

EFFECT OF VISCOPLASTIC FLOW RULES ON STEADY STATE PENETRATION OF THERMOVISCOPLASTIC TARGETS

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Abstract—Steady state thermomechanical deformations of a thick viscoplastic target being penetrated by a fast moving long rigid cylindrical penetrator are analysed by the finite element method. Two different constitutive relations, the Bodner–Partom flow rule, and the Litonski–Batra flow rule, are used to model the viscoplastic response of the material. The two flow rules are calibrated to give essentially similar shear stress–shear strain curves during the overall adiabatic simple shearing deformations of a block deformed at an average strain-rate of 3300 s^{-1} . For the Bodner–Partom flow rule, the effect on target deformations of the penetrator nose shape, penetrator speed, and the variation in the values of material parameters of the target is also studied.

1. INTRODUCTION

Given the penetrator and target geometries, materials, target support conditions, penetrator speed, and the angle of attack, one would like to find out whether or not the target will be perforated, and if yes, the speed of the penetrator when it comes out of the target. If not, the shape and size of the hole made in the target is of interest. A complete analysis of this problem within reasonable resources is still not possible. There have been numerous attempts made to analyze simplified versions of the problem. Backman and Goldsmith [1] have reviewed the open literature on ballistic penetration till 1977. It describes various physical mechanisms involved in the penetration and perforation processes, and also discusses many engineering models. Other review articles and books include those by Wright and Frank [2], Anderson and Bodner [3], Zukas *et al.* [4], Blazynski [5], and Macauley [6]. Ravid and Bodner [7], Ravid *et al.* [8], and Forrestal *et al.* [9] have proposed engineering models of varying complexity.

When a fast moving long rod strikes a very thick target, the deformations of the two appear to be steady to an observer situated at the stagnation point and moving with it after the rod has penetrated into the target a few rod diameters. This steady state lasts until the stagnation point reaches close to the other end of the target. For very high striking speeds, the steady deformations of the target and the penetrator can be assumed to be governed by purely hydrodynamic incompressible flow processes. Tate [10, 11] and Alekseevskii [12] modified this model by incorporating the effects of material strengths of the target and the projectile. These were assumed to equal some multiple of the uniaxial yield stress of the respective materials, but the multiplying factor was unspecified. Tate [13, 14], Pidsley [15], Batra and Gobinath [16], and Batra and Chen [17] have estimated these multiplying factors. Whereas Tate used a solenoidal fluid flow model to simulate the steady state penetration process, the other investigations relied on a numerical solution of the problem.

One of the unresolved issues in penetration mechanics is the choice of the most appropriate constitutive relation for the penetrator and target materials. In order to assess the effect of the constitutive models used for the target material, we presume herein that the penetrator is rigid and use two different constitutive relations for the target material. The two constitutive relations give virtually the same shear stress–shear strain curves during the numerical simulation of overall adiabatic simple shearing of a viscoplastic block deformed at an average strain-rate of 3300 s^{-1} . For the Bodner–Partom flow rule [18], the effect of varying the penetrator nose shape, the penetration speed, and the values of material parameters on the deformations of the target has also been explored. A similar study for the Litonski–Batra flow rule has already been conducted by Batra [19].

2. FORMULATION OF THE PROBLEM

We use a cylindrical coordinate system, with origin at the center of the penetrator nose and moving with it at a uniform speed v_0 and positive z -axis pointing into the target. Equations governing the thermomechanical deformations of the target are:

$$\operatorname{div} \mathbf{v} = 0, \quad (2.1)$$

$$\operatorname{div} \boldsymbol{\sigma} = \rho(\mathbf{v} \cdot \operatorname{grad})\mathbf{v}, \quad (2.2)$$

$$-\operatorname{div} \mathbf{q} + \operatorname{tr}(\boldsymbol{\sigma} \mathbf{D}) = \rho(\mathbf{v} \cdot \operatorname{grad})U, \quad (2.3)$$

$$\mathbf{D} = (\operatorname{grad} \mathbf{v} + (\operatorname{grad} \mathbf{v})^T)/2. \quad (2.4)$$

Equations (2.1)–(2.3), written in the Eulerian description of motion, express, respectively, the balance of mass, balance of linear momentum, and the balance of internal energy. Here we have neglected elastic deformations of the target, and assumed that plastic deformations are isochoric and all of the plastic working, rather than 90–95% of it as asserted by Farren and Taylor [20] is converted into heating. The operators grad and div denote the gradient and divergence operators on fields defined in the present configuration. Furthermore, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, ρ the mass density of the target material, $\mathbf{v}$ the velocity of the target particle relative to the penetrator, $\mathbf{q}$ the heat flux per unit present area, $\mathbf{D}$ the strain-rate tensor, and U the specific internal energy. We need to specify constitutive relations and boundary conditions in order to complete the formulation of the problem.

For the target material, we assume that

$$\mathbf{q} = -k \operatorname{grad} \theta, \quad (2.5)$$

$$U = c\theta \quad (2.6)$$

and either the Litonski–Batra flow rule [19]

$$\boldsymbol{\sigma} = -p\mathbf{1} + 2\mu(I, \theta, \psi)\mathbf{D}, \quad (2.7)$$

$$2\mu(I, \theta, \psi) = \frac{\sigma_0}{\sqrt{3}I} (1 + bI)^e (1 - \nu\theta)(1 + \psi/\psi_0)^q, \quad (2.8)$$

$$\operatorname{tr}(\boldsymbol{\sigma} \mathbf{D}) = \sigma_0 \dot{\psi} (1 + \psi/\psi_0)^q, \quad (2.9)$$

$$I^2 = \frac{1}{2} \operatorname{tr}(\mathbf{D}^2), \quad (2.10)$$

or the Bodner–Partom flow rule [18]

$$\mathbf{s} = \left(z_2 / \left(\sqrt{3} I \left[\frac{2n}{n+1} \ln(D_0/I) \right]^{1/2n} \right) \right) \mathbf{D}, \quad \mathbf{s} = \boldsymbol{\sigma} + p\mathbf{1} \quad (2.11)$$

$$n = \frac{a}{T}, \quad z_2 = z_1 + (z_3 - z_1) \exp(-mW/z_3), \quad (2.12)$$

$$\dot{W} = \operatorname{tr}(\boldsymbol{\sigma} \mathbf{D}) = \operatorname{tr}(\mathbf{s} \mathbf{D}). \quad (2.13)$$

Equation (2.5) is the Fourier law of heat conduction, k the constant thermal conductivity, θ is the change in the temperature of a material particle from that in the undeformed configuration, c the constant specific heat, p the hydrostatic pressure not determined by the deformation history, and σ_0 is the yield stress in a quasistatic simple tension or compression test. The constitutive relation (2.7) was proposed by Batra [19]. It incorporates and generalizes that suggested by Litonski [21] for simple shearing deformations. Batra and his coworkers [19, 22–24] have used it to study thermomechanical penetration problems, and the initiation and growth of shear bands in viscoplastic materials. In equations (2.7) and (2.8), the material parameters b and e characterize the strain-rate sensitivity of the material, ν describes its thermal softening, and ψ_0 and q the strain hardening of the material. Equation (2.9) states that

the rate of increase of ψ at a material point is proportional to the plastic working there. Thus, the present value of ψ depends upon the history of the deformation. The linear dependence of the flow stress upon the temperature rise has been used by Tate [25] in the analysis of a penetration problem, and has been observed by Bell [26], Lin and Wagoner [27], and Lindholm and Johnson [28]. I^2 defined by equation (2.10) equals the second-invariant of the strain-rate tensor, since the deformations are taken to be isochoric. Should the temperature rise at a material point exceed $1/\nu$ so as to make μ negative, we set $\mu = 0$. Thus, the material point will behave like an incompressible fluid. However, no account is taken of the latent heat needed for the phase transformation to occur.

In equations (2.11)–(2.13), $\mathbf{s}$ is the deviatoric stress tensor, T the absolute temperature of a material particle, W the plastic work done, and z_2 may be considered as an internal variable whose present value at a material point depends upon the density of the plastic work done at that point. D_0 is the limiting value of the plastic strain-rate, and is usually assigned a large value. Besides D_0 , we need to specify a , z_1 , z_3 , and m to characterize the material. We identify the parameter a with the melting temperature of the material, and once T equals a , we set $\mathbf{s} = \mathbf{0}$, analogous to what we did for the Litonski–Batra flow rule.

We note that there is no loading or explicit yield surface assumed with either (2.7) or (2.11). The limiting value of $s_e = (1/2 \operatorname{tr} \mathbf{s}^2)^{1/2}$ as I approaches zero is $\sigma_0/\sqrt{3}$ for the Litonski–Batra law, and is zero for the Bodner–Partom law.

Rewriting equation (2.7) as

$$\boldsymbol{\sigma} = -(\bar{p} + \beta K \theta) \mathbf{1} + 2\mu(I, \theta, \psi) \mathbf{D}, \quad (2.14)$$

where β and K equal, respectively, the coefficient of thermal expansion and the bulk modulus of the material, we see that equation (2.7) embodies implicitly thermal stresses caused by the nonuniform temperature rise at different material particles. However, the change in the mass density due to temperature rise of a material particle is not considered. In equation (2.14), $\bar{p}$ is not determined by the deformation history of a material particle, and the addition of a determinate term to it gives rise to p in equation (2.7), which is taken to be an independent variable throughout this work.

