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LETTERS IN APPLIED AND ENGINEERING SCIENCES

HISTORIES OF STRESS, STRAIN-RATE, TEMPERATURE, AND SPIN IN STEADY STATE DEFORMATIONS OF A THERMOVISCOPLASTIC ROD STRIKING A HEMISPHERICAL RIGID CAVITY

R. C. BATRA and PEI-RONG LIN
Department of Mechanical and Aerospace Engineering and Engineering Mechanics, University of Missouri–Rolla, Rolla, MO 65401, U.S.A.

Abstract—Given the velocity field and the values of a field variable f at a large number of discrete points in a bounded 2-dimensional domain, an algorithm has been developed to compute the streamline that passes through a desired point P in the domain, and the time histories of f at the material particle starting from the instant it occupied the point P. Time histories of the effective stress, second invariant of the strain-rate tensor, temperature and the spin for a few material particles in the steady state axisymmetric deformations of a thermoviscoplastic rod striking a rigid hemispherical cavity are presented. This information should help develop appropriate constitutive models for the penetrator material and establish desirable testing regimes for practical problems.

INTRODUCTION

The solution of any mechanics problem necessarily involves choosing for the material of the body a constitutive relation that adequately models its response over the range of deformations anticipated to occur in the problem. However, the computed values of various field variables depend, in a nontrivial way, upon the constitutive assumptions made to solve the problem. One possible resolution of this rather interesting problem is to choose a constitutive relation, solve the problem, check if the constitutive assumptions made are valid over the range of computed deformations, and, if necessary, redo the problem with the modified constitutive relation.

We note that many of the recently proposed theories (e.g. see [1–4]) of large deformation elastoplasticity are based on different kinematic assumptions thus necessitating the hypothesizing of constitutive relations for variables which may not be simply related to each other. In an attempt to determine the most appropriate theory for the analysis of penetration problems and to delimit the range of values of the variables for which the constitutive relation should be valid, we find the histories of the effective stress, second invariant of the strain-rate tensor, the temperature and the spin at a few typical particles on the penetrator. Only steady state axisymmetric deformations of a viscoplastic penetrator striking a rigid hemispherical cavity are studied. The solution of this problem reported earlier by Batra and Lin [5] is presumed to be given. It is hoped that the time histories of various field variables reported herein will help establish desirable testing regimes for practical problems, and assess the efficacy of different plasticity theories for the penetration problem.

Computed results reveal that material particles near the free surface suffer higher values of the peak plastic strain-rate as compared to those near the centroidal axis of the rod. Values of plastic spin are of the same order of magnitude as the second invariant of the strain-rate tensor. The effective stress at material particles initially near the undeformed centroidal axis decreases because of the rise in their temperature, even though the plastic-strain rate stays essentially uniform. This occurs when the particles reach near the cavity bottom. This drop of effective stress suggests that there is a greater likelihood of the formation of an adiabatic shear band near the stagnation point, and at points adjoining the cavity surface.

STATEMENT OF THE PROBLEM

Let f denote one of the quantities such as the second invariant of the strain-rate tensor, temperature or a component of the stress tensor. The problem studied herein may be stated as follows: given the values of the velocity field $(V_x(x^\alpha, y^\alpha),\ V_y(x^\alpha, y^\alpha))$ and $f(x^\alpha, y^\alpha)$, $\alpha = 1, 2, \ldots, M$ at M discrete points (x^α, y^α) in a bounded 2-dimensional domain, find the history of f at a material particle that initially was at any arbitrary place (x_0, y_0). Needless to say, the accuracy with which the problem can be solved depends upon the value of M and the spatial distribution of points where the data is given. The optimum number of points and their spatial distribution required to solve the problem within a prescribed tolerance has not been determined yet. However, in the present case the data is given at numerous points since a very fine finite element mesh was used to solve the steady state penetration problem whose output serves as the data for the current problem.

