), \\ s(-y, t) &= s(y, t), & \psi(-y, t) &= \psi(y, t). \end{aligned} \quad (15)$$

Thus the problem can be studied over the spatial domain $[0, 1]$ and the boundary conditions (13) are replaced by

$$v(1, t) = 1, \quad v(0, t) = 0, \quad q(1, t) = 0, \quad q(0, t) = 0, \quad \sigma(1, t) = 0, \quad \sigma(0, t) = 0. \quad (16)$$

The function $\bar{\theta}$ in equation (14.2) gives the initial temperature perturbation.

COMPUTATION AND DISCUSSION OF RESULTS

In order to solve the problem numerically the governing equations (12) are reduced to first order coupled nonlinear ordinary differential equations by using the Galerkin method. These are then integrated by using the Crank–Nicolson method modified to correct the predicted solution within each time step until a preassigned convergence criterion is satisfied. The details of the solution technique are given in [19].

Following values of material parameters that correspond to a typical hard steel were chosen.

$$\rho = 7,860 \text{ kg/m}^3, \quad k = 49.216 \text{ W/m}^\circ\text{C}, \quad \mu = 80 \text{ GPa},$$

$$\kappa_0 = 333 \text{ MPa}, \quad \nu = 0.00552/^\circ\text{C}, \quad c_v = 473 \text{ J/kg}^\circ\text{C},$$

$$m = 0.025, \quad n = 0.09, \quad \psi_0 = 0.017, \quad b = 10^4 \text{ s}.$$

For this choice of parameters, $\theta_0 = 89.6^\circ\text{C}$. Also we took $H = 2580 \mu\text{m}$ and computed results for $\dot{\gamma}_0 = 500$ and $50,000 \text{ s}^{-1}$. With $\tilde{\theta}(y) = 0$, Fig. 1 shows the stress–strain curve in simple shear for $\dot{\gamma}_0 = 500 \text{ s}^{-1}$ and different choices of the values of parameters ν and b . The peak in the stress–strain curve occurs at a strain of 0.093. At the higher strain-rate of $50,000 \text{ s}^{-1}$ the shear stress peaked out at an average strain of 0.085. The temperature perturbation was introduced when the material just started deforming plastically and the initial stress distribution was adjusted so that all of the material points were on their corresponding yield surfaces. Two temperature perturbations depicted in Fig. 2 were tried; the one with the higher amplitude at $y = 0.06$ is referred to as case 1 and the other one with the higher amplitude at $y = 0.02$ as case 2. In each case results were computed for $\dot{\gamma}_0 = 500$ and $50,000 \text{ s}^{-1}$, and $l = 0.0$ and 0.01 . The finite element mesh had 200 elements, 160 uniform elements over the domain $[0, 0.2]$, and 40 uniform element over $[0.2, 1.0]$. We used $\Delta t = 5 \times 10^{-6}$ in integrating the governing equations. Preliminary computations revealed that the spatial resolution of the domain was more critical in obtaining stable and reliable results than the size of the time increment until the instant the band formed. During the development of the band, Δt should be drastically reduced to compute stable results. However, this was not tried. Also the various deformation fields developed no severe gradients on the domain $[0.2, 1.0]$, and therefore, a coarse mesh could be used over this region.

In Figs 3 and 4 are plotted the results for the two cases when $l = 0.0$ and $\dot{\gamma}_0 = 500 \text{ s}^{-1}$. It is obvious that in case 1 a shear band centered at $y = 0.035$ develops whereas in case 2 its center is at $y = 0.0$. The bands in the two cases form at essentially the same value of the overall strain

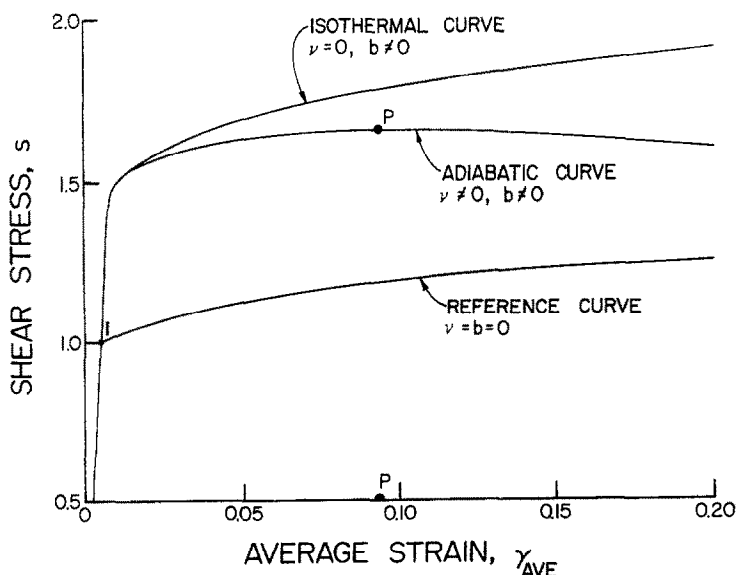


Fig. 1. Average shear stress-average shear strain curve for a typical steel at $\dot{\gamma}_0 = 500 \text{ s}^{-1}$.