We introduce non-dimensional variables, indicated below by a superimposed bar, as follows:

$$\begin{aligned} \bar{\boldsymbol{\sigma}} &= \boldsymbol{\sigma}/\sigma_0, & \bar{p} &= p/\sigma_0, & \bar{\mathbf{v}} &= \mathbf{v}/v_0, & \bar{r} &= r/r_0, \\ \bar{z} &= z/r_0, & \bar{\theta} &= \theta/\theta_0, & \bar{T} &= T/T_0, & \bar{b} &= bv_0/r_0, \\ \bar{v} &= v\theta_0, & \alpha &= \rho v_0^2/\sigma_0, & \bar{k} &= k/(\rho c v_0 r_0), & T_0 &= \theta_0 + 273, \\ \theta_0 &= \sigma_0/(\rho c), & \bar{\mathbf{s}} &= \mathbf{s}/\sigma_0, & \bar{z}_2 &= z_2/\sigma_0, & \bar{z}_1 &= z_1/\sigma_0, \\ \bar{z}_3 &= z_3/\sigma_0, & \bar{W} &= W/\sigma_0, & \bar{D}_0 &= D_0 v_0/r_0, & \bar{a} &= a/\theta_0, \\ & & \bar{h} &= h/(v_0 \rho c). \end{aligned} \quad (2.15)$$

Here r_0 is the radius of the cylindrical part of the penetrator and the heat transfer coefficient h appears in the boundary condition (2.25) below. Substituting from equations (2.5) and (2.6) into equation (2.3), rewriting it and equations (2.1), (2.2), (2.4), and the constitutive relations (2.7) and (2.11) in terms of non-dimensional variables, and dropping the superimposed bars, we arrive at the following set of equations.

$$\operatorname{div} \mathbf{v} = 0, \quad (2.16)$$

$$\operatorname{div} \mathbf{v} = \alpha(\mathbf{v} \cdot \operatorname{grad}) \mathbf{v}, \quad (2.17)$$

$$\operatorname{tr}(\boldsymbol{\sigma} \mathbf{D}) + k \operatorname{div}(\operatorname{grad} \theta) = (\mathbf{v} \cdot \operatorname{grad}) \theta, \quad (2.18)$$

where either

$$\boldsymbol{\sigma} = -p \mathbf{1} + \frac{1}{\sqrt{3} I} (1 + bI)^e (1 - \nu \theta) (1 + \psi/\psi_0)^q \mathbf{D}, \quad (2.19)$$

$$\frac{\operatorname{tr}(\boldsymbol{\sigma} \mathbf{D})}{(1 + \psi/\psi_0)^q} = (\mathbf{v} \cdot \operatorname{grad}) \psi, \quad (2.20)$$

or

$$\boldsymbol{\sigma} = -p\mathbf{1} + \left(z_2 / \left(\sqrt{3} I \left[\frac{2n}{n+1} \ln(D_0/I) \right]^{1/2n} \right) \right) \mathbf{D}, \quad (2.21)$$

$$\text{tr}(\boldsymbol{\sigma}\mathbf{D}) = (\mathbf{v} \cdot \text{grad})W, \quad (2.22)$$

and n , and z_2 are given by expressions (2.12).

We assume that the target/penetrator interface is smooth, and impose on it the following boundary conditions.

$$\mathbf{t} \cdot (\boldsymbol{\sigma}\mathbf{n}) = 0, \quad (2.23)$$

$$\mathbf{v} \cdot \mathbf{n} = 0, \quad (2.24)$$

$$\mathbf{q} \cdot \mathbf{n} = h(\theta - \theta_a), \quad (2.25)$$

where h is the heat transfer coefficient between the penetrator and the target, θ_a is an average temperature of the penetrator, and $\mathbf{n}$ and $\mathbf{t}$ are, respectively, a unit normal and a unit tangent vector to the surface. Equation (2.25) accounts approximately for the heat exchange between the penetrator and the target. At points far away from the penetrator

$$|\mathbf{v} + \mathbf{e}| \rightarrow 0, \quad \theta \rightarrow 0, \quad \psi \rightarrow 0, \quad W \rightarrow 0 \quad \text{as} \quad (r^2 + z^2)^{1/2} \rightarrow \infty, \quad z > -\infty, \quad (2.26)$$

$$|\boldsymbol{\sigma}\mathbf{n}| \rightarrow 0, \quad |\mathbf{q} \cdot \mathbf{n}| \rightarrow 0, \quad \psi \rightarrow 0, \quad W \rightarrow 0 \quad \text{as} \quad z \rightarrow -\infty, \quad r \geq r_0, \quad (2.27)$$

where $\mathbf{e}$ is a unit vector along the positive z -axis. The boundary condition (2.26) implies that target particles at a large distance from the penetrator appear to be moving at a uniform velocity with respect to it, and experience no change in their temperature. Equation (2.27) states that when target particles have moved far to the rear of the penetrator, the surface tractions and heat flux on them vanish. Recalling the constitutive relations (2.7) and (2.11), we see that the vanishing of surface tractions at far away points does not imply that the pressure there vanishes. Ideally, one should specify the rate of decay of quantities in equations (2.26) and (2.27). However, at this time, there is no hope of proving an existence or uniqueness theorem for an analytical solution of the stated problem. We, therefore, gloss over this rather difficult issue. Herein we assume that the aforesaid problem has a solution and seek its approximation by the finite element method.

3. COMPUTATIONAL CONSIDERATIONS

Unless one uses special infinite elements, a numerical solution of the problem necessitates that we consider a finite region. Since the target deformations are assumed to be axisymmetric, only the deformations of the target region R shown in Fig. 1 are studied, and the boundary conditions (2.26) and (2.27) at the far surfaces are replaced by the following conditions (3.1) and (3.3) on the boundary surfaces of the finite region being analyzed.

$$\sigma_{zz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{on the surface } AB, \quad (3.1)$$

$$\sigma_{rz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial r} = 0 \quad \text{on the axis of symmetry } DE, \quad (3.2)$$

$$v_r = 0, \quad v_z = -1.0, \quad \theta = 0, \quad \psi = 0, \quad W = 0 \quad \text{on the boundary surface } EFA. \quad (3.3)$$

Conditions (3.2) follow from the assumed symmetry of deformations. The validity of replacing (2.26) by (3.3), and (2.27) by (3.1), and the accuracy of the computed results depend upon the size of the region R . Keeping DE fixed, we increased the distance BC until the change in the values of solution variables such as the pressure p , velocity $\mathbf{v}$, and temperature θ at points in the vicinity of the target/penetrator interface was less than 0.1%. Then, BC was kept fixed and the size of DE was increased to attain convergence of the solution variables at points adjoining the target/penetrator interface. The region so obtained and its finite element discretization depicted in Fig. 1 were used to compute all of the results presented and discussed below. The

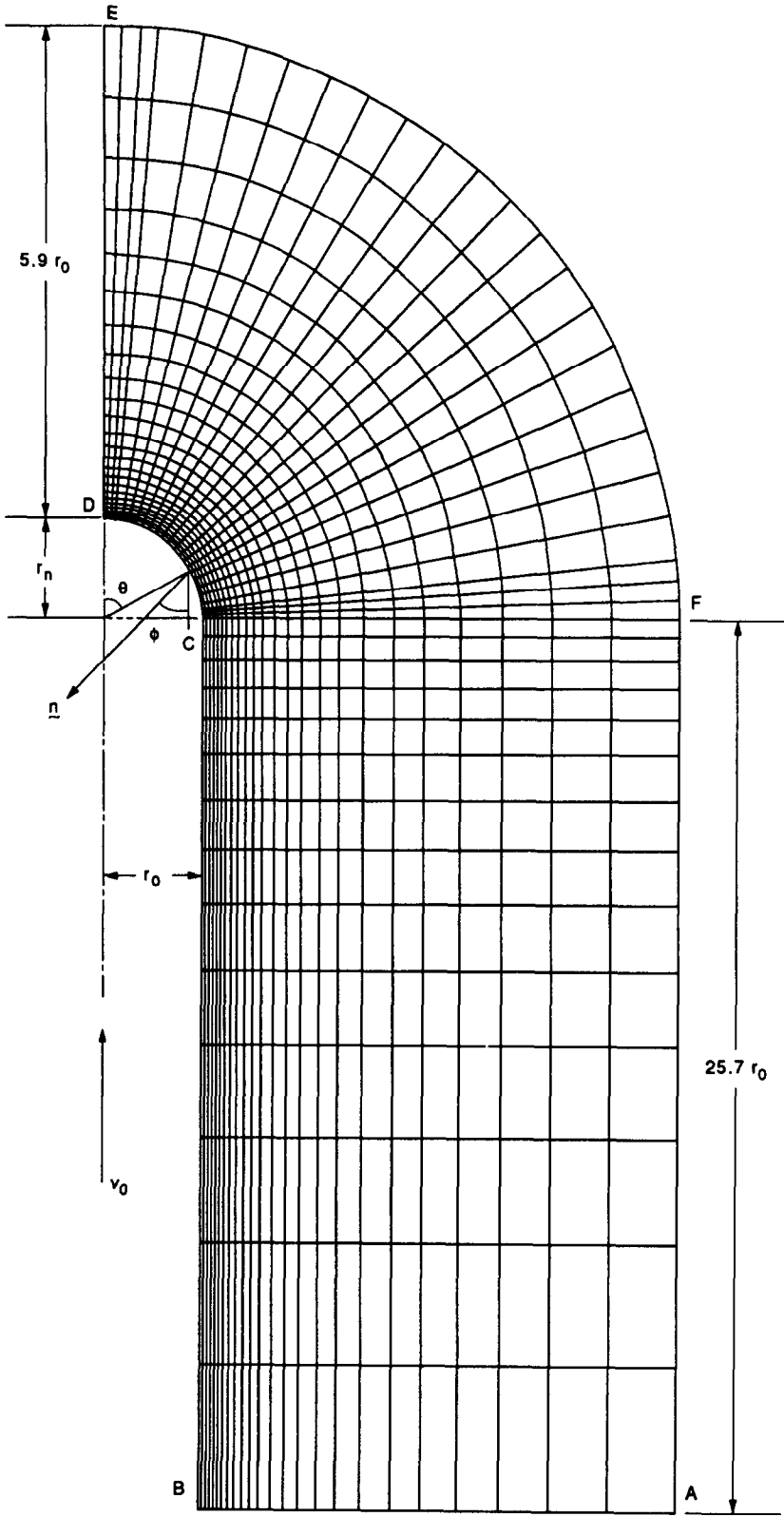


Fig. 1. The finite region studied and its discretization.

finite element mesh is very fine in darker regions. The size of the region considered herein is considerably more than that studied by Batra [19]. An increase in the distance DE resulted in a decrease in the axial resisting force experienced by the penetrator, but an increase in the distance BE increased the axial resisting force acting on the penetrator.

