

STEADY STATE AXISYMMETRIC DEFORMATIONS OF A THICK ELASTIC-THERMOVISCOPLASTIC TARGET

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Abstract—We study thermomechanical deformations of a thick elastic-viscoplastic target being penetrated by a rigid long cylindrical rod, and assume that target deformations as seen by an observer situated at the penetrator nose tip are steady. We presume that the target response can be adequately modeled by the Brown–Kim–Anand flow rule. We analyze the effect of different material parameters on the deformations of the target in order to elucidate their relative importance, and hence enumerate more critical ones. We also study the effect of the penetrator nose shape and the penetrator speed on the deformations of the target.

INTRODUCTION

One of the unresolved issues in penetration mechanics as well as in large deformation elastoplasticity is the choice of an appropriate constitutive relation used to model the finite plastic deformations of a material. Many of the recently proposed theories (e.g. [1–4]) of large deformation elastoplasticity are based on different kinematic assumptions and necessitate the hypothesizing of constitutive relations for variables that are not simply related to each other. Here we use one such theory, namely that due to Brown–Kim–Anand (hereafter referred to as BKA) [4], and study in detail the effect of varying the material parameters in it on the deformations of the target. This should help identify the critical parameters in the constitutive relation, at least for the penetration problem. A similar study was conducted earlier [5, 6] for the Litonski–Batra and the Bodner–Partom flow rules.

We refer the reader to review articles by Backman and Goldsmith [7], Wright and Frank [8], Wright [9], and Anderson and Bodner [10] for a review and discussion of most of the work done on ballistic penetration. Different engineering models have been proposed by Awerbuch [11], Awerbuch and Bodner [12], Ravid and Bodner [13], Ravid *et al.* [14], Forrestal *et al.* [15], and Batra and Chen [16]. For impact velocities in the range of 0.5–10 km/s, Birkhoff *et al.* [17], Pack and Evans [18], Allen and Rogers [19], Alekseevskii [20], and Tate [21] have proposed using the Bernoulli equation or its modification to analyze the impact phenomenon. The last three references introduced a resistive pressure, dependent upon the material strength, in the Bernoulli equation. Tate [22–24], Pidsley [25], Batra and Gobinath [26], Batra and Chen [16], and Jayachandran and Batra [27] have estimated the value of the resistive pressure. Whereas Tate used a solenoid fluid flow model of the steady state penetration process, other investigations used a numerical solution of the problem. Both Pidsley [25] and Wright [9] have pointed out that the transverse gradients of the shear stress evaluated on the axial line make noticeable contributions to the resistive pressure terms in the modified Bernoulli equation. The books by Zukas *et al.* [28], Blazynski [29], and Macauley [30] may be consulted for the available literature on ballistic penetration.

FORMULATION OF THE PROBLEM

With respect to a cylindrical coordinate system with origin at the center of the penetrator nose and positive z -axis pointing into the target, equations governing the target deformations are:

Balance of mass:

$$\operatorname{div} \mathbf{v} = 0, \quad (1)$$

Balance of linear momentum:

$$\operatorname{div} \boldsymbol{\sigma} = \rho(\mathbf{v} \cdot \operatorname{grad})\mathbf{v}, \quad (2)$$

Balance of internal energy:

$$-\operatorname{div} \mathbf{q} + \operatorname{tr}(\boldsymbol{\sigma} \mathbf{D}^p) = \rho \dot{U}, \quad (3)$$

Constitutive relations:

$$\boldsymbol{\sigma} = -p\mathbf{1} + \mathbf{s}, \quad (4.1)$$

$$\dot{\mathbf{s}} = 2G(\mathbf{D} - \mathbf{D}^p), \quad (4.2)$$

$$\mathbf{D}^p = \mathbf{s}/2\mu(I, \theta, g), \quad (4.3)$$

$$\mu(I, \theta, g) = \frac{1}{3} \frac{g}{\xi I} \sinh^{-1}(\phi^m), \quad (4.4)$$

$$\dot{g} = h_0 I \left(\max \left(0, \left(1 - \frac{g}{g^*} \right) \right) \right)^a, \quad (4.5)$$

$$g^* = \bar{g} \phi^n, \quad (4.6)$$

$$\phi = \frac{I}{A} \exp \left(\frac{Q}{RT} \right), \quad (4.7)$$

$$\mathbf{q} = -k \operatorname{grad} \theta, \quad \theta = T - T_0, \quad (5)$$

$$U = c\theta, \quad (6)$$

where

$$2I^2 = \operatorname{tr}(\mathbf{D}^p)^2, \quad (7.1)$$

$$\dot{\mathbf{s}} = (\mathbf{v} \cdot \operatorname{grad})\mathbf{s} + \mathbf{s}\mathbf{W} - \mathbf{W}\mathbf{s}, \quad (7.2)$$

$$2\mathbf{D} = \operatorname{grad} \mathbf{v} + (\operatorname{grad} \mathbf{v})^T, \quad (7.3)$$

$$2\mathbf{W} = \operatorname{grad} \mathbf{v} - (\operatorname{grad} \mathbf{v})^T. \quad (7.4)$$

Equations (1), (2), and (3) are written in the Eulerian description of motion. The operators grad and div denote the gradient and divergence operators on fields defined in the present configuration. In equations (1)–(7), $\mathbf{v}$ is the velocity of a target particle relative to the penetrator, $\boldsymbol{\sigma}$ the Cauchy stress tensor, $\mathbf{s}$ its deviatoric part, p the hydrostatic pressure not determined by the deformation history, an open circle on $\mathbf{s}$ indicates its Jaumann derivative defined by equation (7.2) for the steady stress field, $\mathbf{q}$ is the heat flux vector, and U is the specific internal energy. Furthermore, G is the shear modulus, $\mathbf{D}^p$ the plastic strain-rate, μ defined by equation (4.4) may be interpreted as the shear viscosity of the target material, g is an internal variable whose evolution rate is postulated to be given by equation (4.5), Q the activation energy, R the gas constant, T the absolute temperature, T_0 the ambient absolute temperature, m the strain-rate sensitivity, h_0 a constant rate of athermal hardening, and the quantity g^* represents a saturation value of g associated with given values of the temperature and strain-rate. In order to characterize the viscoplastic response of a material, one needs to assign values to ξ , m , h_0 , a , $\bar{g}$, n , A , Q , and R . Equation (4.2) is Hooke's law in the rate form when the dependence of the shear modulus upon the mass density or the hydrostatic pressure is neglected, equation (5) is the Fourier law of heat conduction, and equation (6) is a constitutive relation for the specific internal energy. The thermal conductivity k and the specific heat c are taken to be constants. The strain-rate tensor $\mathbf{D}$ defined by equation (7.3) is assumed to have additive decomposition into elastic and plastic $\mathbf{D}^p$ parts, and $\mathbf{W}$ defined by equation (7.4) is the spin tensor.

As has been pointed out by Batra and Gobinath [26], the problem formulation does include thermal stresses caused by the uneven temperature rise at different material particles. However, the change in the mass density due to temperature rise of a material particle is not considered.

Henceforth, we use non-dimensional variables defined below and indicated by an overbar.

$$\begin{aligned}\bar{\sigma} &= \frac{\sigma}{\sigma_0}, & \bar{s} &= \frac{s}{\sigma_0}, & \bar{p} &= \frac{p}{\sigma_0}, & \bar{v} &= \frac{v}{v_0}, & \bar{r} &= \frac{r}{r_0}, & \bar{z} &= \frac{z}{r_0}, \\ \bar{I} &= I \frac{r_0}{v_0}, & \bar{b} &= b \frac{v_0}{r_0}, & \bar{\mathbf{D}} &= \mathbf{D} \frac{r_0}{v_0}, & \bar{A} &= A \frac{r_0}{v_0}, & \bar{\theta} &= \frac{\theta}{\theta_0}, \\ \theta_0 &= \frac{\sigma_0}{\rho c}, & \bar{h}_c &= \frac{h_c}{\rho c v_0}, & \bar{g} &= \frac{g}{\sigma_0}, & \bar{\tilde{g}} &= \frac{\tilde{g}}{\sigma_0}, & \bar{h}_0 &= \frac{h_0}{\sigma_0}, & \bar{g}^* &= \frac{g^*}{\sigma_0}.\end{aligned}\quad (8)$$

Here σ_0 is the yield stress of the target material in a quasistatic simple compression test, r_0 is the radius of the cylindrical part of the penetrator, v_0 is the steady speed of penetration, and h_c is the coefficient of heat transfer between the target and the penetrator.