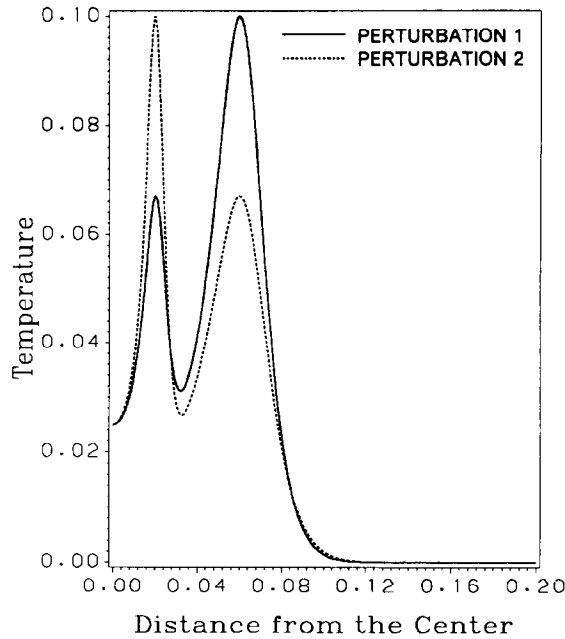


Fig. 2. Temperature perturbations.

of 0.101 in the specimen. We note that when a temperature perturbation with a single bump of amplitude 0.10 centered at $y = 0.0$ was introduced a shear band, $272 \mu\text{m}$ wide, developed at an average strain of 0.0814 [28]. The average strain at which a band develops depends upon the amplitude and the shape of the perturbation. Since the temperature perturbation of Fig. 2 will essentially reduce to that introduced by Batra [28] when there is only one bump of amplitude 0.10 centered at $y = 0.0$, one may conclude that the introduction of a temperature perturbation with two bumps delays slightly the development of a shear band. Because of the assumption (15) we note that in case 1 two bands, one on either side of the center line, grow and in case 2 only one band forms. Following Wright [6] we define the edges of the band to be points where the plastic strain-rate has dropped to one-tenth of its value at the center of the band. The computed width of the single band in case 2 is $216 \mu\text{m}$ and the band width in case 1 could not be computed since the edges of the band could not be delineated according to the stated criterion. In each case the shear stress became uniform throughout the specimen soon after the temperature was perturbed and it increased monotonically till the instant of rapid growth of the band. Since that time the shear stress decreased monotonically and was essentially uniform throughout the thickness of the block. The calculations did not continue long enough for these two cases to observe the stress collapse pointed out by Wright and Walter [18].

Figures 5 and 6 depict the evolution of the velocity, temperature and plastic strain-rate in the specimen for an applied strain-rate of $50,000 \text{ s}^{-1}$. Now there is no band developed at the center of the specimen, rather in each case the center of the band coincides with the center of the temperature bump of higher amplitude. The bands, 14 and 18 microns wide, for cases 1 and 2 develop at an average strain of 0.077 and 0.100 respectively. For the temperature perturbation with a single bump [28] centered at $y = 0.0$, the band, $18 \mu\text{m}$ wide, developed at an average strain of 0.206. The velocity distribution in the specimen at the instant of rapid shear band growth is significantly different at strain rates of 500 and $50,000 \text{ s}^{-1}$. This was also observed by Batra [28] who investigated the effect of applied strain-rates on adiabatic shear banding in nonpolar materials. At the higher strain-rate the velocity field suffers extremely high gradients near the center of the band. Outside the region of shear strain localization the velocity field appears to be varying linearly and the plastic strain-rate is close to zero. Up to the instant results have been computed satisfactorily and plotted here neither of the temperature bumps had died out. Recall that at the lower strain-rate of 500 s^{-1} eventually only one bump emerged with center not coincident with the centers of either of the initial two bumps. For