The finite element code developed by Batra [19] to solve for target deformations when its material is modeled by constitutive relation (2.19) was modified to include the Bodner–Partom flow rule (2.21). A weak formulation of the problem and an iterative technique to solve the nonlinear system of equations is also given in [19]. Whereas Batra [19] used six-noded triangular elements, here we use 9-noded quadrilateral elements to approximate the fields of $\mathbf{v}$, θ , and ψ within an element. The hydrostatic pressure p is assumed to be bilinear on each quadrilateral element, and is defined in terms of its values at the four corner nodes. Batra [29] has shown that when a problem similar to the one being studied here is analyzed by using identical nodal locations but either 6-noded triangular or 9-noded quadrilateral elements, the two sets of results are identical, except that the quadrilateral elements give smoother fields. In either case, *no posteriori* smoothing technique was applied to the computed values of the nodal pressures. In the results presented below, as well as in [19], the solution of the nonlinear system of equations was assumed to have converged, if at each nodal point the norm of the increments in v_r , v_z , and θ differed by less than 2% of the norm of v_r , v_z , and θ . Here v_r and v_z equal, respectively, the r - and z -components of the velocity of a point relative to an observer situated at the stagnation point and moving with it.

4. NUMERICAL RESULTS

4.1 Comparison of predictions from the two constitutive relations

We note that experimental data for the range of deformations expected to occur in the penetration problem under study is not available in the open literature. Batra and Kim [30] determined values of material parameters appearing in the two constitutive relations by ensuring that the computed shear stress–shear strain curve during overall adiabatic simple shearing deformations of a viscoplastic block deformed at an average strain-rate of 3300 s^{-1} matched well with the experimental curve of Marchand and Duffy [31] for a HY-100 structural steel. We use those values, and list them below.

(a) Values same for both constitutive laws

$$\begin{aligned} \rho &= 7860 \text{ kg/m}^3, & \sigma_0 &= 405 \text{ MPa}, & c &= 473 \text{ J/kg } ^\circ\text{C}, \\ K &= 50 \text{ W/m } ^\circ\text{C}, & h &= 20 \text{ W/m}^2 \text{ } ^\circ\text{C}, & \theta_a &= 0, & r_0 &= 2.54 \text{ mm}. \end{aligned}$$

(b) Litonski–Batra flow rule

$$\begin{aligned} \nu &= 6.55 \times 10^{-4} / ^\circ\text{C}, & \psi_0 &= 0.012, & q &= 0.054, & e &= 0.01872, \\ & & b &= 10^4 \text{ s}. \end{aligned}$$

(c) Bodner–Partom flow rule

$$\begin{aligned} a &= 1800^\circ\text{K}, & z_1 &= 3.778, & z_3 &= 3.185, & m &= 2.5, \\ & & D_0 &= 3.3 \times 10^6 \text{ s}^{-1}. \end{aligned}$$

Thus, the reference temperature θ_0 used to non-dimensionalize the temperature rise equals 108.9°C .

Figure 2 depicts the distribution of the normal stress, temperature rise, tangential speed, and I on the nose surface when the penetrator nose is hemispherical and $\alpha = 4.5$. In these plots, the values of the normal stress and the temperature rise have been divided by ten, in order for the curves to fit on the same graph. The values of the tangential speed and the strain-rate measure I as computed with the two constitutive relations come out to be very close to each other. The two temperature distributions agree qualitatively, and seem to differ by essentially a constant value. At first glance it seems that this difference is due to the different scales of temperature in

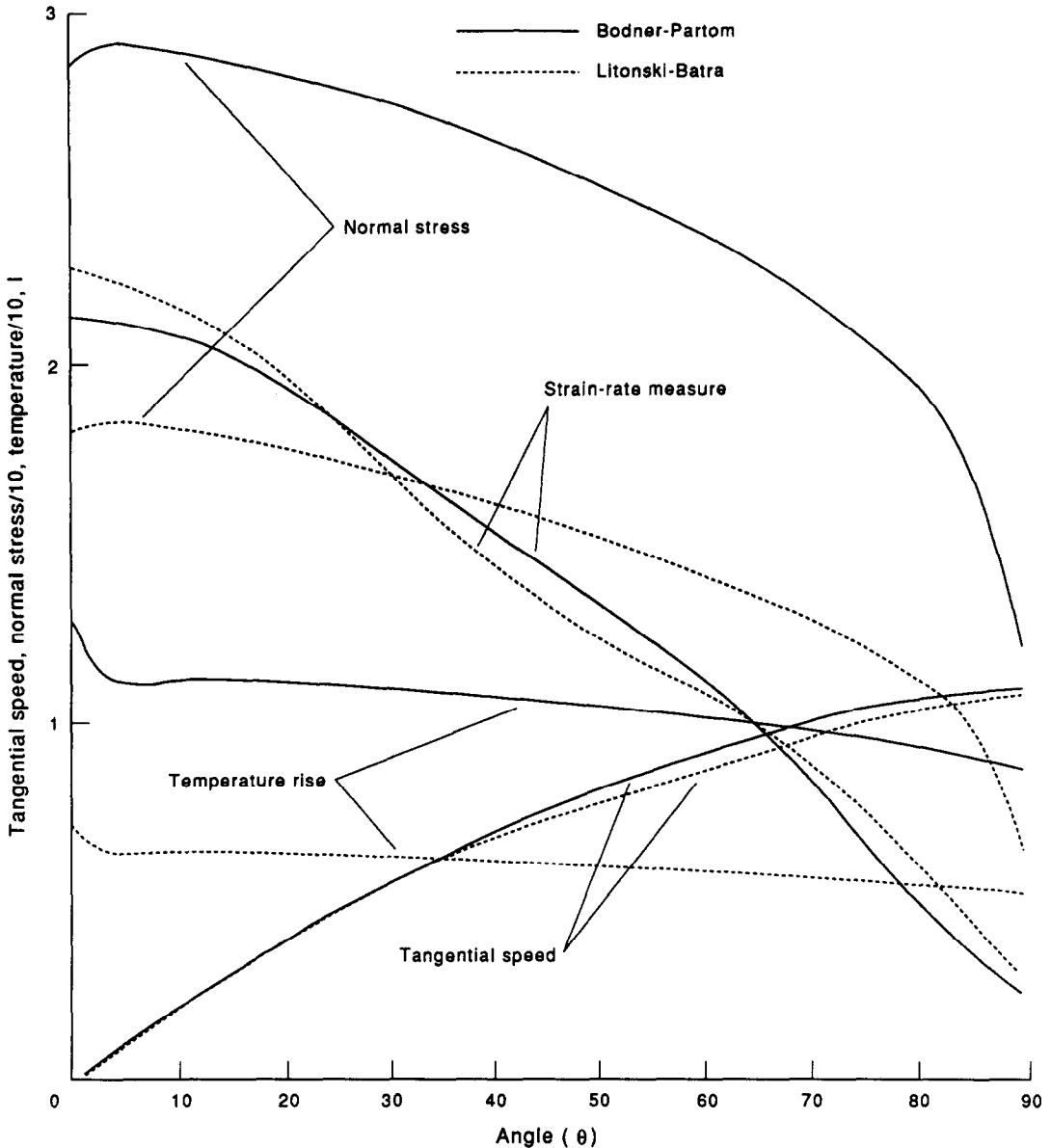


Fig. 2. Comparison of the variation of I , normal stress, tangential speed, and the temperature rise at target particles abutting the penetrator nose surface for the two constitutive relations.

the two constitutive relations. However, this was not found to be the case. Both constitutive relations predict sharply higher values of the temperature rise at target particles near the stagnation point. A possible explanation for this is that, at the stagnation point a considerable amount of heat is generated, but little is conducted away due to the low value of the thermal conductivity, and the heat loss due to convection is also very small because of the relatively small values of the speed of the particles surrounding the stagnation point. As one moves away from the stagnation point, heat loss due to convection increases because of the increased speed of target particles. The distribution of the normal stress on the penetrator/target interface as computed by the two constitutive relations also agrees qualitatively. However, the two normal stress distributions differ quantitatively, mainly because of the difference in the values of the hydrostatic pressure as computed by the two constitutive relations. For example, the peak pressure at or near the stagnation point equalled 18.71 and 30.16, respectively, for the Litonski-Batra and the Bodner-Partom flow rules. Because of the differences in the values of the deviatoric stress tensor s and the strain-rate measure I as computed by the two constitutive relations, the rate of energy dissipated due to plastic working and hence, the resulting

temperature distribution is different in the two cases. We note that the two constitutive relations are calibrated to give identical response in overall adiabatic simple shearing deformations of a viscoplastic block deformed at an average strain-rate of 3300 s^{-1} . The state of deformations at a target particle need not correspond to that of simple shearing. Also, the calibration procedure involves solving a nonlinear initial-boundary-value problem whose solution may be non-unique. Thus, two different sets of values of material parameters may give the same shear stress–shear strain curve. The axial resisting force equalled 15.24 and 25.19, respectively, for the Litonski–Batra flow rule and the Bodner–Partom flow rule.

Figure 3 shows the distribution of $(-\sigma_{zz})$, I , θ , and $(-v_z)$ on the axial line as computed by using the two constitutive relations. Whereas the two sets of values of I and v_z are very close to each other, those of θ and σ_{zz} agree qualitatively. These do indicate that significant deformations occur at target points whose distance from the target/penetrator interface is less than one penetrator diameter. The values of (I, θ) at the stagnation point are found to be (2.09, 7.35) and (2.14, 13.58) for the Litonski–Batra and Bodner–Partom flow rules, respectively. Thus, for the Bodner–Partom law, the temperature at the stagnation point almost equalled the presumed melting temperature of 1800°K .