The governing equations, when written in terms of nondimensional variables, become

$$\operatorname{div} \mathbf{v} = 0, \quad (9.1)$$

$$-\operatorname{grad} p + \operatorname{div} \mathbf{s} = \alpha (\mathbf{v} \cdot \operatorname{grad}) \mathbf{v}, \quad (9.2)$$

$$\mathbf{s} + \beta \gamma [(\mathbf{v} \cdot \operatorname{grad}) \mathbf{s} + \mathbf{s} \mathbf{W} - \mathbf{W} \mathbf{s}] = 2\beta \mathbf{D}, \quad (9.3)$$

$$\operatorname{tr}(\sigma \mathbf{D}^p) + \delta \operatorname{div}(\operatorname{grad} \theta) = (\mathbf{v} \cdot \operatorname{grad}) \theta, \quad (9.4)$$

where

$$\alpha = \frac{\rho v_0^2}{\sigma_0}, \quad \beta = \frac{\mu v_0}{\sigma_0 r_0}, \quad \gamma = \frac{\sigma_0}{G}, \quad \delta = \frac{k}{\rho c v_0 r_0}, \quad (10)$$

are non-dimensional numbers, and we have dropped the overbars. Henceforth, we work in terms of nondimensional variables. The values of α , γ , and δ signify, respectively, the importance of inertia force relative to the flow stress of the material, material elasticity, and the heat transfer due to conduction.

We note that the governing equations are highly nonlinear, and we seek their approximate solution by the finite element method, which necessitates that we consider a finite region. The bounded region R of the target whose deformations are analyzed is depicted in Fig. 1, which also shows the discretization of the domain into finite elements. The boundary conditions imposed on this finite region are

$$\begin{aligned}v_r &= 0, & v_z &= -1.0, & p &= 0, & s_{rr} &= 0, & s_{\theta\theta} &= 0, & s_{zz} &= 0, & s_{rz} &= 0, & \theta &= \theta_0, \\ & & & & & & & & & & & & & & \text{on the boundary surface EFA,}\end{aligned}\quad (11.1)$$

$$\sigma_{rz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad \text{on the axis of symmetry DE,} \quad (11.2)$$

$$\begin{aligned}\mathbf{v} \cdot \mathbf{n} &= 0, & \mathbf{t} \cdot (\sigma \mathbf{n}) &= 0, & \mathbf{q} \cdot \mathbf{n} &= h_c (\theta - \theta_a), \\ & & & & & & & & \text{on the target/penetrator interface DCB,}\end{aligned}\quad (11.3)$$

$$\sigma_{zz} = 0, \quad v_r = 0, \quad \frac{\partial \theta}{\partial z} = 0, \quad \text{on the bounding surface BA.} \quad (11.4)$$

The validity of replacing boundary conditions at the boundaries of the semi-infinite target region by conditions (11.1) and (11.4) on the bounding surfaces EFA and BA of the finite target region studied was established by analyzing the problem over successively larger regions

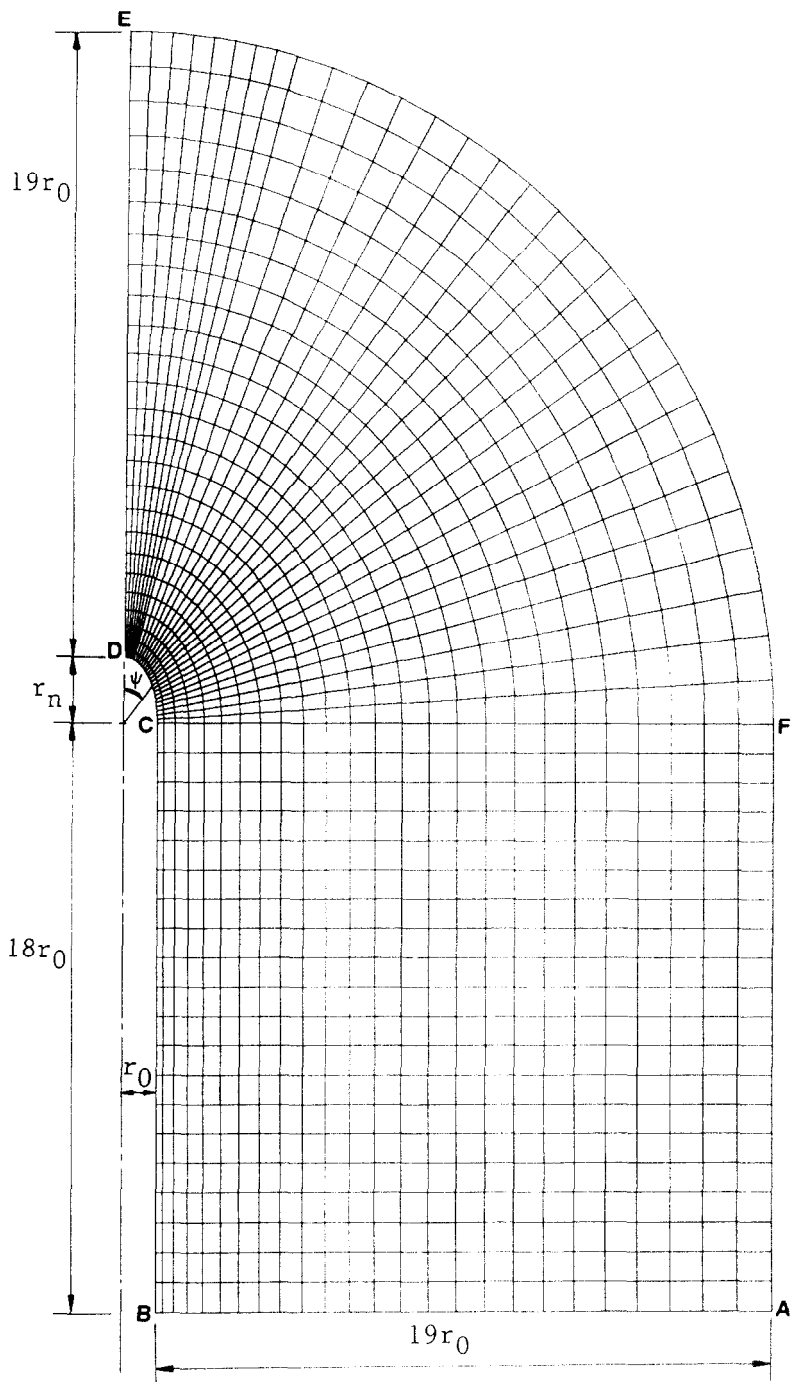


Fig. 1. Finite target region studied and its discretization.

each new region containing the previous one, until the solution variables at points on the target/penetrator interface DC changed by less than 4.7%. Of the variables p , v , θ , s , and g of interest, the variation in s was the largest and it occurred at points near the nose periphery. The boundary conditions on p and components of s on the surface EFA are needed, since here we need to solve equation (9.3) for s , along with equations (9.1), (9.2), and (9.4) for p , v and θ ; e.g. see Shimazaki and Thompson [31]. Conditions (11.2) follow from the assumed axisymmetric nature of deformations. The imposed boundary conditions (11.3) on the target/penetrator interface DCB imply that there is no interpenetration of the target material

into the penetrator, the contact surface is smooth, and the heat flux between the target and the penetrator is proportional to the temperature difference between the two; θ_a being the average temperature of the penetrator. The boundary conditions (11.4) on BA are a good approximation, provided that the surface BA is far removed from DC, as is the case here.

COMPUTATIONAL CONSIDERATIONS

As shown in Fig. 1, the region R is divided into quadrilateral elements, the elements being smaller near the target/penetrator interface DC and gradually becoming larger as we move away from this surface. There was no attempt made to optimize the mesh for a given number of nodes. However, keeping R fixed, results were computed with three successively finer meshes containing 700, 900, and 1250 elements. The peak values of p , θ , and I for these three meshes were found to be (12.07, 3.42, 1.38), (12.17, 3.69, 1.52), and (12.10, 3.67, 1.53), respectively. Henceforth, we employ the mesh with 1250 elements.

Within each element the values of v_r , v_z , s_{rr} , s_{rz} , s_{zz} , $s_{\theta\theta}$, θ , and g are approximated by bilinear polynomials expressed in terms of their values at the four corner nodes, and the hydrostatic pressure p is assumed to be constant. The discontinuous pressure field $\bar{p}$ thus computed is smoothened *a posteriori* by using

$$\sum_{j=1}^M \left(\int_R N_i N_j dV \right) p_j = \int_R N_i \bar{p} dV, \quad i = 1, 2, \dots, M \quad (12)$$

where M equals the number of nodes, and $N_1, N_2, \dots, N_M$ are the piecewise bilinear finite element basis functions.

We used the Petrov–Galerkin formulation [32] of equations (9.3) and (9.4), and the Galerkin approximation [32] of equations (9.1) and (9.2). The iterative process used to solve the resulting nonlinear algebraic equations was stopped when

$$\left(\sum \|\phi^m - \phi^{m-1}\|^2 \right)^{1/2} \leq 0.01 \left(\sum \|\phi^m\|^2 \right)^{1/2} \quad (13)$$

where the summation sign implies the sum of the value of the indicated quantity at all nodes, $\|\phi\|^2 = \text{tr}(\phi\phi^T)$ when ϕ is a second order tensor, $\|\phi\|^2 = \phi_r^2 + \phi_z^2$ when ϕ is a vector, and $\|\phi\| = |\phi|$ when ϕ is a scalar. The convergence criterion (13) is applied to $\mathbf{v}$, $\mathbf{s}$, θ , p , and g . This convergence criterion is weaker than that used by Batra [5], who checked for the convergence of the solution variables at each node. Batra used 6-noded triangular elements and approximated the solution variables, except for p , by piecewise quadratic polynomials, and p by piecewise linear polynomials defined in terms of their values at the three corner nodes.