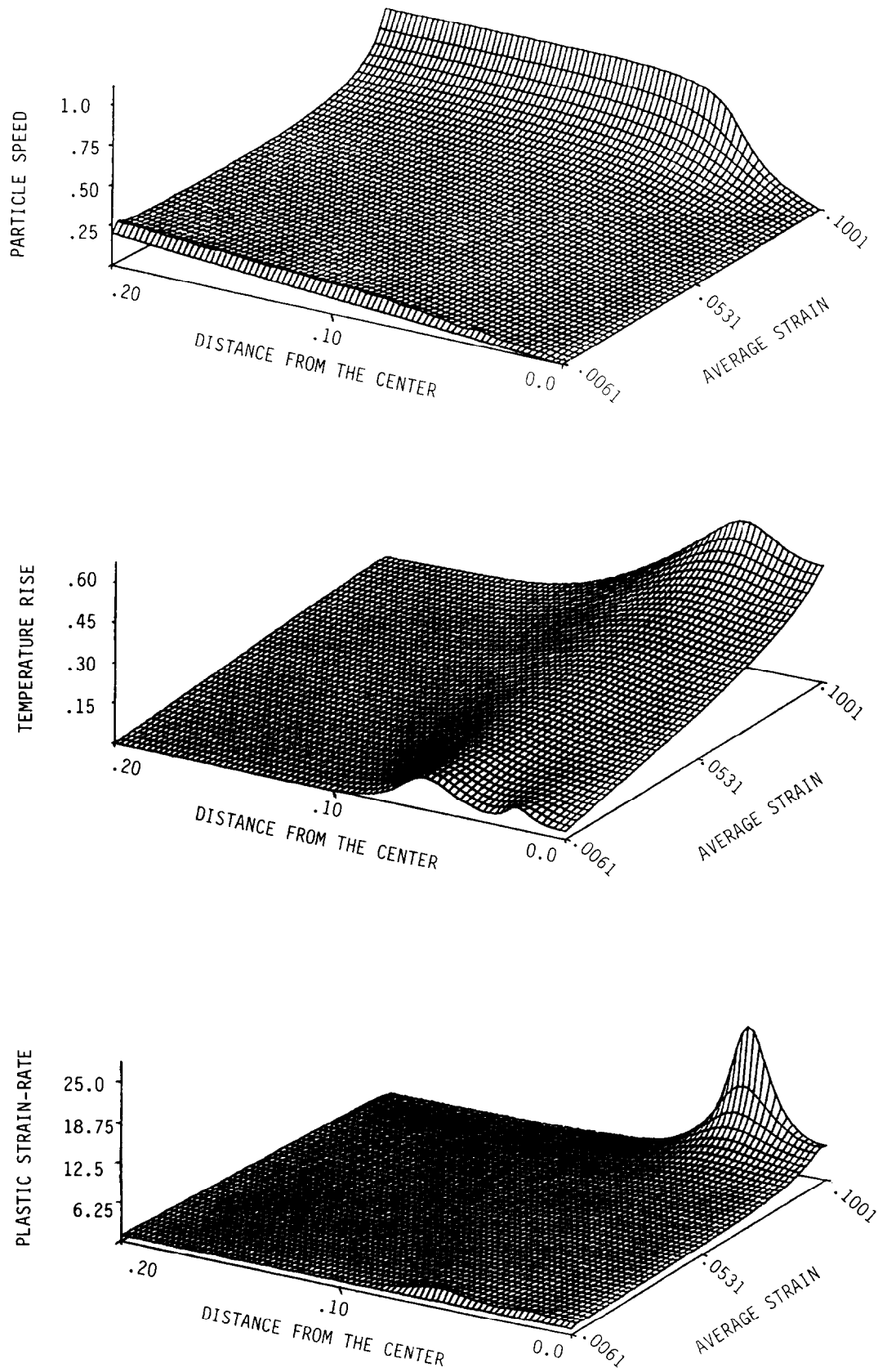


Fig. 3. Evolution of the velocity, temperature and plastic strain-rate fields for nonpolar materials and for temperature perturbation 1 at $\dot{\gamma}_0 = 500 \text{ s}^{-1}$.

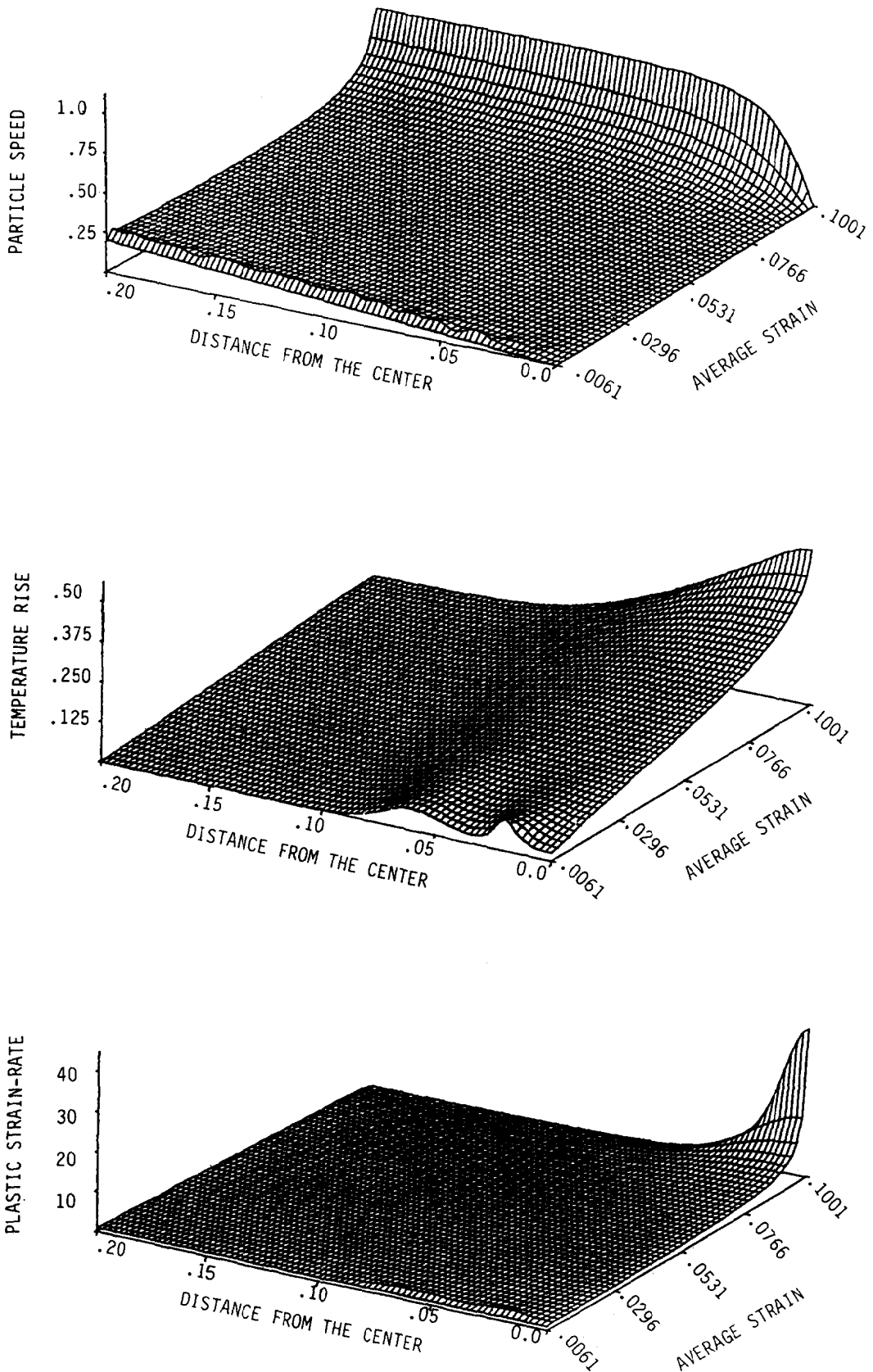


Fig. 4. Evolution of the velocity, temperature and plastic strain-rate fields for nonpolar materials and for temperature perturbation 2 at $\dot{\gamma}_0 = 500 \text{ s}^{-1}$.