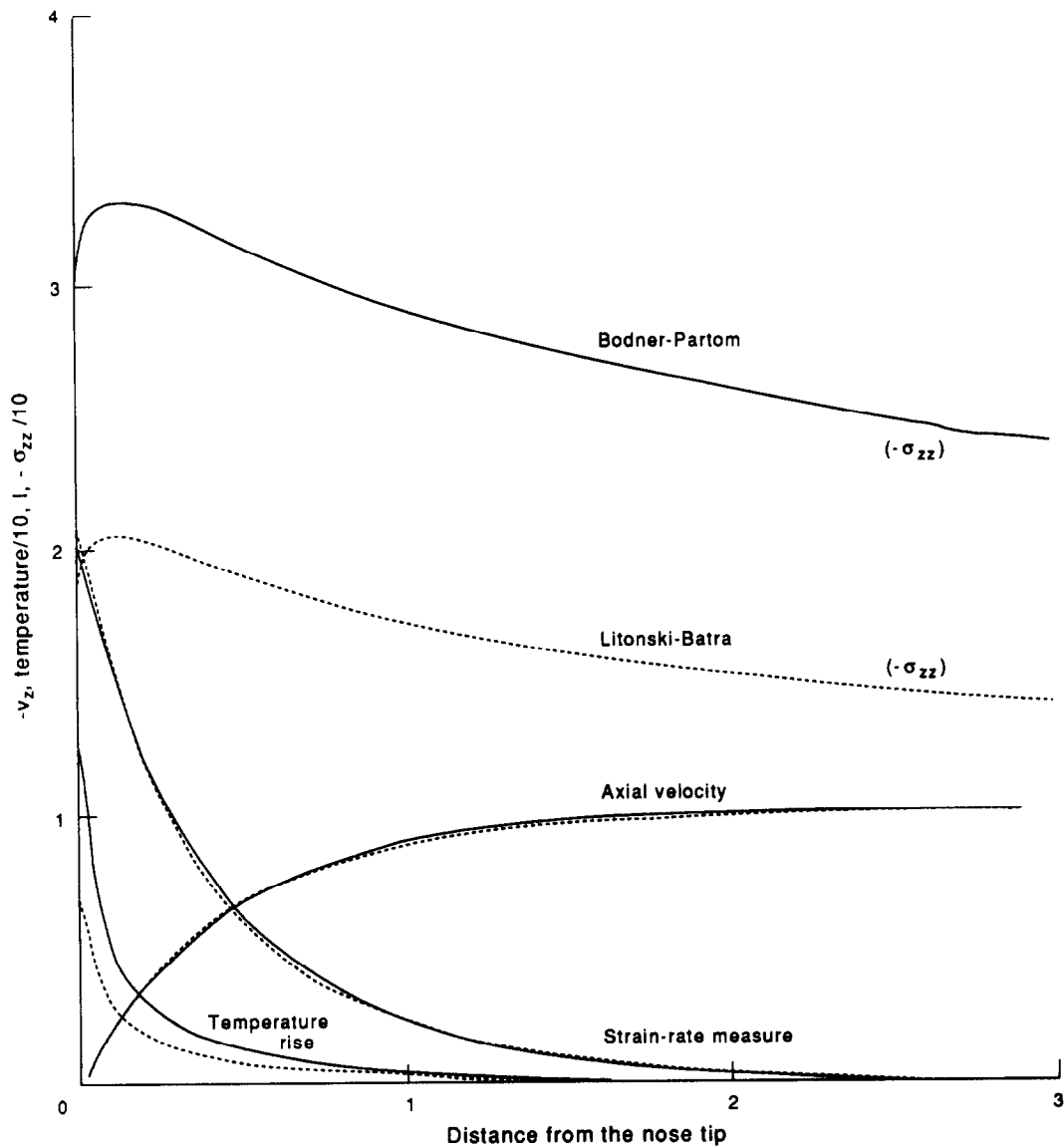


Fig. 3. Comparison of the variation of $(-\sigma_{zz})$, I , θ , and $(-v_z)$ on the axial line for the two constitutive relations.

4.2 Results for the Bodner–Partom flow rule

We now study the effect of different material parameters in the Bodner–Partom law on the deformations of the target. This will elucidate the relative importance of various material parameters and hence help design experiments for the precise determination of more critical ones. Since we are interested in the parametric study, the values of different parameters used is of less significance. The range of values of material parameters considered herein is probably more than that likely to be encountered for any real material. We have assigned the following values to various non-dimensional material and geometric parameters.

$$D_0 = 6, \quad z_1 = 1.505, \quad z_3 = 1.236, \quad m = 5, \quad r_n = 1.0, \quad \alpha = 2.1 \quad (4.1)$$

Except when studying the effect of changes in the melting temperature a of the material, it was set equal to 1200°K. The variables that are assigned values different from those given above are so indicated in the figures, along with their new values. In (4.1), $2r_n$ equals the length of the principal axis of the ellipsoidal nose in the z -direction.

In Fig. 4, we have plotted the variation of the normal stress, strain rate measure I , the

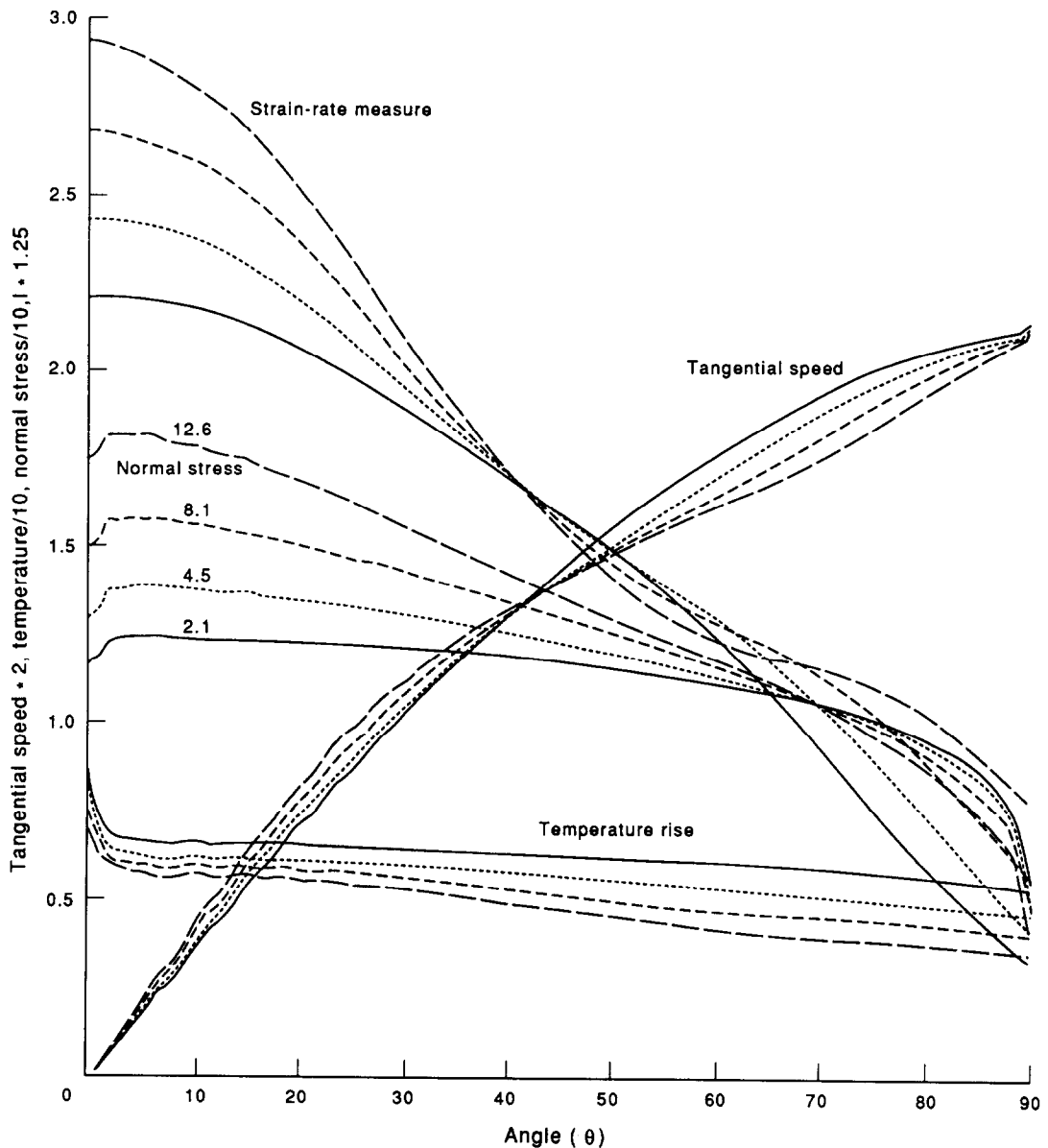


Fig. 4. Variation of I , normal stress, the tangential speed, and the temperature rise at target particles adjoining the penetrator nose surface for different values of α .