The first step in the solution of the problem is to find the streamline that passes through the point $P(x_0, y_0)$. If the velocity field were given as a continuous function of position, then the streamline can be found by integrating the given ordinary differential equations and finding their solution that passes through P. However, for the present problem such is not the case. When the data is given at a discrete set of points, Lin and Batra [6] have developed an algorithm to compute the streamline through P and the time history of the field variable f for a material particle that once occupied the place (x_0, y_0). We use the same computer code to find the histories of the effective stress, second invariant of the strain-rate tensor, spin, hydrostatic pressure and the temperature for a few material particles during the steady state axisymmetric deformations of a penetrator striking a rigid hemispherical cavity.

RESULTS

Below we present streamlines and histories of various field variables for the steady state axisymmetric deformations of a thermoviscoplastic rod striking a hemispherical rigid cavity [5]. In cylindrical coordinates and in terms of non-dimensional variables, the governing equations are:

$$\operatorname{div}\mathbf{v} = 0,$$

$$\operatorname{div}\boldsymbol{\sigma} = \frac{\bar{\rho}v_0^2}{\sigma_0}(\mathbf{v}\cdot\operatorname{grad})\mathbf{v},$$

$$\operatorname{tr}(\boldsymbol{\sigma}\mathbf{D}) + \beta\operatorname{div}(\operatorname{grad}\theta) = (\mathbf{v}\cdot\operatorname{grad})\theta,$$

$$\frac{\operatorname{tr}(\boldsymbol{\sigma}\mathbf{D})}{(1+\psi/0.017)^{0.01}} = (\mathbf{v}\cdot\operatorname{grad})\psi,$$

where

$$\boldsymbol{\sigma} = -p\mathbf{1} + \frac{1}{\sqrt{3}\,I}\left(1 + 10^4\frac{v_0}{r_0}I\right)^{0.025}(1 - 0.000556\theta_0\theta)\left(1 + \frac{\psi}{0.017}\right)^{0.01},$$

$$2I^2 = \operatorname{tr}(\mathbf{D}^2), \qquad 2\mathbf{D} = \operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^T,$$

$$\boldsymbol{\sigma} = \bar{\boldsymbol{\sigma}}/\sigma_0, \qquad p = \bar{p}/\sigma_0, \qquad \mathbf{v} = \bar{\mathbf{v}}/v_0, \qquad r = \bar{r}/r_0, \qquad z = \bar{z}/r_0,$$

$$\theta = \bar{\theta}/\theta_0, \qquad \beta = \bar{k}/(\bar{\rho}\bar{c}v_0r_0), \qquad \theta_0 = \sigma_0/(\bar{\rho}\bar{c}).$$

Here the dimensional quantities are indicated by a superimposed bar, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\bar{\rho}$ is the mass density of a penetrator material particle, $\bar{k}$ is the thermal conductivity, σ_0 is the yield stress in a quasistatic simple tension or compression test, $\bar{c}$ is the specific heat, θ is the temperature rise, the internal parameter ψ describes the work hardening of the material, v_0 is the speed of the penetrator and r_0 its radius. Values assigned to different parameters are

$$\bar{c} = 473\ \mathrm{J/kg\,^\circ C}, \qquad \bar{k} = 48\ \mathrm{W/m\,^\circ C}, \qquad \bar{\rho} = 7800\ \mathrm{kg/m^3}, \qquad \sigma_0 = 180\ \mathrm{MPa},$$

$$v_0 = 340\ \mathrm{m/s}, \qquad r_0 = 2.54\ \mathrm{mm}.$$

This choice of values gives $\theta_0 = 48.9^\circ\mathrm{C}$. Results presented below are in terms of non-dimensional variables and the multiplying factors in order to obtain their dimensional counterparts are given in Table 1.