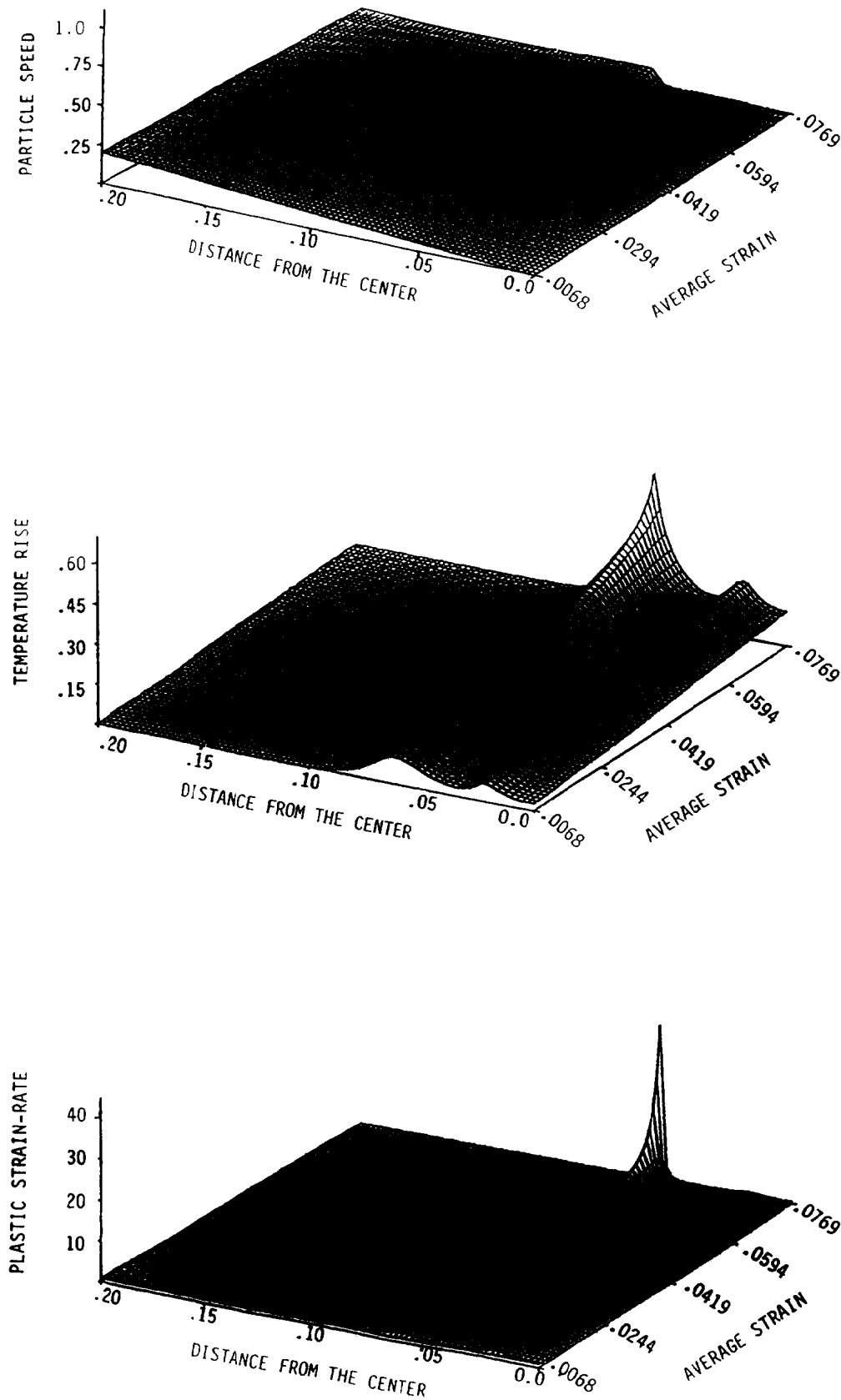


Fig. 5. Evolution of the velocity, temperature and plastic strain-rate fields for nonpolar materials and for temperature perturbation 1 at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$.