The code developed by Jayachandran and Batra [27] was modified to solve the present problem. The boundary condition (11.3)₁ on the target/penetrator interface is satisfied by using the method of Lagrange multipliers.

RESULTS AND DISCUSSION

When computing results, we assigned the following values to various material and geometric parameters.

$$\begin{aligned} \rho &= 7860 \text{ kg/m}^3, & \sigma_0 &= 405 \text{ MPa}, & G &= 80 \text{ GPa}, & c &= 473 \text{ J/kg } ^\circ\text{C}, \\ k &= 50 \text{ W/m } ^\circ\text{C}, & h_c &= 20 \text{ W/m}^2 \text{ } ^\circ\text{C}, & \theta_a &= 0^\circ\text{C}, & r_0 &= 10 \text{ mm}, \\ A &= 6.346 \times 10^{15} \text{ s}^{-1}, & Q &= 275 \text{ kJ/mol}, & \tilde{g} &= 405 \text{ MPa}, & h_0 &= 5000 \text{ MPa}, \\ \xi &= 3.25, & m &= 0.1, & n &= 0.002, & a &= 1.5, & \alpha &= 10, & r_n &= 1.0. \end{aligned} \quad (14)$$

Here $2r_n$ equals the length of the principal axis of the elliptical penetrator nose in the z -direction. For the values of ρ , σ_0 , and c given above, the reference temperature θ_0 used to nondimensionalize the temperature rise equals 108.9°C . Since we are interested in delineating the effect of different values of material and geometric parameters in the BKA flow rule on the deformations of the target, the base values assigned as in (14) to different parameters are of less significance. The present study should enumerate the relative importance of various material parameters and hence help design experiments for the precise determination of more critical ones. The range of values of material parameters considered herein is probably more than that likely to occur for any real material. The variables that are assigned values different from those given above are so indicated in the figures along with their new values.

All of the results presented below and values of variables indicated in figures, unless stated otherwise, are non-dimensional.

(a) Effect of penetration speed

Figure 2 depicts, for $\alpha = 2, 6, 8$, and 10 , the variation of the normal stress, strain-rate measure I , tangential speed, and the temperature rise at target particles abutting the hemispherical penetrator nose surface. The angular position ψ , indicated in Fig. 1, is measured from the centroidal axis. In this and other figures, various quantities have been scaled appropriately so as to fit on the same plot. The normal stress on the penetrator nose surface is independent of α for $\psi \approx 45^\circ$, and the normal stress at a point on the penetrator nose surface increases with α for $\psi < 45^\circ$ and the reverse happens for $\psi > 45^\circ$. Such a behavior was also observed by Batra and Wright [33] for a rigid plastic target, by Jayachandran and Batra [27] for

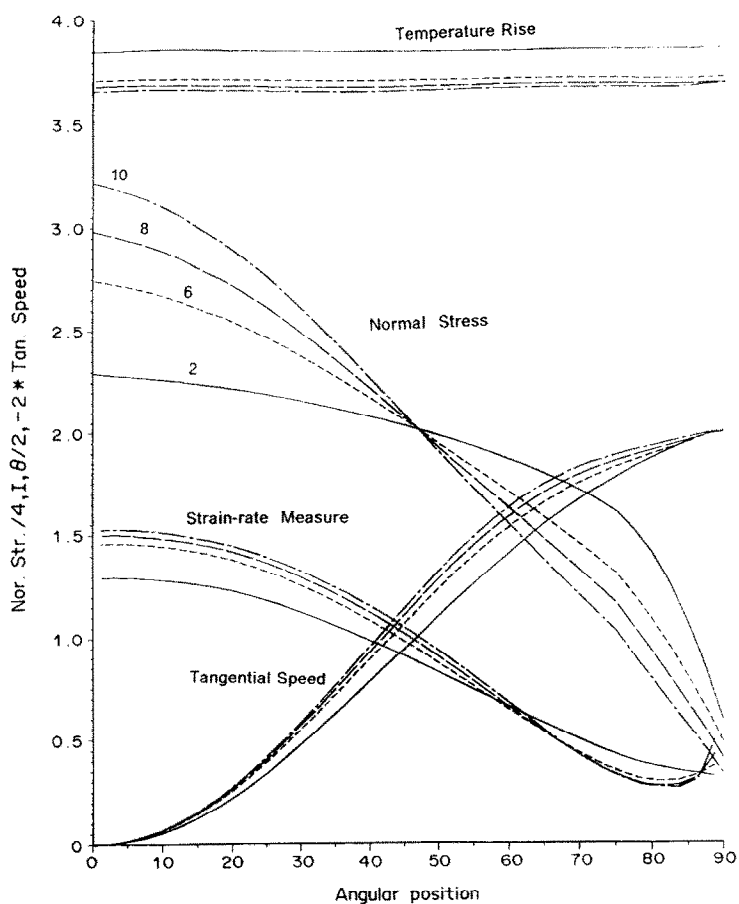


Fig. 2. Distribution of the tangential speed, strain-rate measure, normal stress, and the temperature rise at target particles on the penetrator nose surface for $\alpha = 2, 6, 8$, and 10 .

an elastic–plastic target, by Batra [5] for a viscoplastic target obeying the Litonski–Batra flow rule, and by Batra and Adam [6] for a viscoplastic target following the Bodner–Partom flow law. The values of the nondimensional strain-rate measure I at and near the penetrator nose tip decrease with an increase in the value of α . We note that the change in the dimensional values of I is more than that in the nondimensional values since the former are obtained by multiplying the latter by v_0/r_0 . However, the temperature rise is nearly uniform over the penetrator nose surface, and its value decreases with an increase in the value of α , mainly because more heat is transferred due to convection at higher speeds. We note that Batra [5], and Batra and Adam [6] observed similar behavior for the Litonski–Batra and Bodner–Partom flow rules, respectively.

In Fig. 3 we have plotted the variation of the strain-rate measure I , temperature rise θ , $(-\sigma_{zz})$, and the axial velocity for $\alpha = 2, 6, 8$, and 10 . Whereas the strain-rate measure I and the temperature rise drop sharply as one moves away from the nose tip, the value of $(-\sigma_{zz})$ decreases slowly to zero mainly because the hydrostatic pressure p drops off slowly with the axial distance from the nose surface. For $\alpha = 10$, $v_0 = 771$ m/s, and the peak values of I and the temperature rise equal, respectively, $3.55 \times 10^5 \text{ s}^{-1}$ and 408°C . It seems that there is a thin layer of material, of thickness nearly $0.2r_0$, around the penetrator nose surface in which the temperature rise is quite high. Severe deformations of the target material occur at particles situated within a distance of $3r_0$ from the penetrator nose surface. Thus, the target region studied is quite adequate.

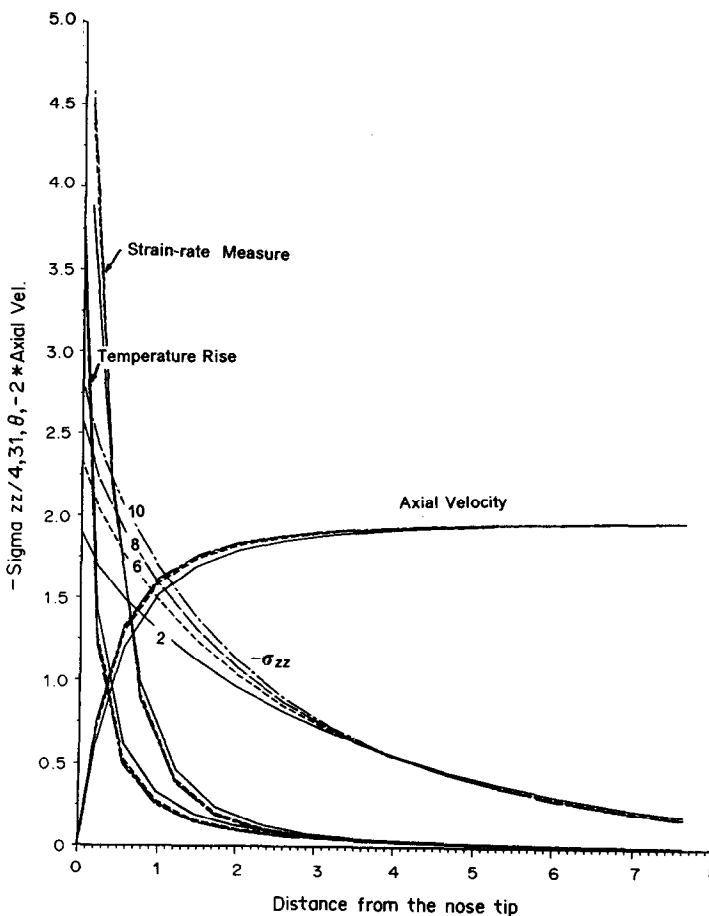


Fig. 3. Variation of the strain-rate measure, temperature rise, axial velocity, and $(-\sigma_{zz})$ at points on the axial line for $\alpha = 2, 6, 8$, and 10 .