Figure 1 depicts the streamlines emanating from the points (0.05, 3.0), (0.10, 3.0), (0.90, 3.0) and (0.95, 3.0). We have not plotted the streamlines that start from the points (0.0, 3.0) and

Table 1

Quantity	Multiplying factor
Speed (m/s)	340
r or z-coordinate (mm)	2.54
Hydrostatic pressure (MPa)	180
Effective stress (MPa)	180
Strain-rate invariant (s^{-1})	1.34×10^5
Spin (s^{-1})	1.34×10^5
Temperature rise (°C)	48.9

(1.0, 3.0). The former coincides with the centroidal axis of the rod and the latter with the free surface. The shape of the free surface for different values of the penetrator speed is shown in Fig. 3 of the paper by Batra and Lin [5]. Whereas the streamlines passing through the points (0.90, 3.0) and (0.95, 3.0) stay essentially parallel to each other, the distance between those originating from (0.05, 3.0) and (0.10, 3.0) decreases sharply after they reach the area near the stagnation point and turn around. In Fig. 2 we have plotted the location of these material particles at different times and their speed, the time is reckoned from the instant these material particles were on the surface $z = 3$. The speed of particles near the free surface decreases from 1.0 to about 0.45 when they are at the bottom-most point on the free surface and then stays essentially constant. However, the speed of material particles initially near the centroid axis undergoes a significant change, their speed decreases from 1.0 to about 0.15 when they reach the area near the stagnation point and then increases gradually to about 0.74. As expected the speed of the particle initially closer to the centroidal axis decreases more than that of the particles initially farther from the centroidal axis.

Figure 3 shows the time histories of the spin and the second invariant I of the strain-rate tensor. Since we have neglected elastic deformations and studied the axisymmetric problem, there is only one non-zero component of the plastic spin tensor. These plots reveal that the values of the plastic spin and I are higher for material particles near the free surface as compared to those for material particles near the centroidal axis. Whereas the peak values of I for the four material particles considered are of the same order of magnitude those of the plastic spin are not. The maximum value of the plastic spin for the material particles near the centroidal axis is an order of magnitude lower than that of the particles near the free surface of the rod.

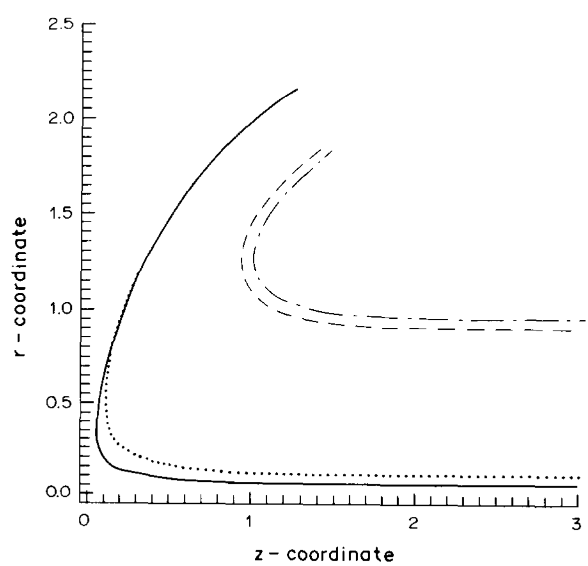

Fig. 1. Streamlines emanating from four points on the incoming penetrator. The originating point for the four curves are: —— (0.05, 3.0); . . . . . . (0.10, 3.0); - - - - - - (0.90, 3.0); —·—·— (0.95, 3.0).

