

# Addendum to “On Extensional Oscillations and Waves in Elastic Rods”

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In a previous paper on infinitesimal waves in an infinite elastic Cosserat rod [1], we proposed values for various material parameters, hereafter referred to as KB2. These parameters were determined by matching the response of the Cosserat rod with that of a three-dimensional elastic cylinder. Subsequently, we have studied extensional vibrations in rods of finite length [2]. For a small range of intermediate frequencies, it was found that a slightly different set of parameters yielded more accurate results. This latter set referred to as KB1 is the same as KB2, except that  $\alpha_2$  of KB1 is computed with  $\beta$  being the first nonzero root, rather than the second, of  $(1 - 2\nu)J_1(\beta a) = (1 - \nu)\beta a J_0(\beta a)$ . Here,  $a$  is the radius of the rod,  $\nu$  is Poisson's ratio, and  $J_n$  is the Bessel function of order  $n$ .

The phase and group speeds of the Pochhammer-Chree solution are commonly plotted only for the first two or three modes. Figures 1 and 2 depict these speeds for the first nine modes. We compare the dispersion curves (Figs. 3–8) and group speeds (Figs. 9–11) arising from the Cosserat theory using KB1 and KB2, with those from the Pochhammer-Chree solution. In these figures, results for the Cosserat theory using material parameters GN, given by Green and Naghdi, and reported in [1], are also included. The nondimensional wave numbers  $\hat{k}$  and  $\tilde{k}$  are defined by  $\hat{k} = ka/2\pi$  and  $\tilde{k} = ka/d$ , where  $k$  is the wave number, and  $d$  is the first nonzero root of  $J_1(x) = 0$  (see Onoe et al. [3]). Both the phase speed  $c$  and group speed  $c_g$  are nondimensionalized by the classical bar wave speed  $c_0$  and are denoted by a superimposed hat. All plots have been generated for  $\nu = 0.29$ .

We discuss some of the salient features of these results. First, we note that all the qualitative features of the Pochhammer-Chree solution are captured by the Cosserat solution; the only exception is that the Cosserat solution seems to smear out the rapid variation of  $\hat{c}_g$  for the two higher modes in a small range of low  $\hat{k}$ . The low-frequency limit of Pochhammer-Chree solution for the first mode is  $\hat{c} = 1$ . The higher two modes have no cutoff frequency. In the high-frequency limit, the phase speed of the first mode approaches the Rayleigh wave speed  $c_R$ , whereas the second mode tends to the shear wave speed  $c_S$ .

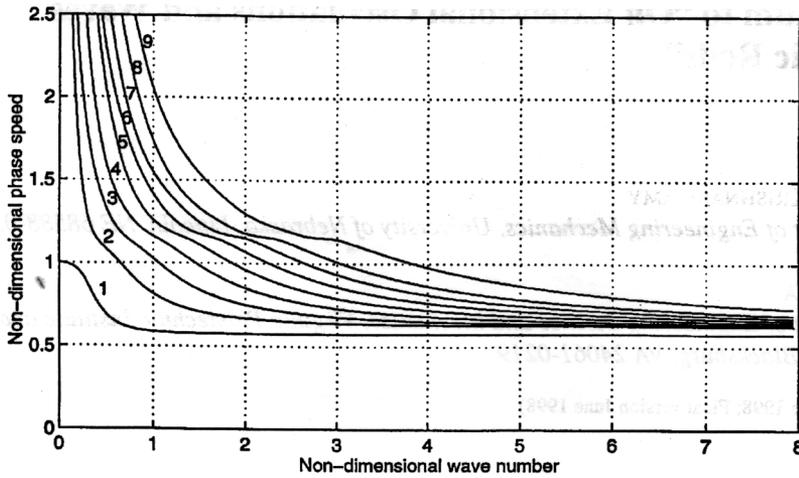


Fig. 1. Phase speed  $\hat{c}$  versus wave number  $\hat{k}$ : first nine modes of the Pochhammer-Chree solution.

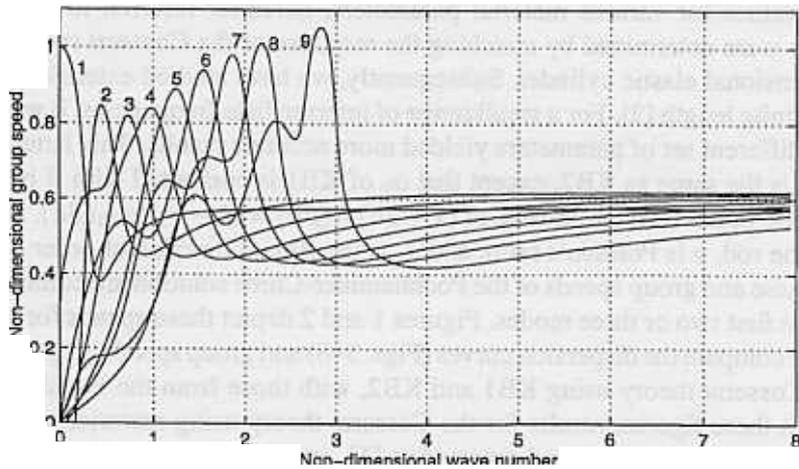


Fig. 2. Group speed  $\hat{c}_g$  versus wave number  $\hat{k}$ : first nine modes of the Pochhammer-Chree solution.