tangential speed, and the temperature rise on the penetrator/target interface for four different values of α . Note that these variables are multiplied by different numbers so that the same vertical scale could be used. As expected, the normal stress on the target/penetrator interface increases with an increase in the value of α . However, for every value of α considered, it does drop off quite rapidly near the periphery of the penetrator nose, and seems to be independent of α at the point for which the angle $\theta \simeq 70^\circ$. A similar behavior at $\theta \simeq 45^\circ$ was observed by Batra [19] for the Litonski-Batra flow rule. The values of I for $\theta \leq 40^\circ$ and $\theta \geq 70^\circ$ increase with an increase in the value of α , but at many points for which $40^\circ < \theta < 70^\circ$, they exhibit the opposite trend. As the penetration speed v_0 is varied, the dimensional values of I change more than the non-dimensional ones, since the latter need to be multiplied by v_0/r_0 to obtain the former. The same is true about the tangential speed on the target/penetrator interface. However, with an increase in the value of α , the tangential speed increases at points on the target/penetrator interface that are near the axial line, but decreases at points near the nose periphery. It would appear from the distributions of the normal stress and I on the penetrator/target interface that the temperature rise at target particles abutting the penetrator nose should increase with an increase in the value of α . However, the temperature rise at the nose surface decreases with an increase in the penetration speed, because at higher speeds, the

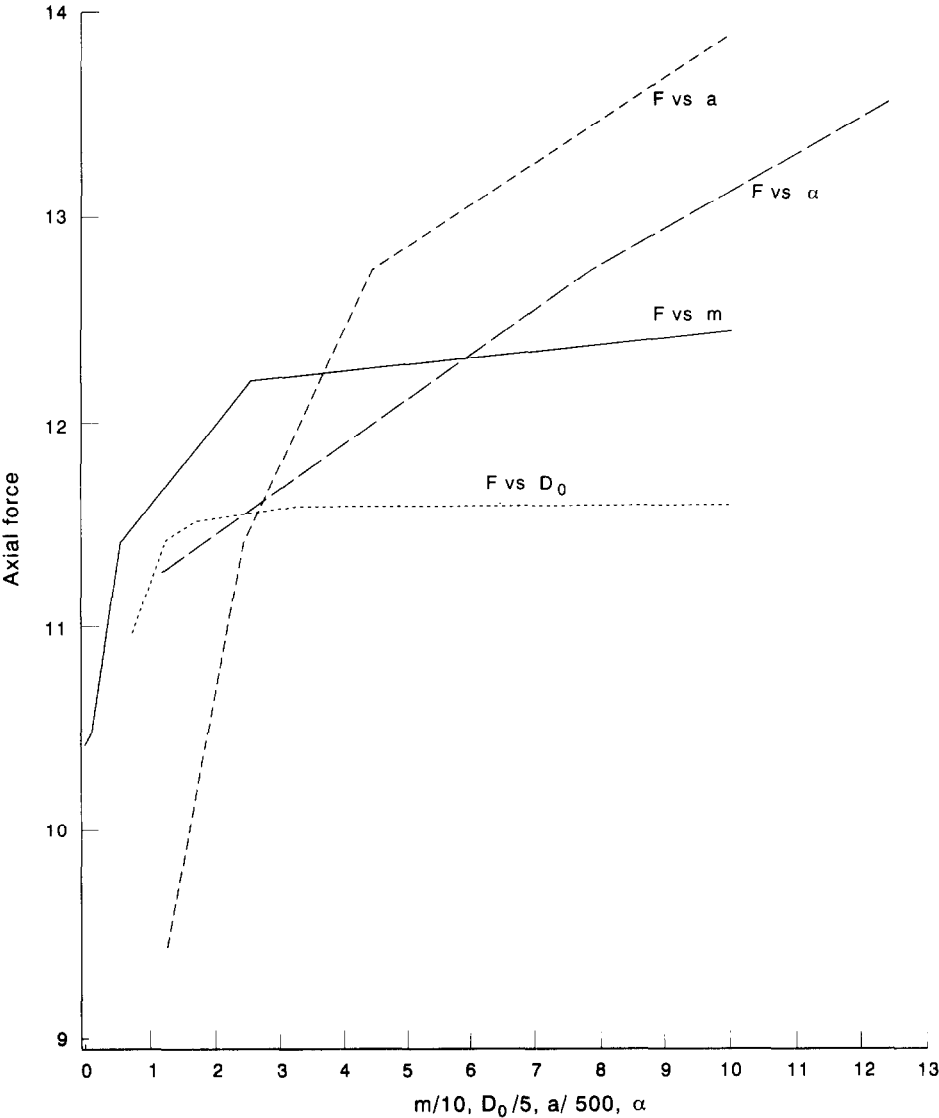


Fig. 5. Dependence of the axial resisting force upon various parameters.

heat loss due to convection increases significantly. A similar trend in the temperature distribution was computed by Batra [19] with the Litonski-Batra flow rule.

The axial resisting force F is given by

$$F = 2 \int_0^{\pi/2} (\mathbf{n} \cdot \boldsymbol{\sigma} \mathbf{n}) \frac{\cos \phi \sin \theta [\sin^2 \theta + (1/r_n)^4 \cos^2 \theta]^{1/2}}{[\sin^2 \theta + (1/r_n)^2 \cos^2 \theta]^2} d\theta, \quad (4.2)$$

$$\cos \phi = \frac{z/r_n^2}{[r^2 + (z/r_n^2)^2]^{1/2}}, \quad (4.3)$$

where the angle θ is defined in Fig. 1, and (r, z) are the coordinates of a point on the penetrator/target interface. The corresponding axial force in physical units is given by $(\pi r_0^2 \sigma_0)F$. We note that the expression given by Batra [19] for the axial force, except for the hemispherical nose shape, is in error. The dependence of the axial force upon α is exhibited in Fig. 5; the axial force depends upon α rather weakly, and the relation between the two is

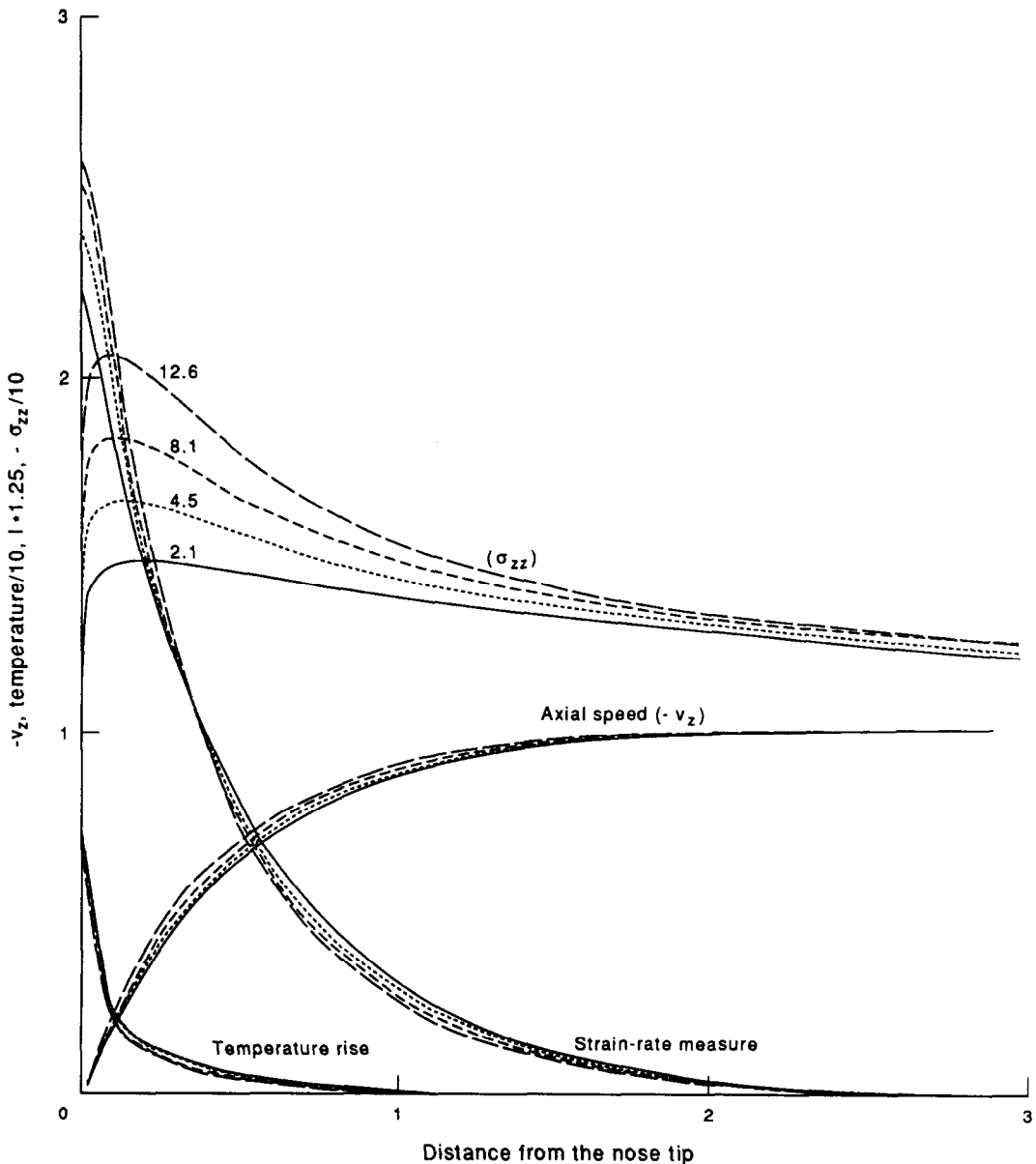


Fig. 6. Dependence of the temperature rise, I , σ_{zz} , and v_z at target particles on the axial line upon α .

nearly affine. Because of the increase in F with α , for the same initial kinetic energies of penetrators, those moving at higher speeds will give lower values of the penetration depth.

The variation of the temperature rise θ , I , σ_{zz} , and v_z along the axial line for the four different values of α considered is shown in Fig. 6. These plots vividly reveal that severe deformations of the target occur in the vicinity of the target/penetrator interface. The values of I and θ drop to zero rather quickly, and stay at zero for $z \geq 2.0$. This ensures the adequacy of the region considered. The values of σ_{zz} decay slowly, mainly because the hydrostatic pressure which contributes noticeably to σ_{zz} drops off slowly.