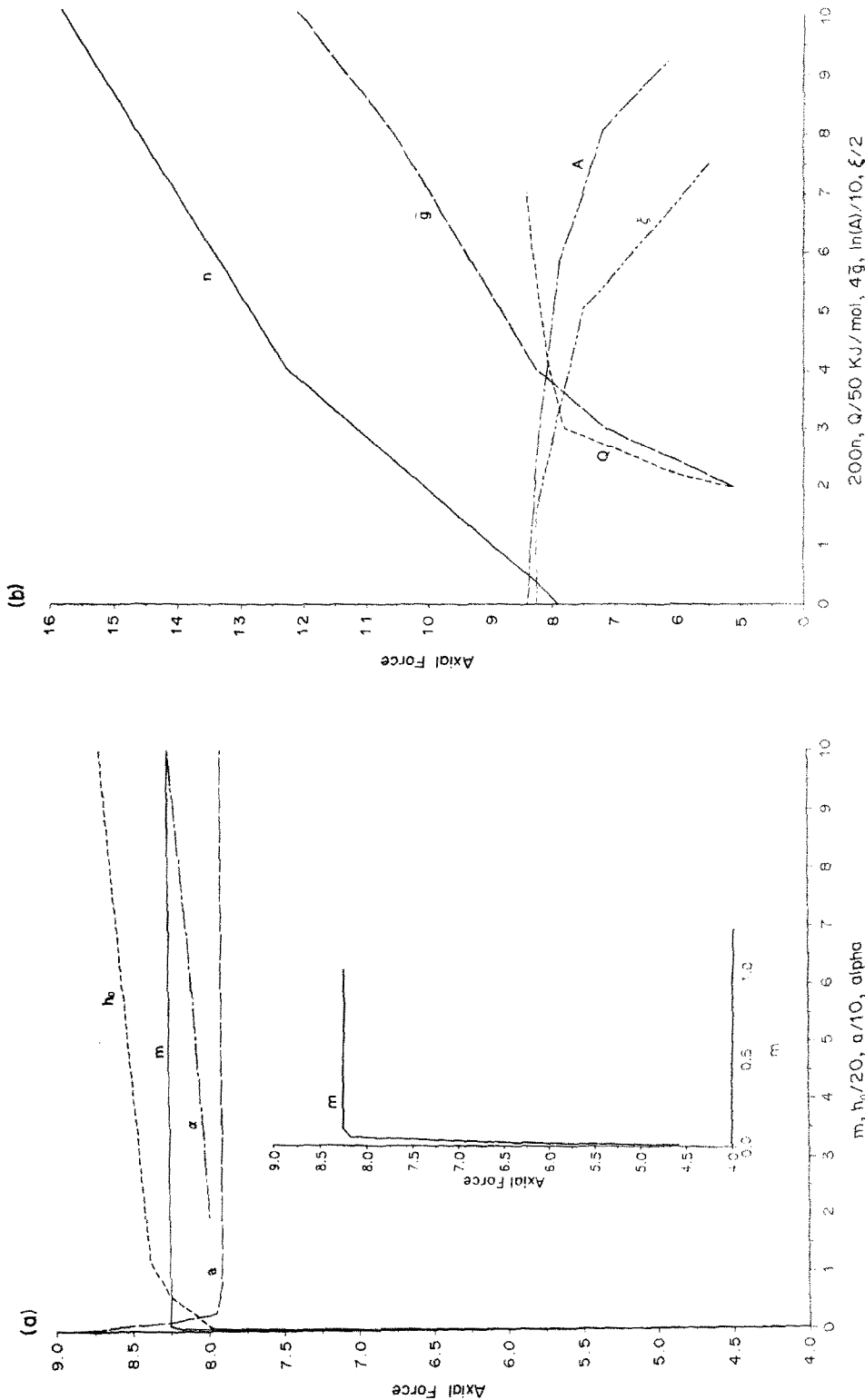


Fig. 4. Dependence of the axial resisting force upon (a) m , a , h_0 and α , and (b) n , $\bar{g}$, Q , A and ξ .

For the hemispherical nosed penetrator, the axial resisting force F is given by

$$F = \int_0^{\pi/2} (\mathbf{n} \cdot \boldsymbol{\sigma} \mathbf{n}) \sin 2\theta \, d\theta. \quad (15)$$

The corresponding axial force in physical units is given by $(\pi r_0^2 \sigma_0)F$. The dependence of the axial force upon α , exhibited in Fig. 4(a), reveals that F depends upon α weakly. A least squares fit to the computed values of F gives

$$F = 7.92 + 0.033\alpha. \quad (16)$$

Thus, the dependence of F upon α is quite weak.

(b) *Effect of penetrator nose shape*

In Fig. 5 we have plotted the distribution of the normal stress, temperature rise, tangential speed, and the strain-rate measure I on the penetrator nose surface for three different nose shapes with $r_n/r_0 = 0.2, 1.0$, and 2.0 . As expected, the normal stress stays essentially uniform on the nose surface of a blunt nosed ($r_n/r_0 = 0.2$) penetrator, and it suddenly drops to zero near the nose periphery. The curve for the normal stress vs the angular position ψ is concave downward for the hemispherical nosed penetrator, but is concave upward for the ellipsoidal nosed penetrator. The peak values of I occur at the nose tip or the stagnation point for both hemispherical nosed and ellipsoidal nosed penetrators, but at points on the nose periphery for a blunt nosed penetrator. The peak value of I for a blunt nosed penetrator is nearly twice that

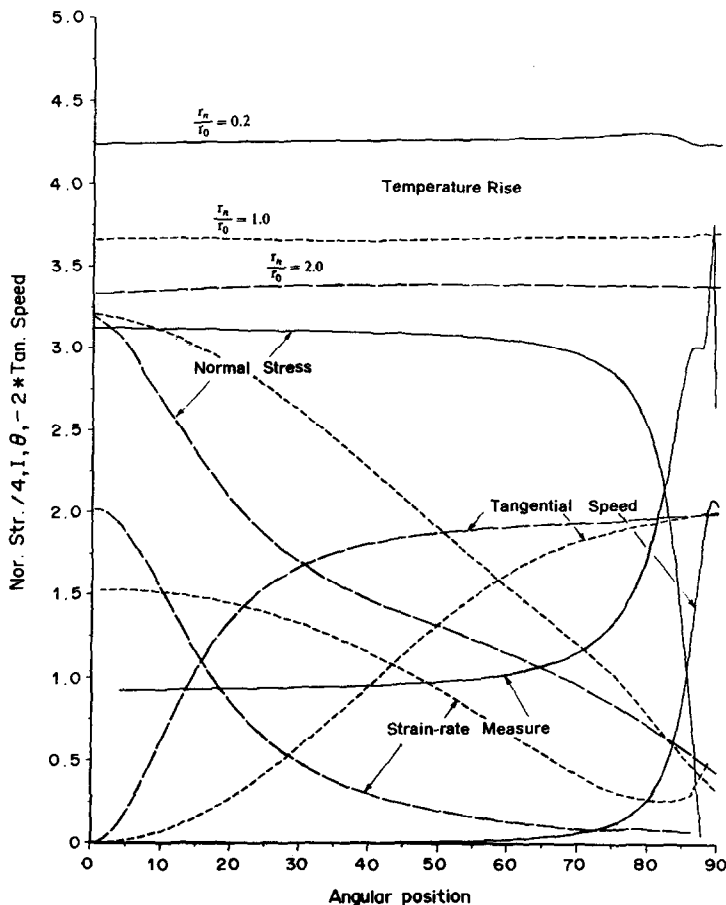


Fig. 5. Distribution of the normal stress, strain-rate measure, tangential speed, and the temperature rise on the penetrator nose surface for three different nose shapes (— blunt nose, ---- hemispherical nose, and ——— ellipsoidal nose).