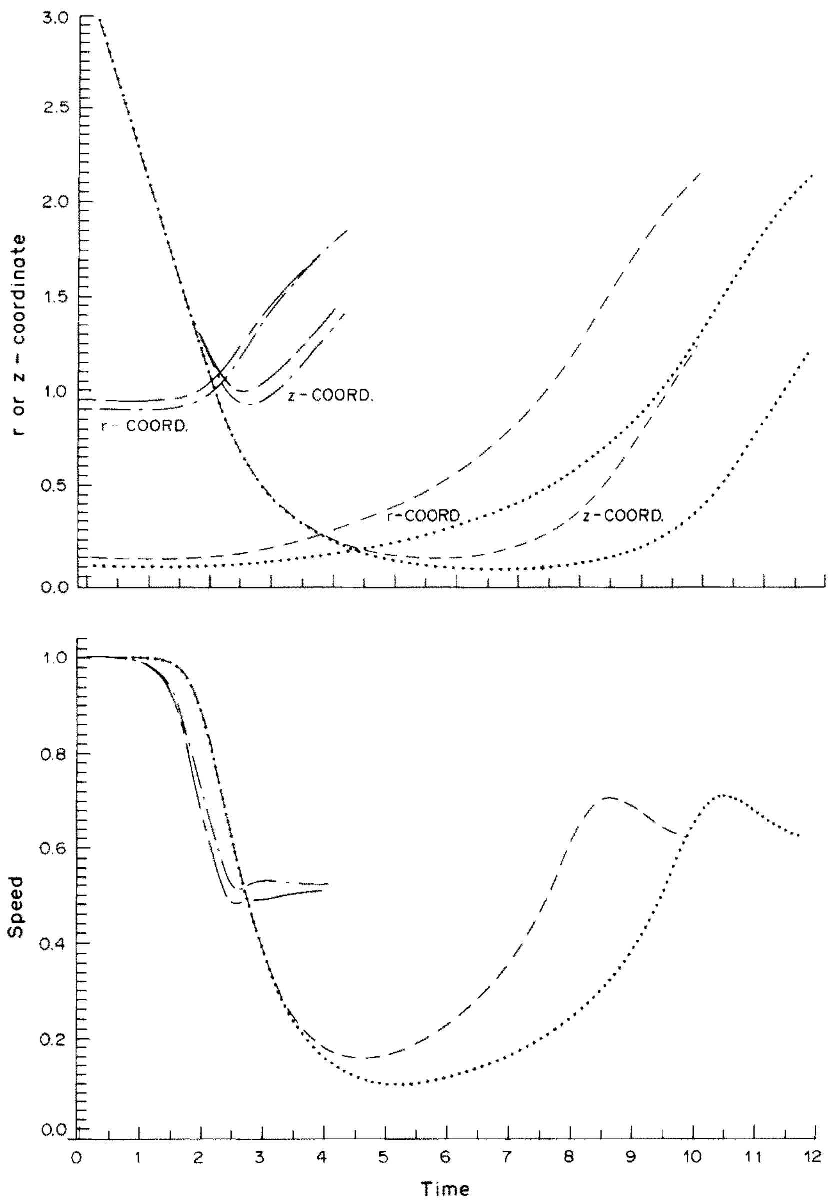

Fig. 2. The variation with time of the speed, and r-, z-coordinates of the four material particles. The initial location of the material particle for the four curves are: ——— (0.95, 3.0); – – – – (0.90, 3.0); ------ (0.10, 3.0); ········ (0.05, 3.0).

The time histories of the temperature and the effective stress s_e defined as

$$s_e = \left(\frac{1}{2}\operatorname{tr}(\boldsymbol{\sigma} + p\mathbf{1})^2\right)^{1/2}$$

for the four material particles considered are plotted in Fig. 4. Note that the peak value of the effective stress experienced by the four material particles is essentially the same. However, the time histories of the temperature and I for these particles are noticeably different. The temperature rise for material particles near the free surface is considerably less as compared to that for material particles near the centroidal axis. The temperature rise for the latter particles is nearly one-third of the presumed melting temperature of the material. At a higher speed, these values will be even higher. The plots of the strain-rate invariant I in Fig. 3 and of the temperature rise and s_e in Fig. 4 reveal that the values of s_e for material particles near the