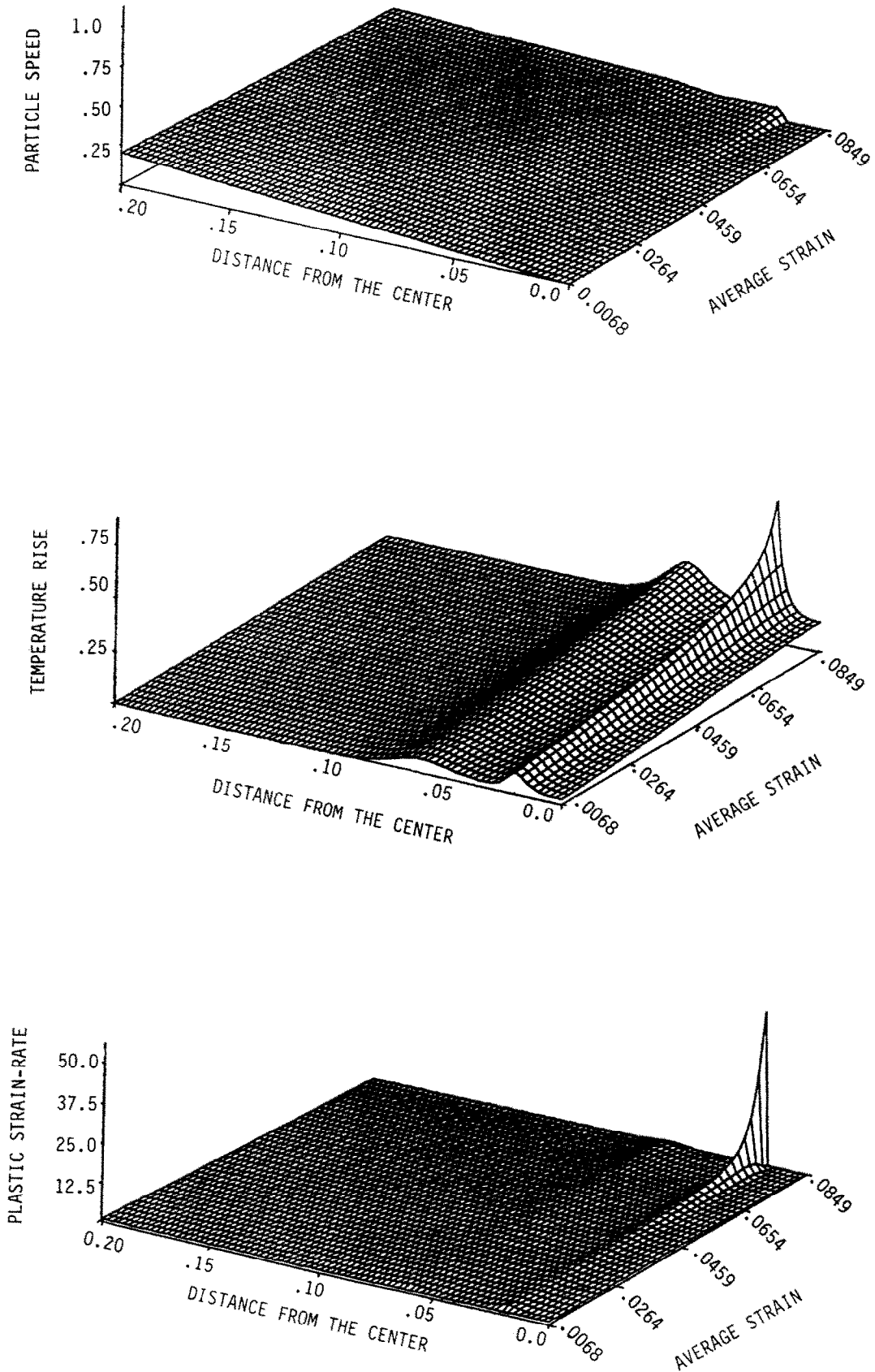


Fig. 6. Evolution of the velocity, temperature and plastic strain-rate fields for nonpolar materials and for temperature perturbation 2 at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$.

$\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$, the shear stress showed a sudden drop at the center of the shear band when the band began to grow very rapidly. Until that time the shear stress stayed uniform throughout the thickness of the specimen and increased gradually with increasing strain.

The above results may be summarized as follows. If two bands, one primary and the other secondary (with the smaller amplitude) initiate simultaneously, then they will merge together and the growth of the single band is delayed as compared to the case when only one band initiates to start with. The center of the merged band need not coincide with that of the primary band. The shift, if any, in the center of the merged band away from that of the primary band depends possibly upon the thermal conductivity, strain and strain-rate hardening characteristics, the value of the thermal softening coefficient for the material and the overall applied strain-rate. Note that for the material parameters selected here, the non-dimensional mass density ρ increases from 3.928×10^{-5} to 3.928×10^{-1} and the non-dimensional thermal conductivity k decreases from 3.978×10^{-3} to 3.978×10^{-5} when $\dot{\gamma}_0$ is increased from 500 to $50,000 \text{ s}^{-1}$. Since ρ is a measure of the effect of inertia forces relative to that of the flow stress of the material and k of the length over which heat conduction effects are important, at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$ the inertia effects play a dominant role and the process is essentially adiabatic locally. This should explain, at least partially, the different behaviors observed at the two strain-rates.

Results for the dipolar material with $l = 0.01$, $\dot{\gamma}_0 = 500 \text{ s}^{-1}$ and the two temperature perturbations are plotted in Figs 7 and 8. For cases 1 and 2 only one band $272 \mu\text{m}$ wide centered at the centerline of the specimen develops at an average strain of 0.15. The two temperature bumps soon die out and eventually the temperature peaks out at the centre of the specimen. As was also observed by Wright and Batra [17] and Batra [19] the growth of the shear band is considerably delayed for dipolar materials as compared to that for nonpolar materials. The two temperature perturbations give rise to essentially similar bands. The velocity distribution within the region $0.0 \leq y \leq 0.20$ exhibits a little bit oscillatory behavior, especially for case 2, till the time the band begins to grow. Once the band forms, the spatial oscillations in the velocity field die out. The peak value 104°C of the temperature rise at the center of the specimen when the band has developed is nearly twice of that for nonpolar materials at approximately the same stage of the development of the band.

Figures 9 and 10 depict results for dipolar materials deformed at the higher strain-rate of $50,000 \text{ s}^{-1}$ and $l = 0.01$. In this case the initiation and growth of the shear band is enormously delayed as compared to those for nonpolar materials at the same strain-rate and dipolar materials deformed at the lower strain-rate. As for $\dot{\gamma}_0 = 500 \text{ s}^{-1}$, only one band $216 \mu\text{m}$ wide located at the center of the specimen forms at an average strain of 0.301. Whereas for nonpolar materials the band developed at the location of the center of the temperature bump with the higher amplitude and was very narrow as compared to the one formed at the lower strain-rate, for dipolar materials the band width decreased only slightly with an increase in the applied strain-rate and its location remained unaffected by the applied strain-rate. Also unlike the case for dipolar materials at the lower strain-rate, the speed at any point in the region $0 \leq y \leq 0.2$ did not exceed that of the boundary point $y = 1.0$. The peak value 134°C of the temperature rise at the center of the band was a little higher than the 104°C computed at the lower strain-rate.

Dipolar stresses contribute significantly to the value of Λ in equation (12.6) and hence to the plastic strain rate at a point since the peak value of σ at the time a shear band developed in case 2 at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$ was nearly equal to the peak value of s . It thus seems that for $l = 0.01$ the dipolar effects dominate over the effects of inertia forces and of heat conduction. Certainly, the computed results depend upon the value of l . The effect of dipolar stresses decreases with a decrease in the value of l .