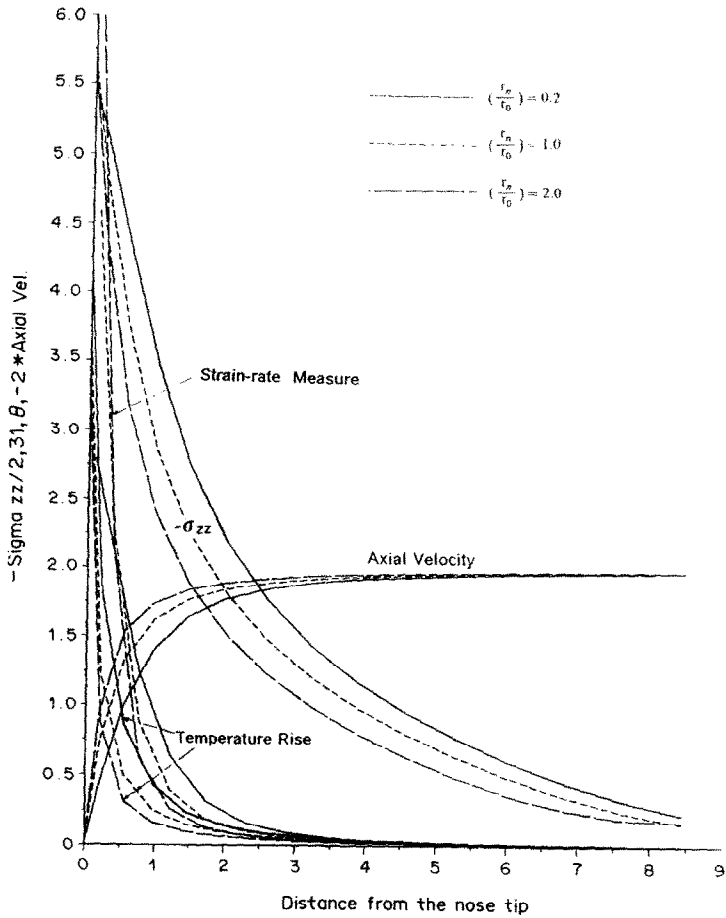


Fig. 6. Variation of the axial velocity, $(-\sigma_{zz})$, temperature rise, and the strain-rate measure at points on the axial line for three different nose shapes.

for an ellipsoidal nosed penetrator. The temperature distribution is essentially uniform over the nose surface for each one of the three nose shapes considered; the temperature rise being highest for an ellipsoidal nosed penetrator.

Figure 6 exhibits the variation of I , $(-\sigma_{zz})$, the temperature rise θ , and the z -velocity on the axial line versus the distance from the penetrator nose tip. The values of I , θ , and $(-\sigma_{zz})$ decrease slowly and the magnitude of the relative z -velocity increases less rapidly for the blunt nosed penetrator as compared to the other two nose shapes. Knowing the value of σ_{zz} at the stagnation point, one can find the value of the strength parameter R_t in the modified Bernoulli equation. The values of R_t for hemispherical, blunt and ellipsoidal noses were found to be 7.89, 7.63, and 7.83, respectively. Thus, the value of R_t is independent of the nose shape, as was assumed by Tate. For the hemispherical nosed penetrator, a least-squares fit to the computed values of R_t for $\alpha = 2, 6, 8$ and 10 gave

$$R_t = 8.25 - 0.038\alpha \tag{17}$$

implying that the dependence of R_t upon α is weak, again confirming Tate's assertion.

(c) Effect of variation in material parameters

In order to see which material parameters are likely to affect significantly the target deformations, we have plotted in Fig. 7, for various values of material parameters, the equivalent deviatoric stress s_e vs the equivalent strain ϵ_e curves for a block made of the target material and deformed in plane strain compression at an average strain-rate of 3300 s^{-1} . We realize that the strain-rate in the penetration problem varies from almost zero to 10^5 s^{-1} .

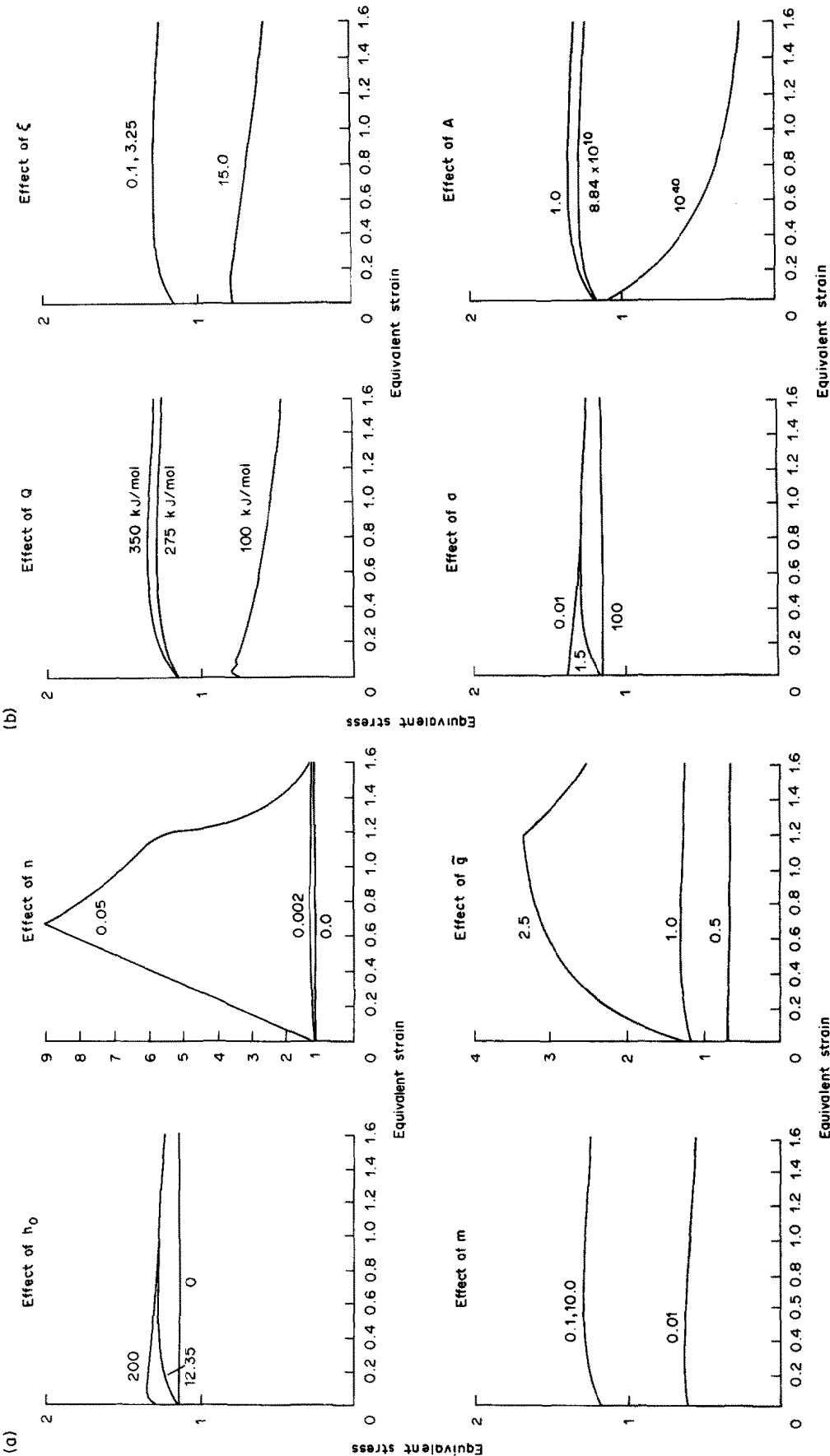


Fig. 7. Effect of the values of material parameters in the BKA flow rule upon the stress-strain curves for a block deformed in plane strain compression at an average strain-rate of 3300 s^{-1} .

However, it is hoped that the plotted s_e vs ϵ_e curves will delineate the relative importance of various parameters. The equivalent deviatoric stress and the equivalent strain are defined as

$$s_e = \sqrt{\frac{3}{2} \text{tr}(\mathbf{ss}^T)^{1/2}}, \tag{18.1}$$

$$\epsilon_e = \frac{2}{\sqrt{3}} \ln\left(\frac{l_0}{l}\right), \tag{18.2}$$

where l_0 and l are the heights of the block in the undeformed and deformed configurations, respectively. The values considered herein are the lower limit, base value listed in (14), and the upper limit value of the variable used to study the penetration problem. It is evident that an increase in the value of h_0 , n , m , $\tilde{g}$, and Q enhance the hardening of the material, and an increase in the value of ξ , a , and A results in the softening of the material. Here enhanced hardening of the material implies that the maximum value of s_e increases. The value of the equivalent strain at which the equivalent stress attains its peak value changes noticeably with the value of a material parameter. The curves in Fig. 7 reveal that all material parameters, except possibly a and h_0 , affect the s_e vs ϵ_e curves appreciably, and will very likely have a significant effect on the deformations of the target.

Even though we studied the penetration problem by varying one material parameter at a time, we present below results that elucidate the dependence of the computed quantities upon n , m , a , and A . The results obtained by varying h_0 , $\tilde{g}$, and Q are similar to those computed by changing n and m , and the variation in the value of ξ gives results similar to those obtained by varying a and A . In this parametric study, the penetrator nose was taken to be hemispherical.