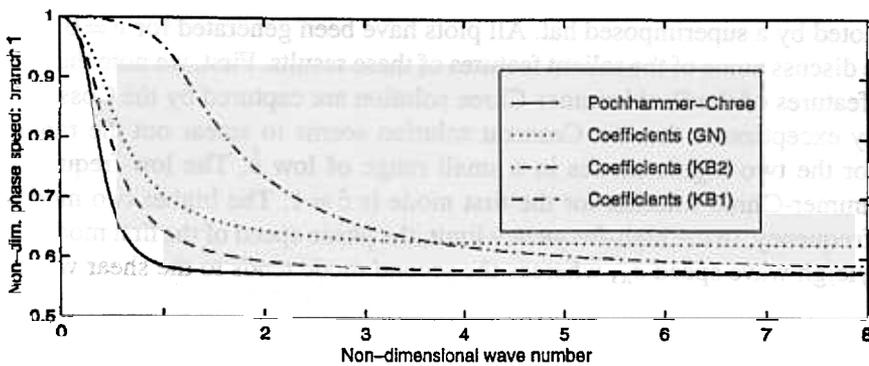


Fig. 3. Phase speed  $\hat{c}$  versus wave number  $\hat{k}$ : first mode.

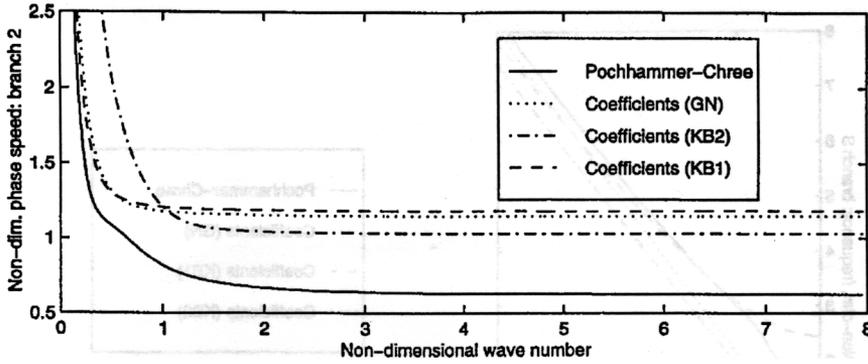


Fig. 4. Phase speed  $\hat{c}$  versus wave number  $\hat{k}$ : second mode.

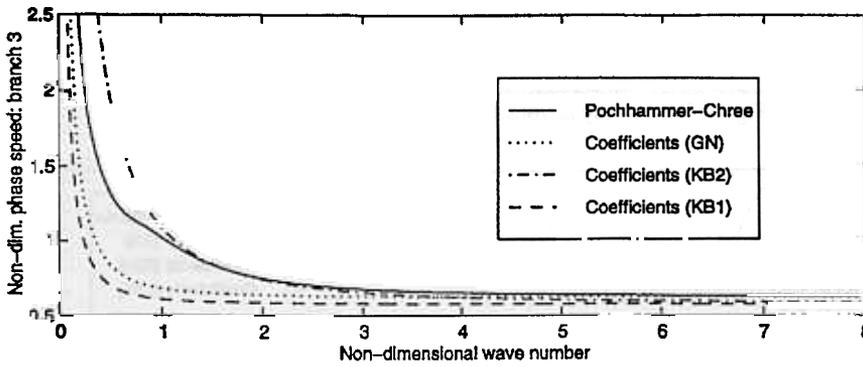


Fig. 5. Phase speed  $\hat{c}$  versus wave number  $\hat{k}$ : third mode.

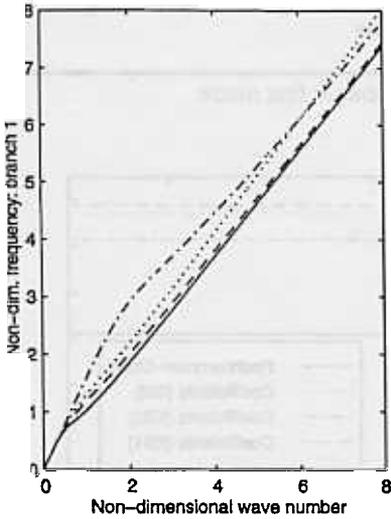


Fig. 6. Frequency  $\hat{\omega}$  versus wave number  $\hat{k}$ : first mode.