Figure 7 depicts the distribution of the tangential speed, normal stress, temperature rise, and I on the target/penetrator interface for several values of m . For larger values of m , the value of z_2 approaches the saturation value z_1 for smaller values of the plastic work density W . At a target particle abutting the penetrator nose, the values of the normal stress and the temperature rise increase monotonically with an increase in the value of m , those of I do not show any definite trend. The values of the tangential speed do not change that much when m is varied. The

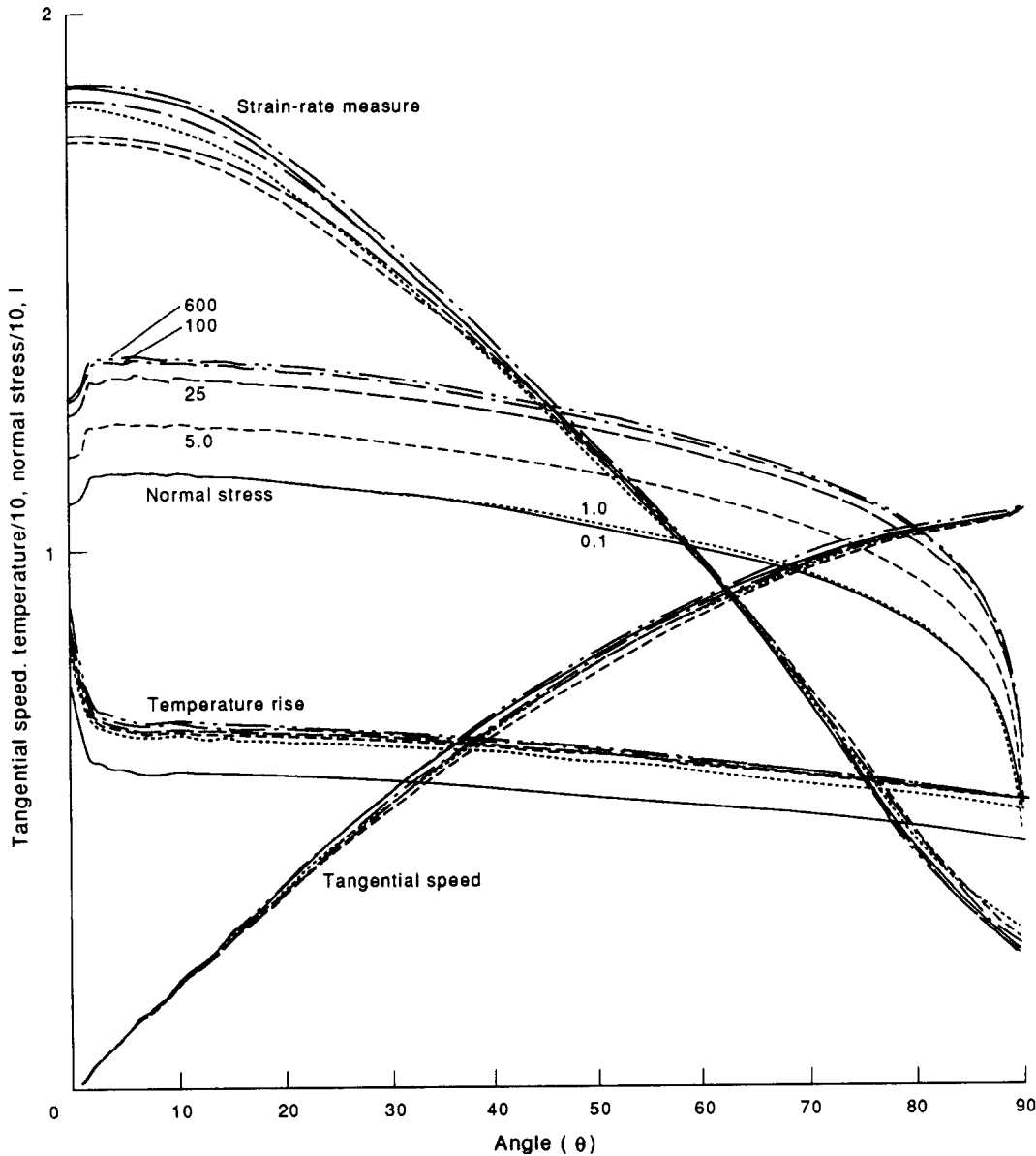


Fig. 7. Effect of m on the normal stress, temperature rise, and I at target particles on the penetrator nose surface.

distributions of the normal stress and the temperature rise are affected very little when m is increased from 100 to 600.

The effect of changing the non-dimensional value of D_0 on the distribution of the normal stress, temperature rise θ , the tangential speed, and I on target particles adjoining the hemispherical penetrator nose is shown in Fig. 8. The values of the normal stress and I increase with an increase in the value of D_0 . Recall that D_0 defines the limiting value of I . The peak values of I at a target particle located near the stagnation point keep on increasing with D_0 , albeit slowly; those of the normal stress increase even more slowly. For none of the values of D_0 considered, does the computed peak value of I equal D_0 . The values of θ decrease with an increase in D_0 , possibly because of the increase in the rate of heat loss due to convection. The values of the tangential speed change very little with D_0 . The axial resisting force experienced by the penetrator, plotted in Fig. 5, first increases with D_0 , and then levels off.

Figure 9 depicts the effect of varying the value of a upon the distribution of the normal stress, strain rate measure I , temperature rise θ , and the tangential speed at target particles

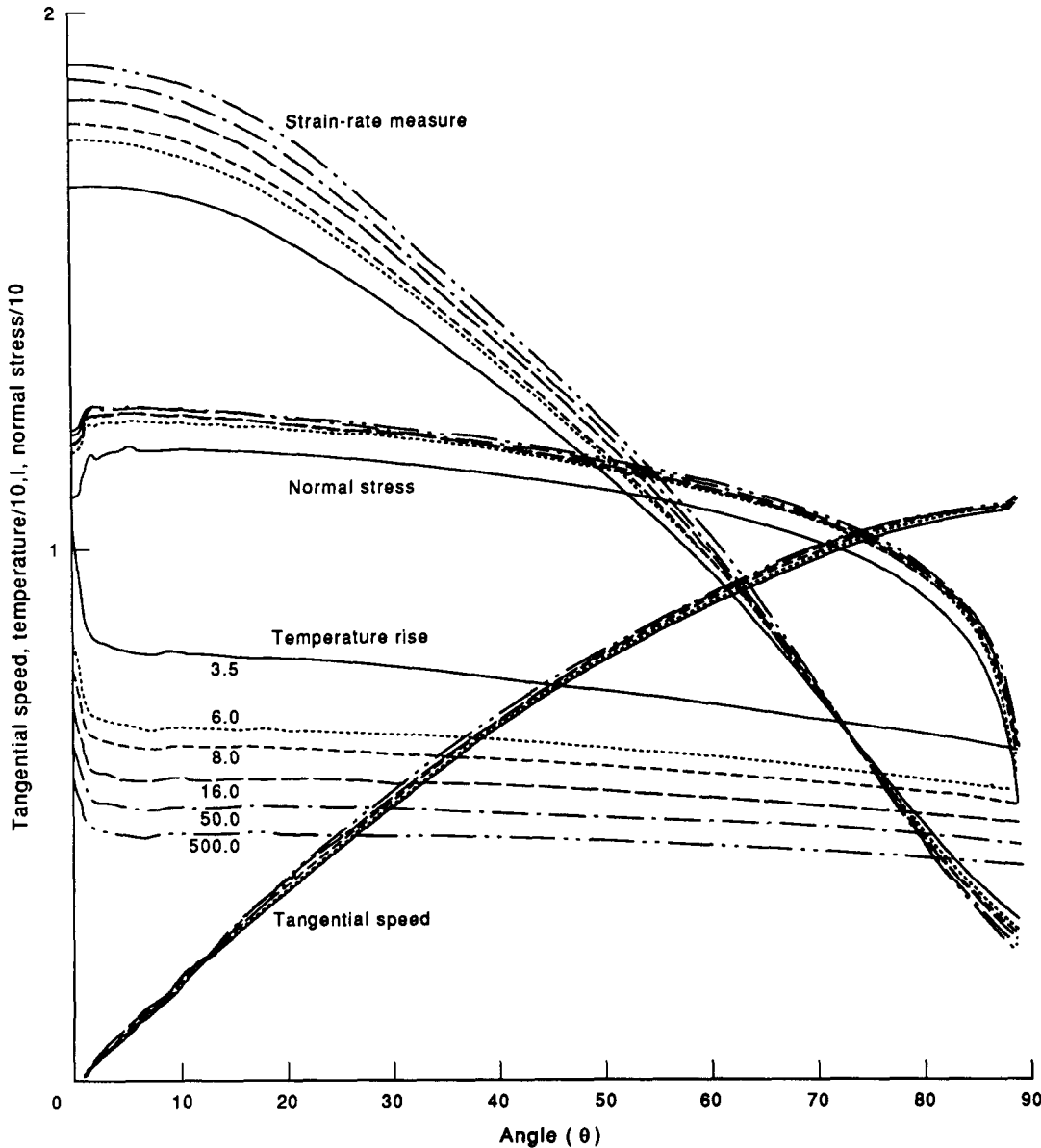


Fig. 8. Effect of D_0 on the normal stress, temperature rise, and I at target particles on the penetrator nose surface.