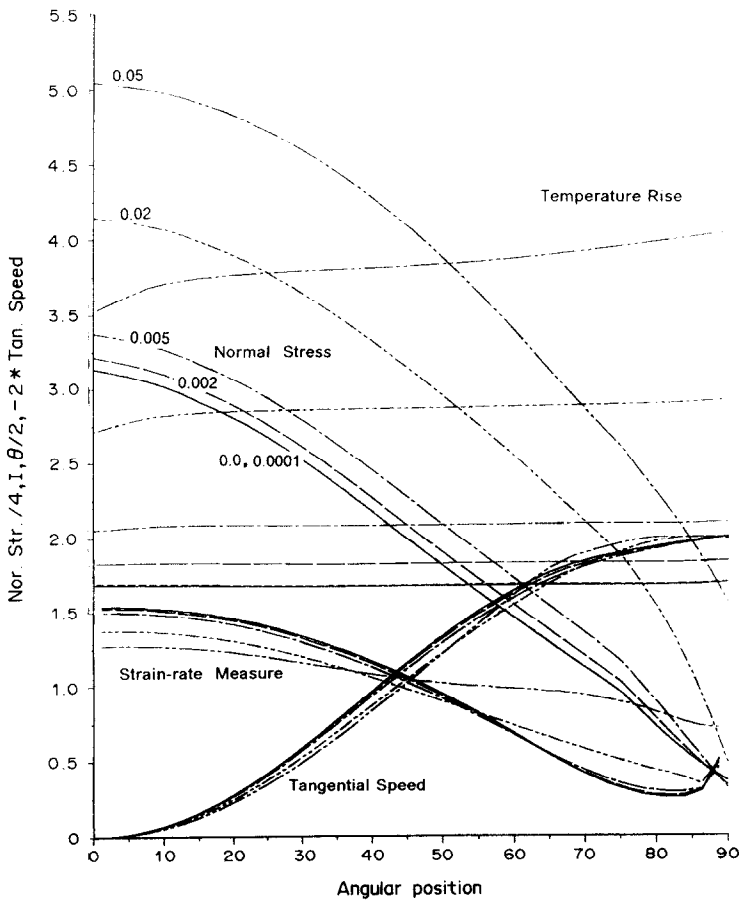


Fig. 8. Distribution of the temperature rise, normal stress, strain-rate measure, and the tangential speed on the penetrator nose surface for different values of n .

Figure 8 depicts the distribution of the normal stress, strain-rate measure I , tangential speed, and the temperature rise at target particles abutting the nose surface for $n = 0.0, 0.0001, 0.002, 0.005, 0.02$, and 0.05 . The parameter n influences the saturation value g^* of the internal variable g associated with a given temperature/strain-rate pair. The equivalent stress vs equivalent strain curve of Fig. 7(b) elucidates that the peak value of s_e increases appreciably when n is increased from 0 to 0.05. For the penetration problem, this is reflected in the enhanced values of the normal stress on the target/penetrator interface. In order to see how much of this increase is due to the higher values of the hydrostatic pressure, we note that the values of p at the stagnation point for the above referenced values of n equal, respectively, 11.80, 11.81, 12.10, 12.77, 16.23, and 20.39. Thus, the hardening of the material due to an increase in the value of n also contributes to the increase in the value of the normal stress. The value of n affects minimally the tangential speed on the penetrator nose surface, but the value of the strain-rate measure I near the stagnation point decreases, and that near the nose periphery increases with an increase in the value of n . The higher values of the temperature rise for larger values of n suggest an increase in the value of the heat generated due to plastic working. As n increases, the temperature at the nose periphery increases more than that at the stagnation point, in part due to an increase in the value of I at the nose periphery and decrease in the value of I at the stagnation point.

The dependence of the axial force F upon n , plotted in Fig. 4(b), suggests that F increases sharply with an increase in the value of n . The variation of various solution variables on the axial line for different values of n is shown in Fig. 9. The values of $(-\sigma_{zz})$ and the temperature rise at target particles located within $2r_0$ of the penetrator nose surface are noticeably affected

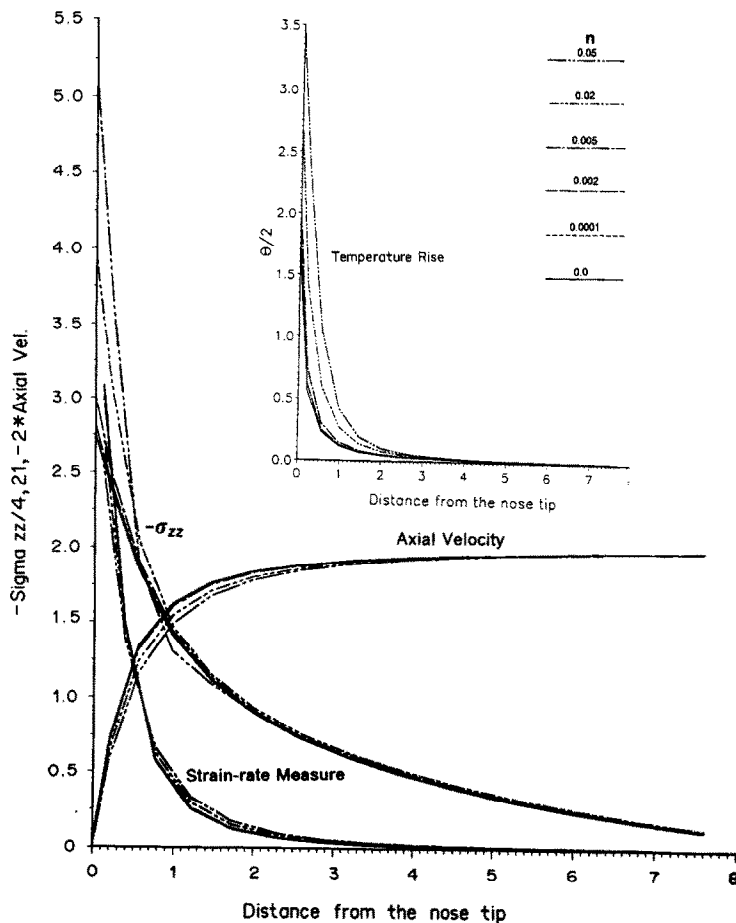


Fig. 9. Variation of the axial velocity, $(-\sigma_{zz})$, temperature rise, and the strain-rate measure at points on the axial line for different values of n .

by the value of n . The dependence of the tangential speed upon n is rather weak, as is the dependence of all quantities upon n at points situated more than $2r_0$ from the penetrator nose tip. For each value of n considered herein, the axial stress decays slowly with the distance from the nose tip.

In Fig. 10 we have plotted the variation, on the penetrator nose surface, of the tangential speed, strain-rate measure I , normal stress and the temperature rise for strain-rate hardening parameter $m = 0.01, 0.02, 0.05, 0.1, 1.0$, and 10.0 . The curves for $m \geq 0.1$ overlap each other, indicating the saturation of the strain-rate hardening effects. Whereas the values of I and the tangential speed decrease those of the normal stress and the temperature rise increase with an increase in the value of m . For $m = 0$, the target particles tended to separate away from the penetrator nose surface near the nose periphery. Hence, results for this case are not included herein. For the aforestated values of m , the hydrostatic pressure p at the stagnation point equals 8.87, 10.63, 12.04, 12.10, 12.10, and 12.10, respectively. Thus, an increase in the value of the normal stress is due to both an increase in the value of p and the enhanced hardening of the material for higher values of m in the range $0.01 \leq m \leq 0.1$. In this case the temperature rise is almost evenly distributed on the target/penetrator interface. On the axial line (cf. Fig. 11), the values of I and the tangential speed seem not to depend upon m . However, the values of the temperature rise and the axial stress ($-\sigma_{zz}$) are influenced by the value of m considered. The curves depicting the variation of the axial stress and the temperature rise with z , the distance from the nose tip, for $m = 0.05, 0.1$, and 10.0 essentially coincide with each other.