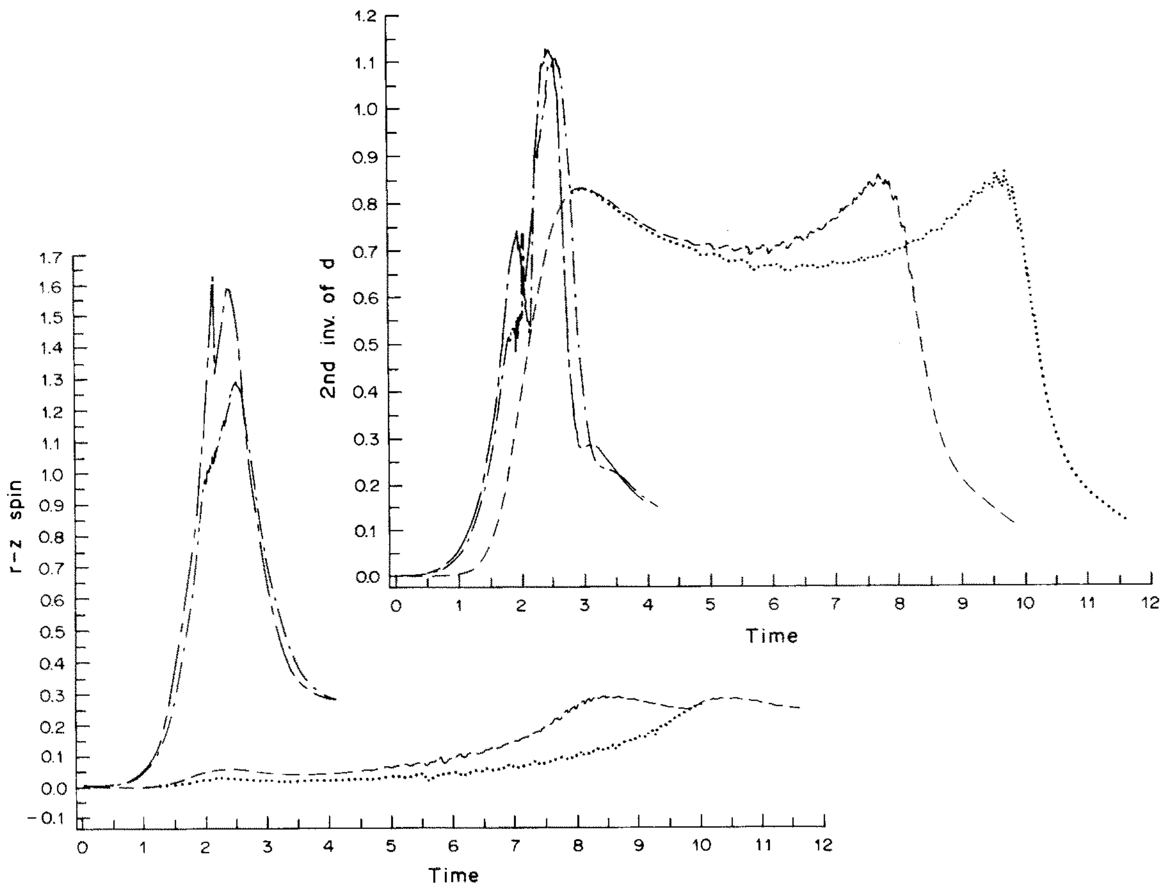

Fig. 3. Time histories of the spin and the second invariant of the strain-rate tensor for the four material particles. See Fig. 2 for explanations.

centroidal axis decrease due to the fact that softening of the material caused by its heating has overcome the combined effects of strain-rate and work hardening. The effect of work-hardening represented herein by the parameter ψ is not that significant because of the relatively small value (0.01) of the exponent selected in the constitutive relation. That is why we have not included the time history of ψ in our results. If plotted, the time histories of ψ for these material particles mimic those of the temperature rise except for a change of scale. The results of Fig. 4 suggest that, in this problem, a shear band is likely to form near the cavity surface for higher values of the penetrator speed.