CONCLUSIONS

On the hypothesis that a temperature perturbation simulates material inhomogeneities present in the material, a shear band develops in nonpolar materials at the location of the stronger inhomogeneity at the higher strain-rate considered but somewhere between the two

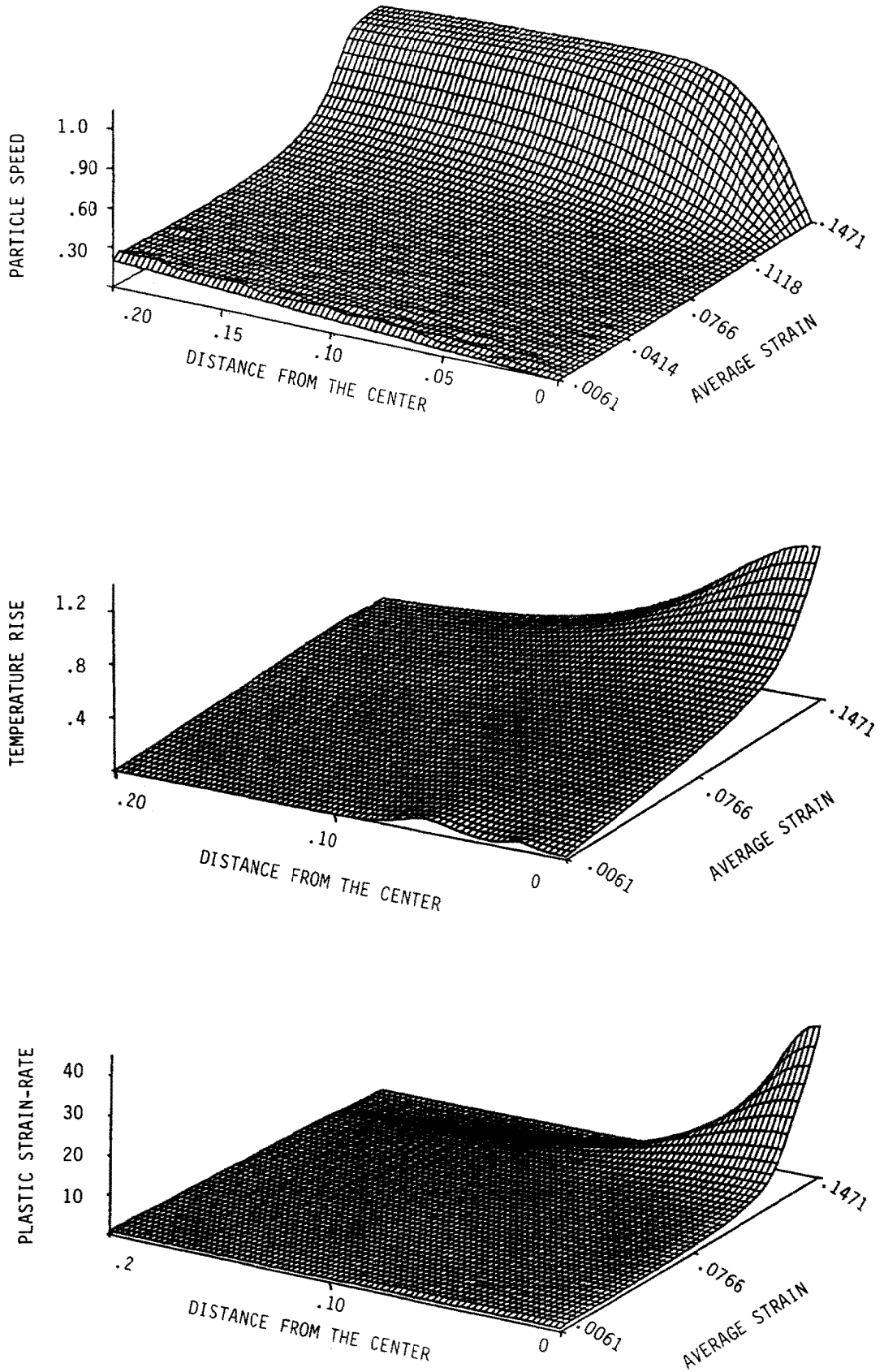


Fig. 7. Evolution of the velocity, temperature and plastic strain-rate fields for dipolar materials ($l = 0.01$) and temperature perturbation 1 at $\dot{\gamma}_0 = 500 \text{ s}^{-1}$.