The axial resisting force F (cf. Fig. 4) experienced by the penetrator increases sharply when

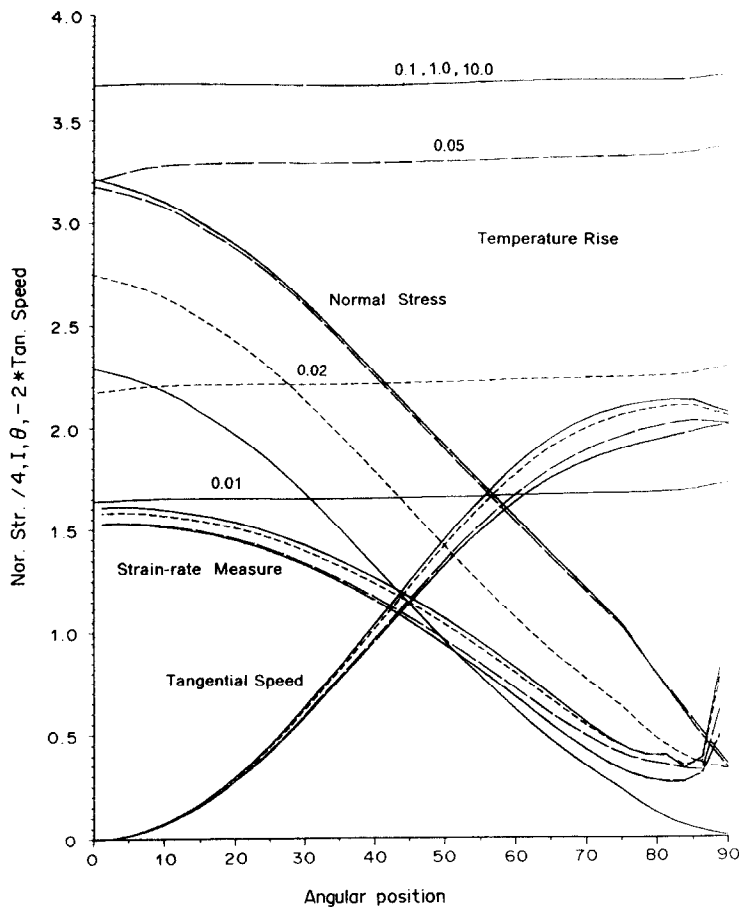


Fig. 10. Effect of different values of the strain-rate hardening parameter m on the distribution of the normal stress, tangential speed, temperature rise, and the strain-rate measure on the penetrator nose surface.

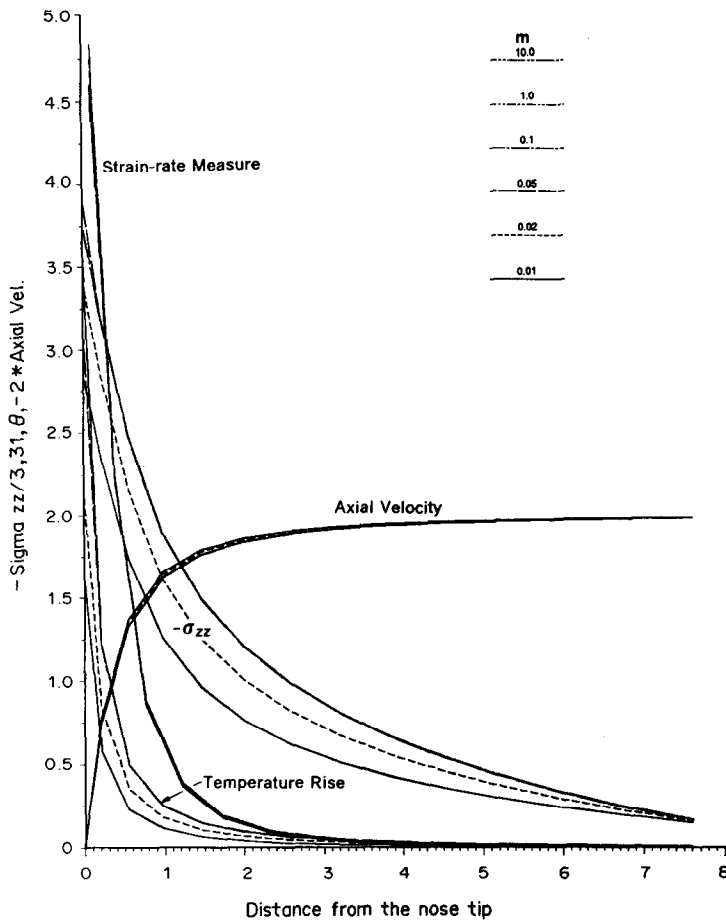


Fig. 11. Effect of m on the variation of the normal stress, axial velocity, temperature rise, and the strain-rate measure on the axial line.

the value of m is increased from 0.01 to 0.1, but levels off subsequently. Out of the other parameters, h_0 , $\bar{g}$, and Q that increase the hardening of the material, F depends strongly upon $\bar{g}$ and rather weakly upon h_0 . The values of F increase rapidly with an increase in the value of Q for $Q \leq 150$ kJ/mol, and then quite slowly.

Figure 12 depicts the distribution of the strain-rate measure I , tangential speed, normal stress, and the temperature rise on the penetrator nose surface for $a = 0.01, 0.1, 1.5, 3, 10$, and 100. The values of I and the tangential speed are affected very little by the value of a . Also, the values of $a \leq 0.1$ and $a \geq 3.0$ have negligible effect on target deformations. The normal stress and the temperature rise decrease with an increase in the value of a from 0.1 to 3.0. We note that the value of a is usually selected to represent best the experimental hardening curves. The axial resisting force F experienced by the penetrator decreases when a is increased from 0.1 to 3.0, and stays constant subsequently. As evidenced by the results plotted in Fig. 13, the values, at points on the axial line, of the temperature rise, strain-rate measure, axial speed, and the axial stress are affected very little by the value of a . The effect of changing the value of the pre-exponential factor A on the distribution of the aforementioned four variables is shown in Fig. 14. The values of the temperature rise and the normal stress decrease, and those of the tangential speed and the strain-rate measure I increase with an increase in the value of A . Equations (4.6) and (4.7) suggest that an increase in the value of A reduces the saturation value g^* of g . When g equals g^* , further resistance to flow does not develop. The results plotted in Fig. 4 indicate that the axial resisting force experienced by the penetrator decreases noticeably with an increase in the value of A .

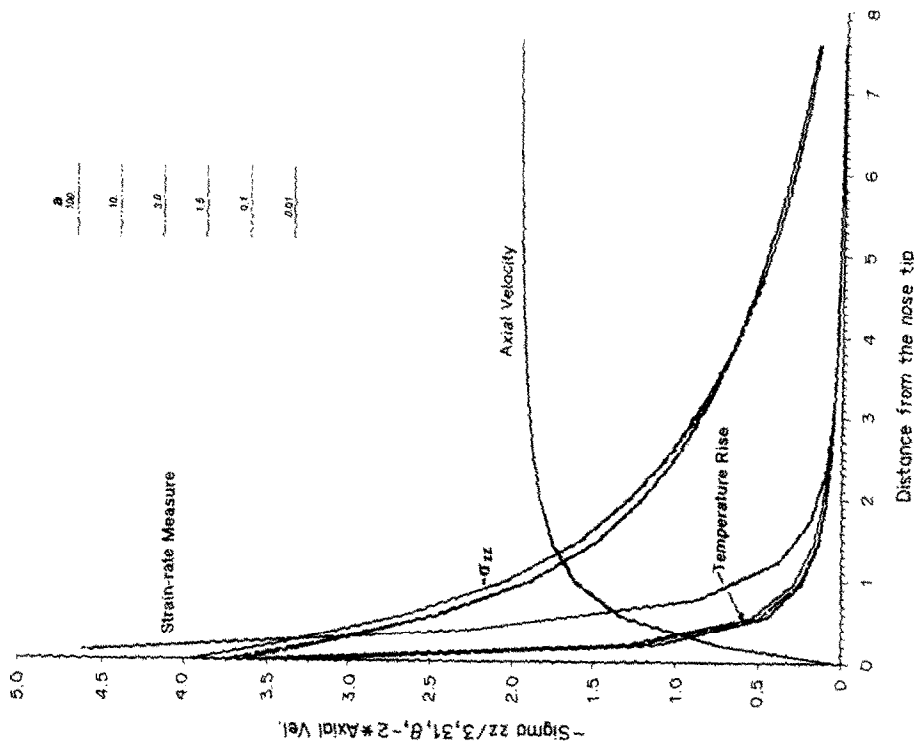


Fig. 13. Effect of α on the variation of the normal stress, axial velocity, temperature rise, and the strain-rate measure on the axial line.