CONCLUSIONS

During the steady state axisymmetric thermomechanical deformations of a viscoplastic rod striking a rigid hemispherical cavity, the plastic spin for material particles near the free surface is an order of magnitude higher than that for material particles near the centroidal axis. The values of the second invariant of the strain-rate tensor at these particles are not that much different and are of the same order of magnitude as the peak value of the plastic spin. Material particles initially near the centroidal axis of the rod are heated considerably more than those near the free surface of the rod. The time histories of the temperature, strain-rate invariant I and the effective stress seem to suggest that there is a greater likelihood of the formation of an adiabatic shear band near the cavity surface.

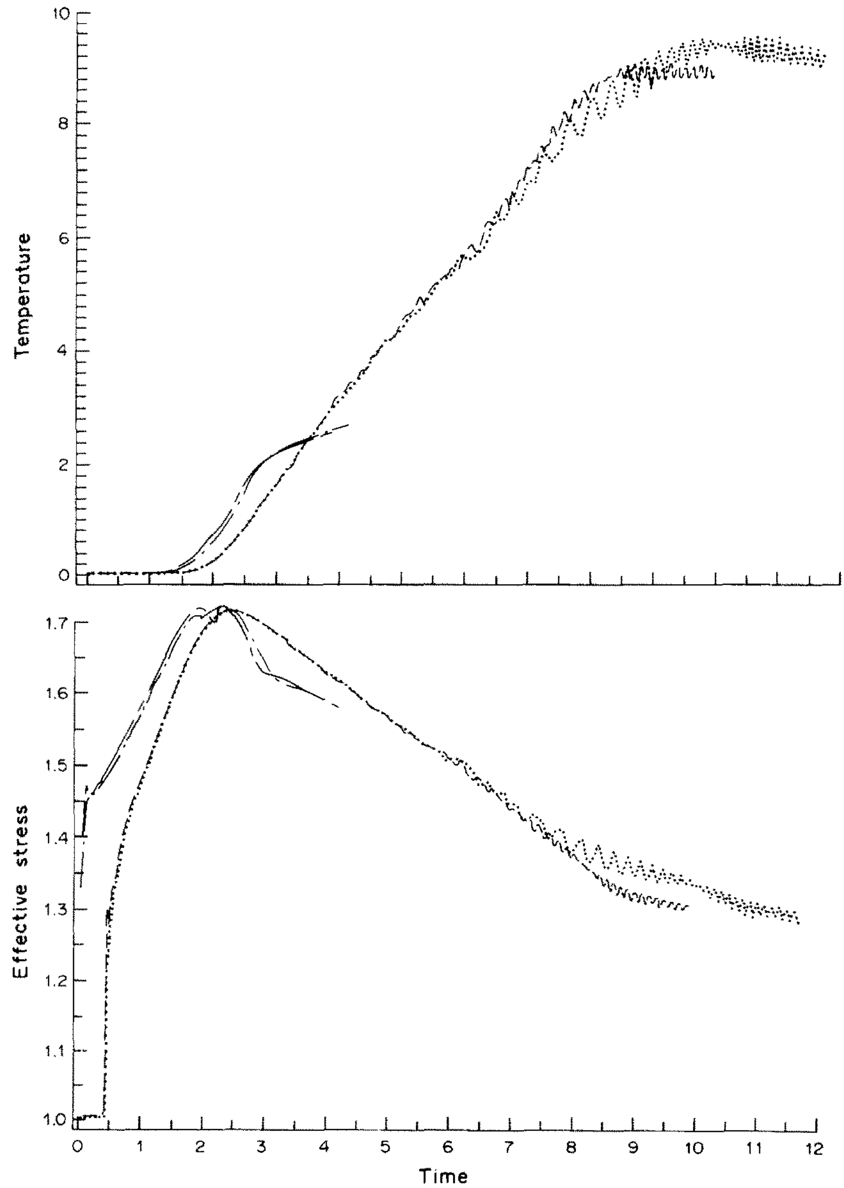

Fig. 4. Time histories of the temperature and the effective stress for the four material particles. See Fig. 2 for explanations.

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