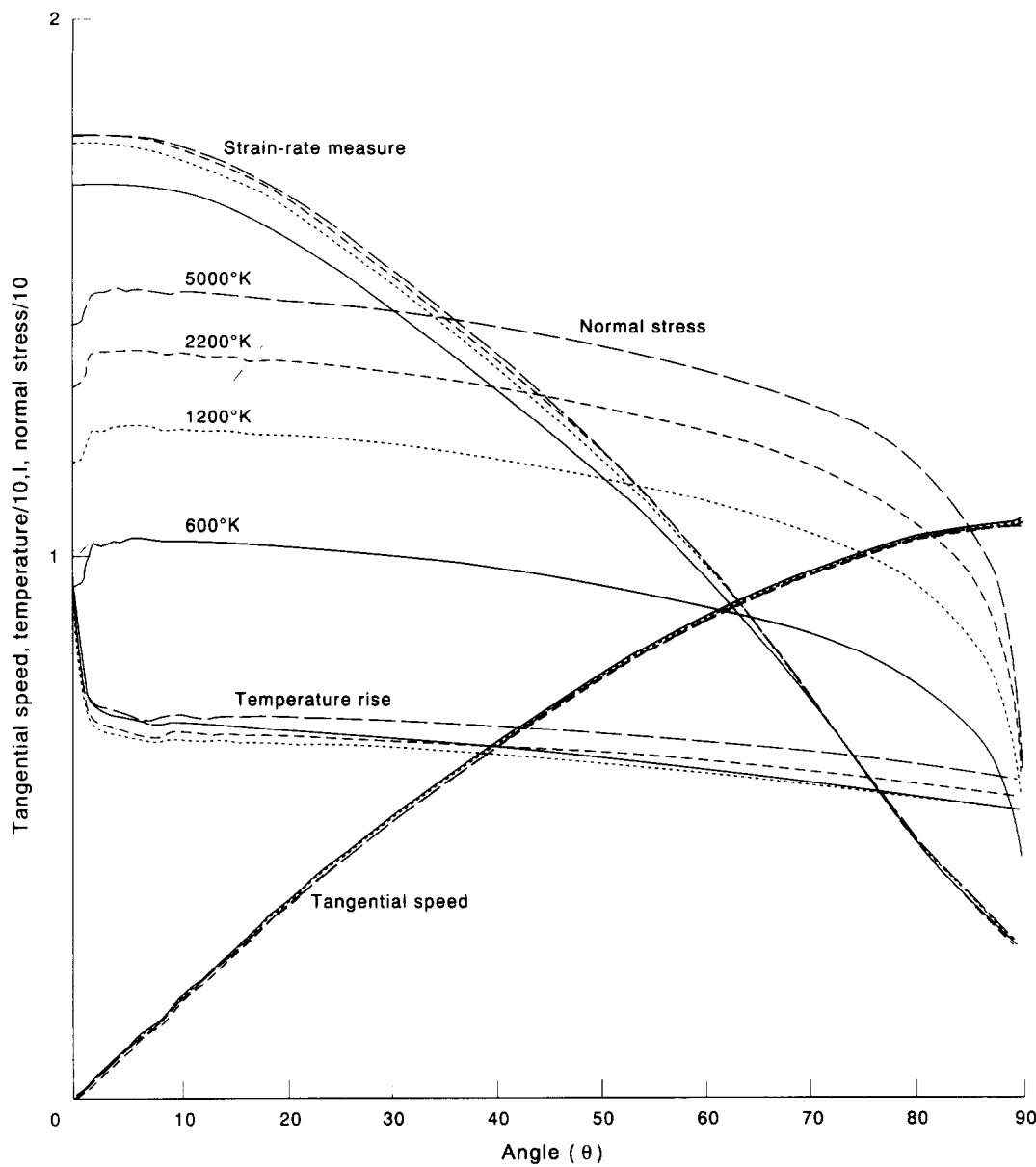


Fig. 9. Effect of a on the normal stress, temperature rise, and I at target particles on the penetrator nose surface.

adjacent to the penetrator nose surface. Whereas both the normal stress and I increase with an increase in the value of a , the temperature rise at a point does not show any clear trend. The values of the tangential speed seem to be unaffected by the value of a . Higher values of a imply that the material will thermally soften less for the same temperature rise. Consequently, it will offer more resistance to penetration as suggested by the larger values of the normal force acting on the target/penetrator interface. The axial resisting force experienced by the penetrator keeps on increasing with a , but the rate of increase drops off at larger values of a .

The distribution of the normal stress, strain-rate measure I , and the tangential speed at target particles abutting the penetrator nose surface for three different nose shapes, i.e. $r_n/r_0 = 2.0, 1.0$, and 0.5 , is plotted in Fig. 10. The curves representing the normal stress distribution when $r_n/r_0 = 2.0$ and 1.0 have curvature of opposite signs. For the penetrator nose shape with $r_n/r_0 = 0.5$, the normal stress changes very little over the region $10^\circ \leq \theta \leq 45^\circ$. At any particular location on the penetrator nose surface, the tangential speed decreases with a decrease in the value of r_n/r_0 . For the long tapered nosed penetrator, the strain-rate measure I

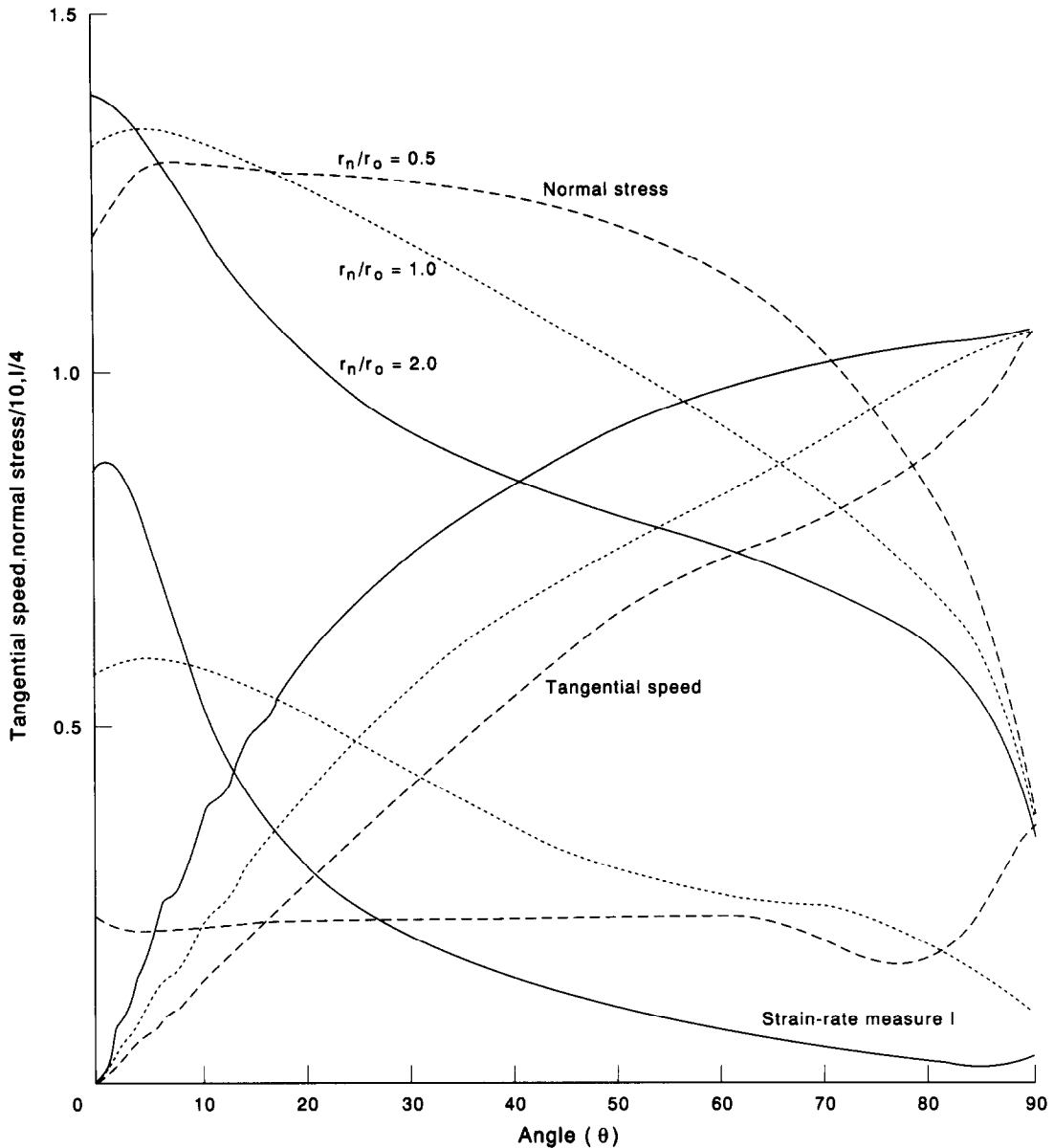


Fig. 10. Effect of nose shape on the normal stress, strain-rate measure I , and the tangential speed at target particles on the penetrator nose surface.

assumes its peak value at a target particle near the stagnation point. For a somewhat blunt nosed penetrator, the strain-rate measure I stays constant over most of the penetrator nose surface, and increases near the nose periphery. We recall that the results [19] computed with the Litonski-Batra flow rule agree qualitatively with the ones given in Fig. 10. For the Bodner-Partom flow rule, the convergence of the solution for the case when $r_n/r_0 = 0.2$ necessitated an increase in the value of D_0 , presumably because the peak value of I near the nose periphery approached D_0 and the term $\ln(D_0/I)$ in the denominator of the right-hand side of equation (2.21) became negative, and the denominator in equation (2.21) could not be evaluated. Thus, results for this case are not included herein. One way to get around this problem is to increase D_0 . Results plotted in Fig. 11 reveal that at target particles on the axial line, the rate of decrease of $(-\sigma_{zz})$ and I with the distance from the penetrator nose tip becomes less as the value of r_n/r_0 is decreased. For the somewhat blunt nosed penetrator ($r_n/r_0 = 0.5$), the target particles deform less severely, but more of the target material is deformed as compared to that for the long tapered nosed penetrator.