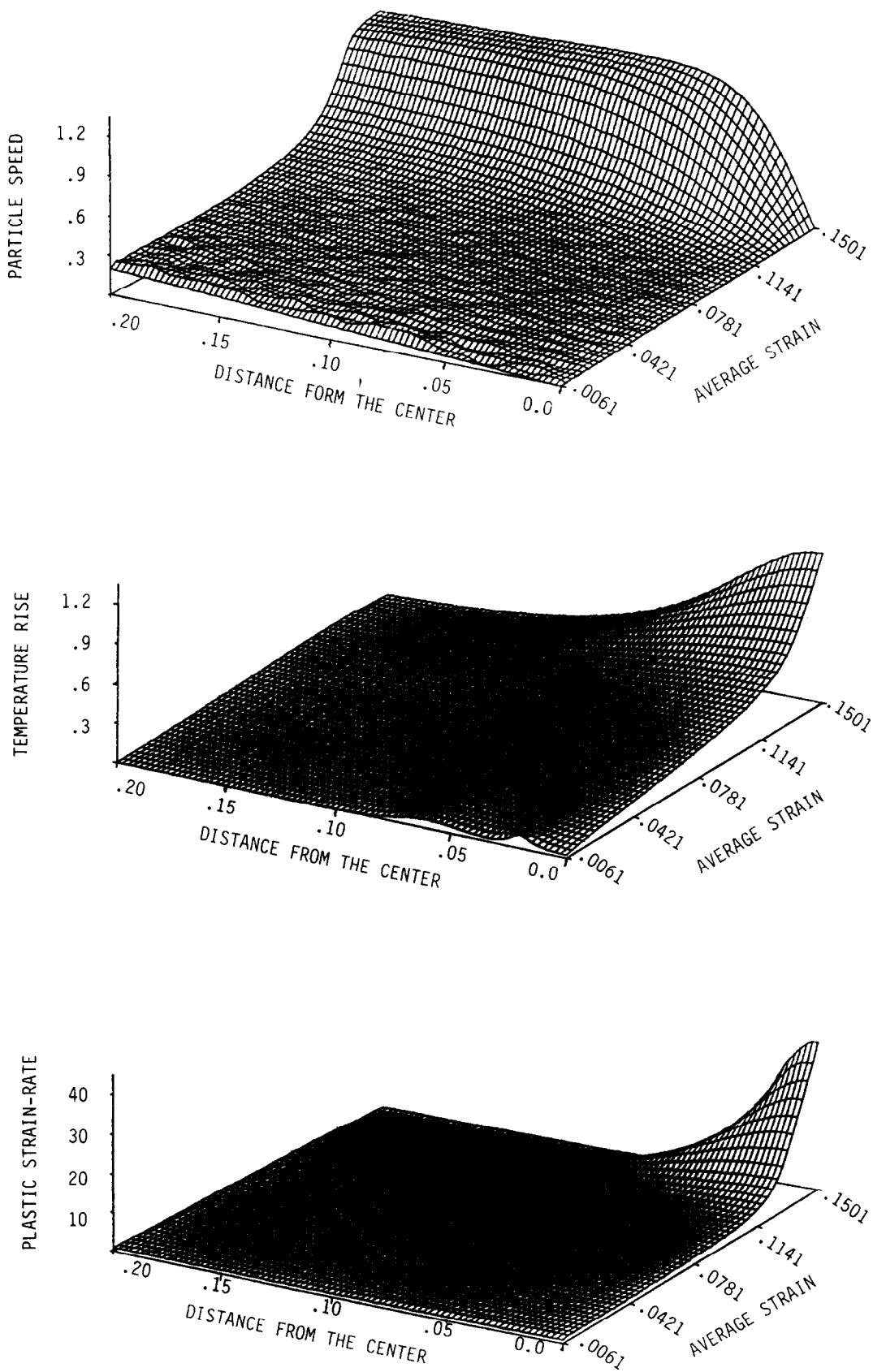


Fig. 8. Evolution of the velocity, temperature and plastic strain-rate fields for dipolar materials ($l = 0.01$) and temperature perturbation 2 at $\dot{\gamma}_0 = 500 \text{ s}^{-1}$.

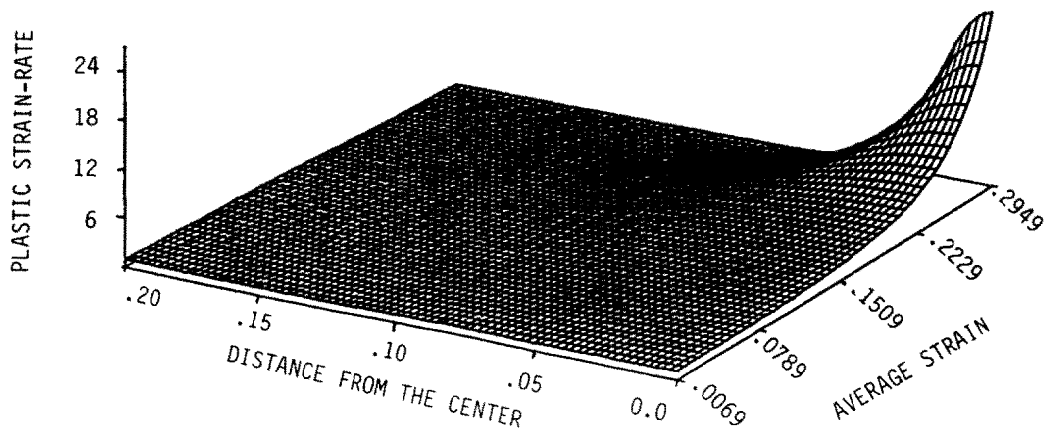
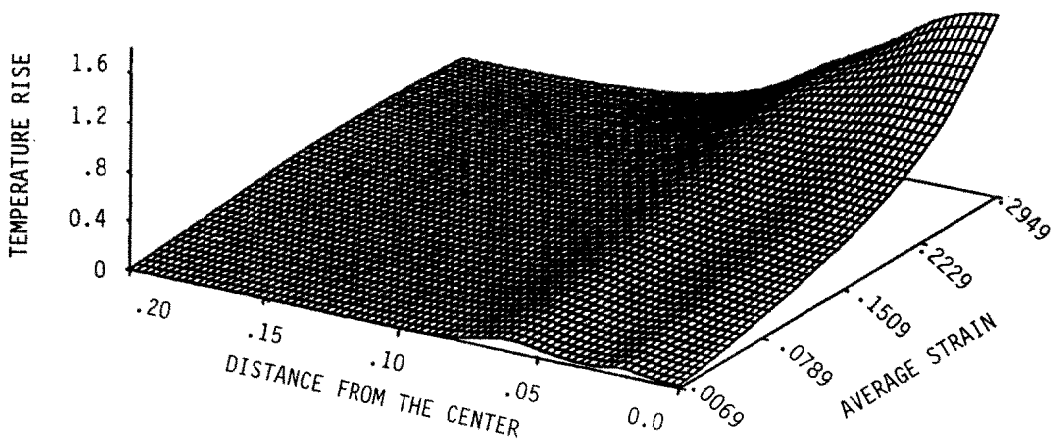
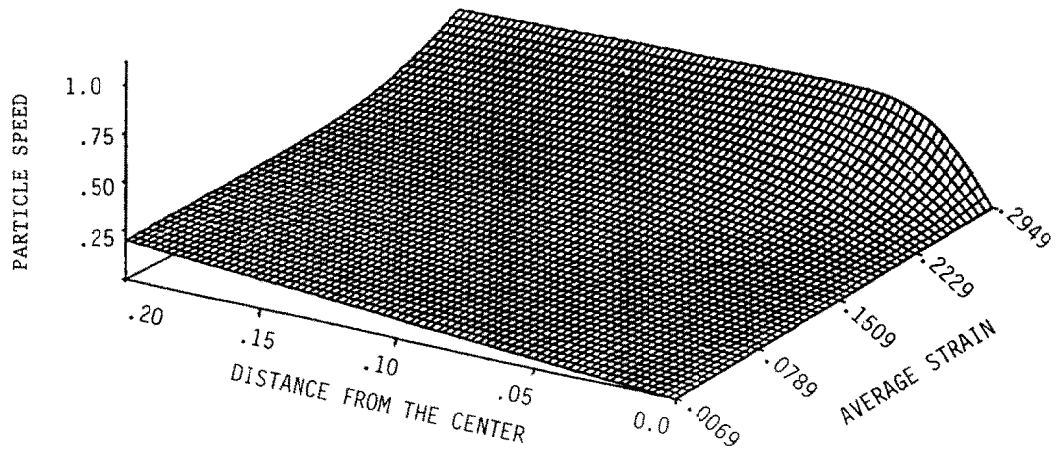