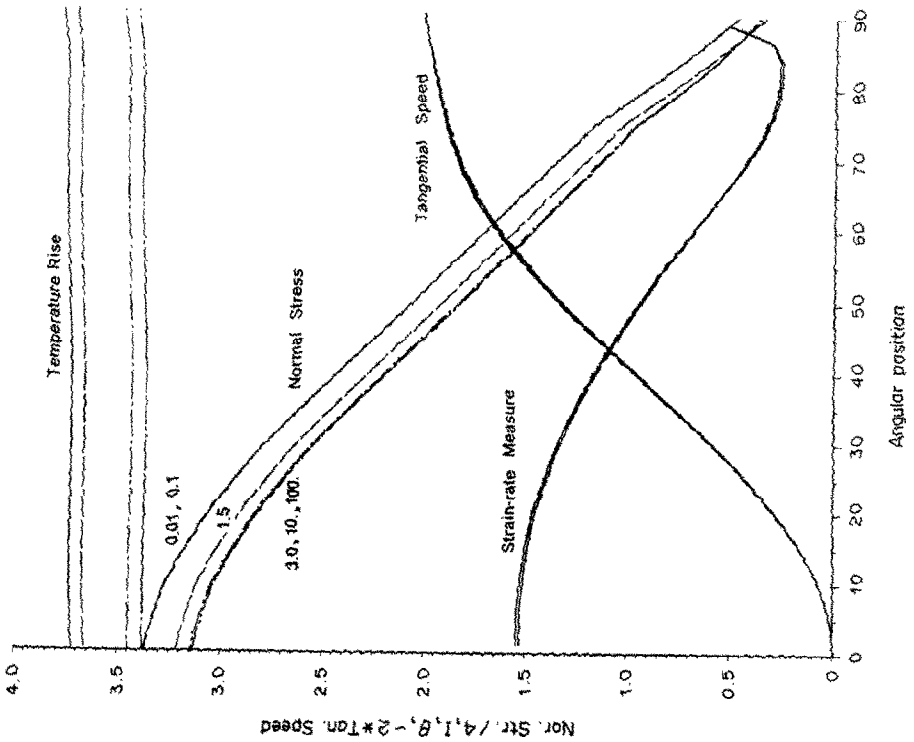


Fig. 12. Distribution of the temperature rise, normal stress, strain-rate measure, and the tangential speed on the penetrator nose surface for different values of α .

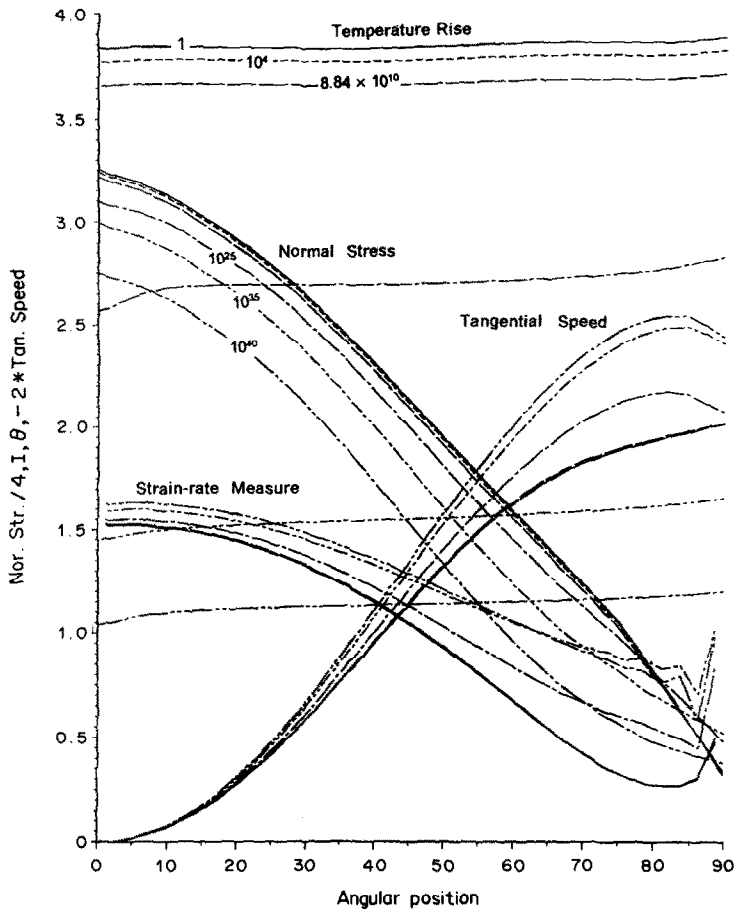


Fig. 14. Effect of different values of A on the distribution of the temperature rise, normal stress, strain-rate measure, and the tangential speed on the penetrator nose surface.

CONCLUSIONS

We have studied the steady state axisymmetric deformations of a thermoelastic-viscoplastic target being penetrated by a long rigid cylindrical rod by the finite element method, using bilinear quadrilateral elements. The target material is modeled by a viscoplastic flow rule proposed recently by BKA. Within each element, the fields of the velocity, deviatoric stress tensor, temperature, and an internal variable are approximated by bilinear polynomials, and the hydrostatic pressure is assumed to be constant. It is found that the axial resisting force experienced by the penetrator depends weakly upon the penetration speed. The material strength parameter, introduced by Tate, in the modified Bernoulli equation, is found not to depend upon the penetrator nose shape. The material parameters that enhance the hardening of the material in plane strain compression of a block made of the target material increase the axial resisting force acting on the penetrator. The severe deformations of the target are confined to material particles situated at most one penetrator diameter away from the target/penetrator interface. Thus, the target region analyzed herein is adequate, and the computed results do not depend upon the extent of the domain analyzed.

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REFERENCES

- [1] E. H. LEE, *J. Appl. Mech.* **36**, 1 (1969).
- [2] S. NEMAT-NASSER, in *Proc. Symp. Plasticity Today* (Edited by A. SAWCZUK), Elsevier, London (1983).
- [3] Y. F. DAFALIAS, *Proc. Current Theories of Plasticity and Their Applications*. Univ. of Oklahoma-Norman, Okla (1984).
- [4] S. B. BROWN, K. H. KIM and L. ANAND, *Int. J. Plasticity* **5**, 95 (1989).
- [5] R. C. BATRA, *Comp. Mech.* **3**, 1 (1988).
- [6] R. C. BATRA and A. ADAM, *Int. J. Engng Sci.* **29**, 1391 (1991).
- [7] M. E. BACKMAN and W. GOLDSMITH, *Int. J. Engng Sci.* **16**, 1 (1978).
- [8] T. W. WRIGHT and K. FRANK, *Approaches to Penetration Problems, SMIRT Symposium*, No. 14, Impact. Lausanne, Switzerland (1987).
- [9] T. W. WRIGHT, in *Lecture Notes in Engineering*, Vol. 3, Computational Aspects of Penetration Mechanics (Edited by J. CHANDRA and J. FLAHERTY). Springer, New York (1983).
- [10] C. E. ANDERSON and S. R. BODNER, *Int. J. Impact Engng* **7**, 9 (1988).
- [11] J. AWERBUCH, Technion-Israel Institute of Technology, MED Report No. 28 (1970).
- [12] J. AWERBUCH and S. R. BODNER, *Int. J. Solids and Structures* **10**, 671 (1974).
- [13] M. RAVID and S. R. BODNER, *Int. J. Engng Sci.* **21**, 577 (1983).
- [14] M. RAVID, S. R. BODNER and I. HOLCMAN, *Int. J. Engng Sci.* **25**, 473 (1987).
- [15] M. J. FORRESTAL, K. OKAJIMA and V. K. LUK, *J. Appl. Mech.* **55**, 755 (1988).
- [16] R. C. BATRA and XINGJU CHEN, *Int. J. Engng Sci.* **28**, 1347 (1990).
- [17] G. BIRCKOFF, D. P. MCDUGALL, E. M. PUGH and G. TAYLOR, *Proc. Phys. Soc. Lond.* **57**, 147 (1945).
- [18] D. C. PACK and W. M. EVANS, *Proc. Phys. Soc. Lond.* **B64**, 298 (1951).
- [19] W. A. ALLEN and J. W. ROGERS, *J. Franklin Inst.* **272**, 275 (1961).
- [20] V. P. ALEKSEEVSKII, *Comb. Expl. Shock waves* **2**, 63 (1966). (Translation from Russian, Faraday Press, New York.)
- [21] A. TATE, *J. Mech. Phys. Solids* **15**, 387 (1967).
- [22] A. TATE, *J. Mech. Phys. Solids* **17**, 141 (1969).
- [23] A. TATE, *Int. J. Mech. Sci.* **28**, 535 (1986).
- [24] A. TATE, *Int. J. Mech. Sci.* **28**, 599 (1986).
- [25] P. H. PIDSLEY, *J. Mech. Phys. Solids* **32**, 315 (1984).
- [26] R. C. BATRA and T. GOBINATH, *Int. J. Impact Engng* **11**, 1 (1991).
- [27] R. JAYACHANDRAN and R. C. BATRA, *Acta Mech.* In press (1992).
- [28] J. A. ZUKAS *et al.*, *Impact Dynamics*. Wiley-Interscience, New York (1982).
- [29] T. Z. BLAZYNSKI, in *Materials at High Strain Rates* (Edited by T. Z. BLAZYNSKI). Elsevier, London (1987).
- [30] M. MACAULEY, *Introduction to Impact Engineering*. Chapman & Hall, London (1987).
- [31] Y. SHIMAZAKI and E. G. THOMPSON, *Int. J. Num. Meth. Engng* **17**, 97 (1981).
- [32] T. J. R. HUGHES, *The Finite Element Method. Linear Static and Dynamic Problems*. Prentice-Hall, Englewood Cliffs, New York (1987).
- [33] R. C. BATRA and T. W. WRIGHT, *Int. J. Engng Sci.* **24**, 41 (1986).

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