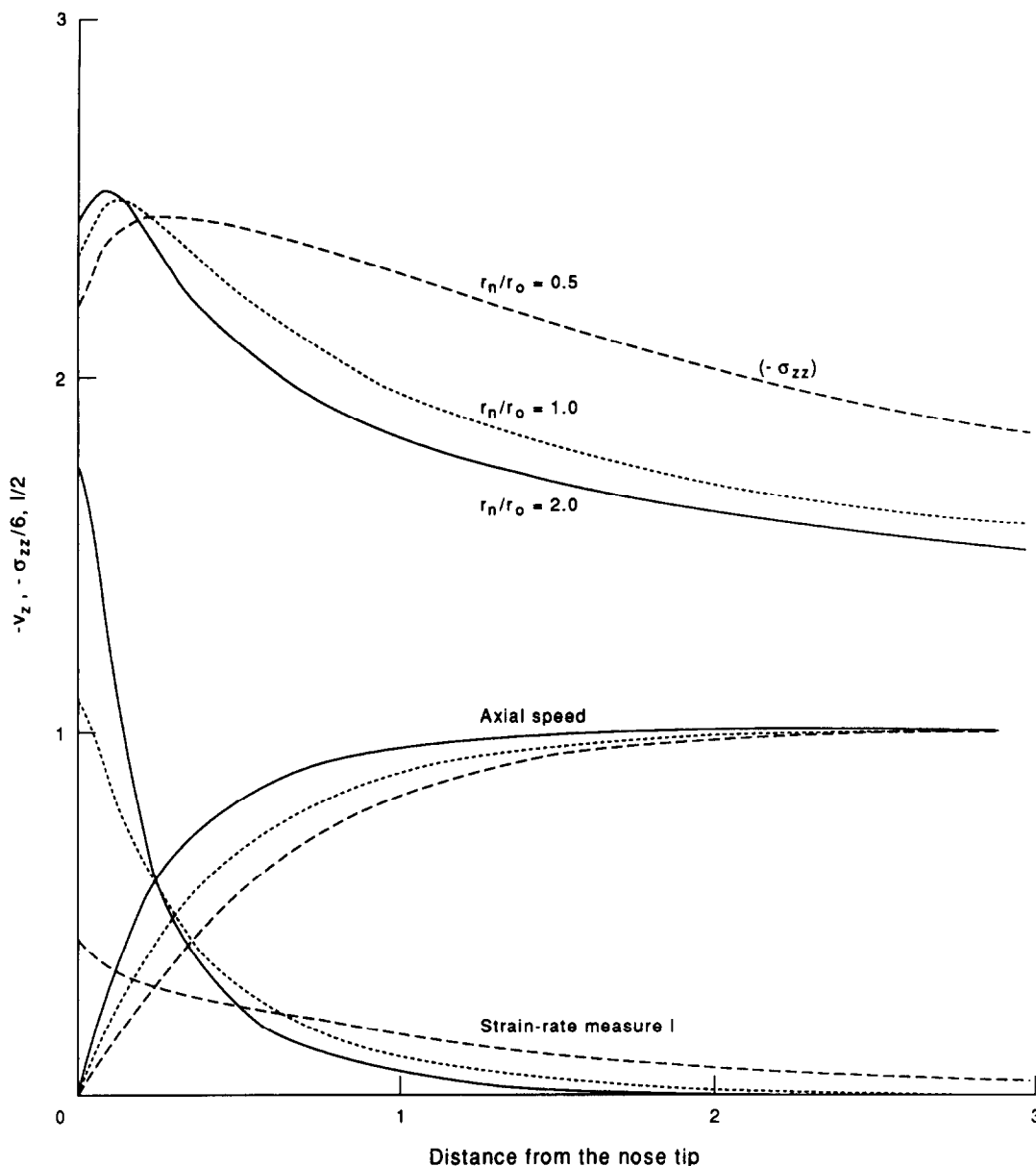


Fig. 11. Distribution of $(-\sigma_{zz})$, I , and the axial speed at target particles on the axial line for three different nose shapes.

The axial resisting force acting on the penetrator increases sharply as r_n/r_0 is decreased from 2.0 to 0.5; this is plotted in Fig. 5. Figure 12 depicts the variation of the axial speed of the target material flowing rearward and instantaneously lying on the planes $z = 0$ and $z = -1.0$. It is clear that the target material adjacent to the sides of the penetrator appears to extrude rearward as a uniform block that is separated from the bulk of the stationary target by a relatively narrow region with a sharp velocity gradient. This calculation of backward extrusion of a uniform block provides a partial justification to the velocity field assumed by Ravid and Bodner [7] in their work involving penetration of targets of finite thickness. There is no experimental data available in the open literature that proves or disproves the validity of results presented herein.

5. CONCLUSIONS

We have studied thermomechanical deformations of a thick viscoplastic target being penetrated by a long rigid cylindrical penetrator. Results computed when the target material is

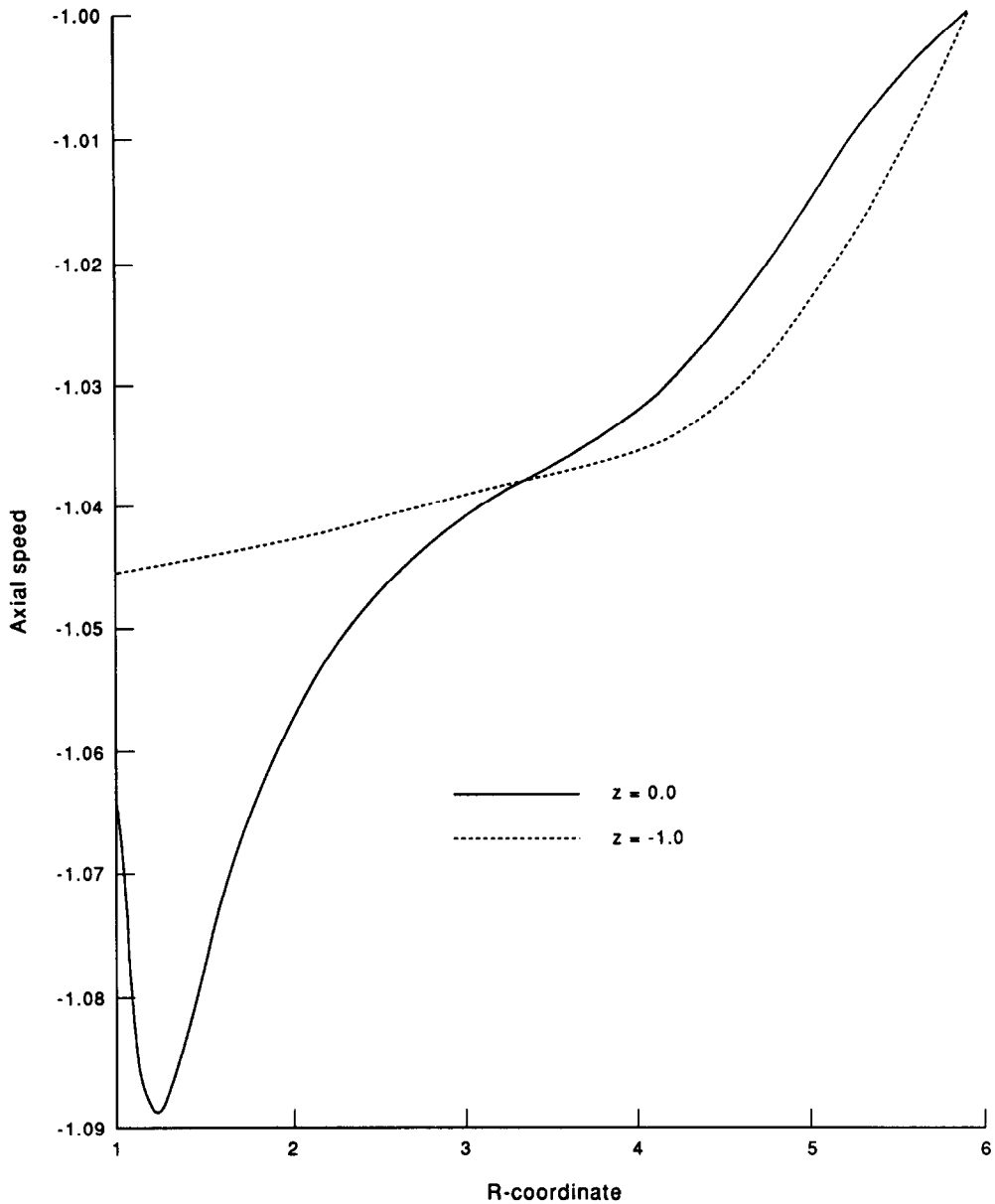


Fig. 12. Variation of the axial speed on planes $z = 0$ and $z = -1.0$.

modeled by the Litonski-Batra flow rule or the Bodner-Partom flow rule agree with each other qualitatively, but differ quantitatively. The material constants in the two constitutive relations were determined by requiring that the shear stress-shear strain curve in overall adiabatic simple shearing deformations of a block made of the target material were essentially similar. We note that the method used to determine the parameter values is not unique. The quantitative difference in the results computed with the two flow rules could also be due to the more complex state of deformations prevailing in the target than that in the simple shearing problem. The peak hydrostatic pressure for the Bodner-Partom flow rule is considerably more than that computed with the use of the Litonski-Batra flow rule.

We have also investigated the effect of the variation in the values of various parameters appearing in the Bodner-Partom flow rule. The range of values of parameters considered is more than that likely to be determined for one material. It is found that all of the parameters appearing in the Bodner-Partom flow rule influence strongly the deformations of the target. Significant deformations of the target occur at target particles ahead of the penetrator nose,

and distant less than one penetrator diameter from the penetrator nose surface. More severe deformations occur at target particles in the vicinity of the stagnation point for a long tapered nosed penetrator than for other nose shapes. However, for a blunt nosed penetrator, severest deformations occur at target particles near the nose periphery.

Acknowledgements—This work was supported by the U.S. Army Research Office Contract DAAL03-89-K-0050 to the University of Missouri-Rolla. Some of the computations were done on the NSF sponsored supercomputer center in Urbana, Illinois.

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(Revision received 12 February 1991; accepted 15 March 1991)