Fig. 9. Evolution of the velocity, temperature and plastic strain-rate fields for dipolar materials ($l = 0.01$) and temperature perturbation 1 at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$.

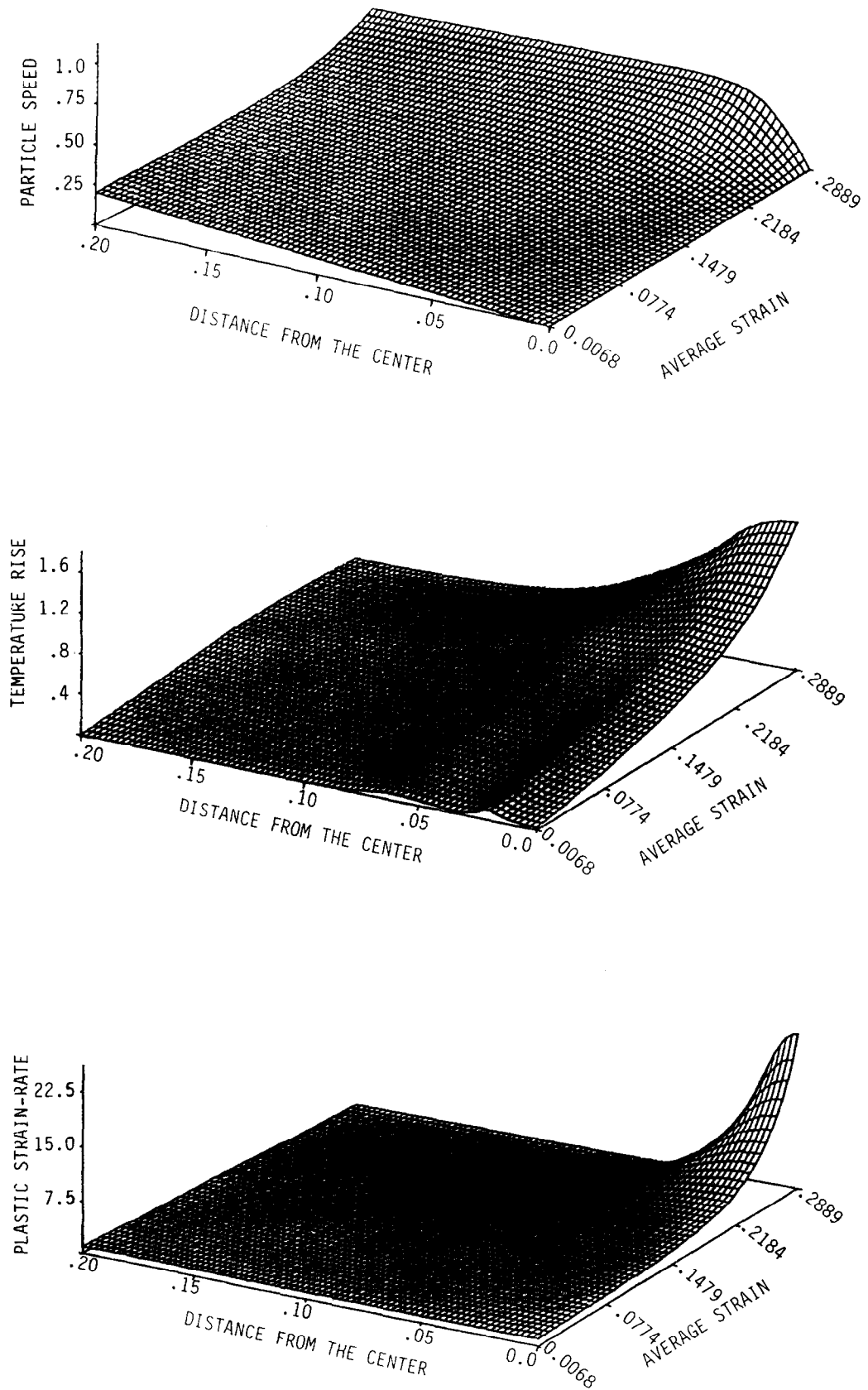


Fig. 10. Evolution of the velocity, temperature and plastic strain-rate fields for dipolar materials ($l = 0.01$) and temperature perturbation 2 at $\dot{\gamma}_0 = 50,000 \text{ s}^{-1}$.

inhomogeneities at the lower strain-rate. However, for dipolar materials at both strain-rates, a shear band develops at the center of the specimen which is not the location of the stronger inhomogeneity. Also for dipolar materials, the shear band is considerably wider than that for nonpolar materials, especially at the higher strain-rate. Both for simple and dipolar materials the initiation of the bands is significantly delayed at the higher strain-rate possibly due to the dominant effect of inertia forces. The rate of growth of a shear band is gradual for dipolar materials but is quite fast for nonpolar materials. Thus in the presence of multiple inhomogeneities of varying strengths, whether or not a shear band initiates at the site of the strongest inhomogeneity depends upon the overall applied strain-rate, values of various material parameters and the consideration of dipolar effects.

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