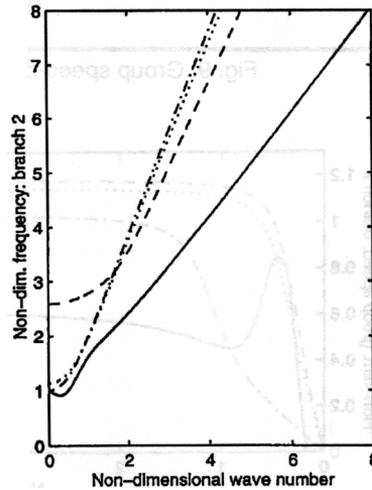


Fig. 7. Frequency  $\hat{\omega}$  versus wave number  $\hat{k}$ : second mode.

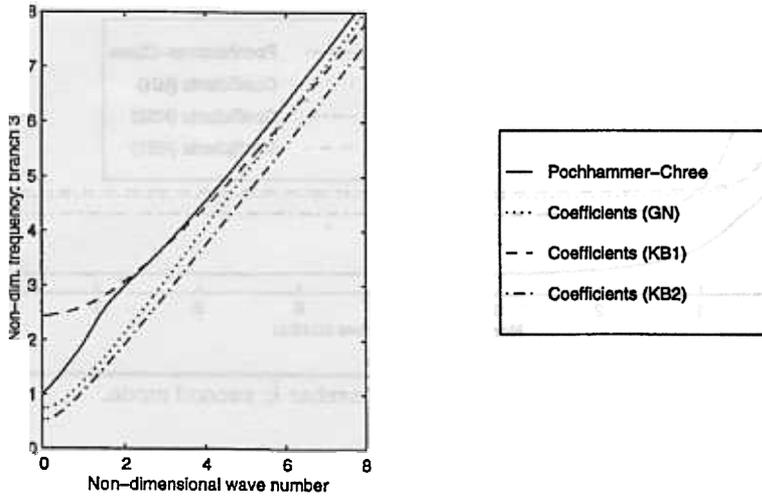


Fig. 8. Frequency  $\hat{\omega}$  versus wave number  $\hat{k}$ : third mode.

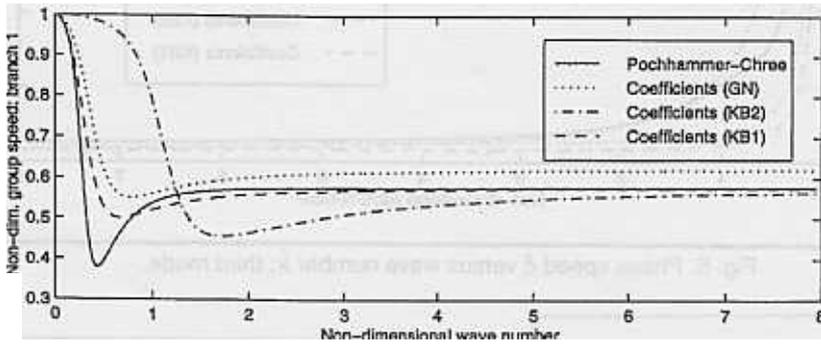


Fig. 9. Group speed  $\hat{c}_g$  versus wave number  $\hat{k}$ : first mode.

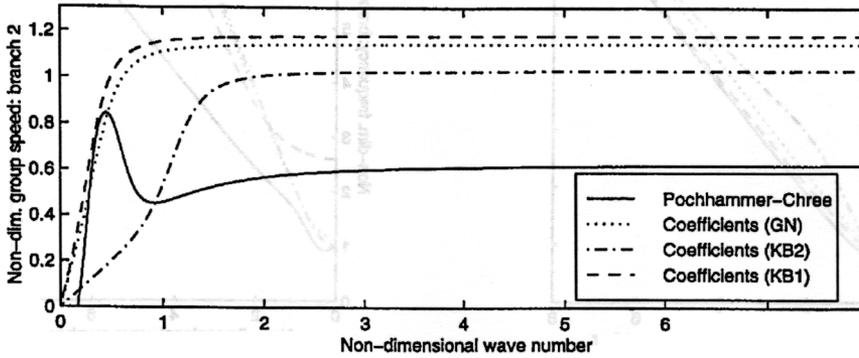


Fig. 10. Group speed  $\hat{c}_g$  versus wave number  $\hat{k}$ : second mode.

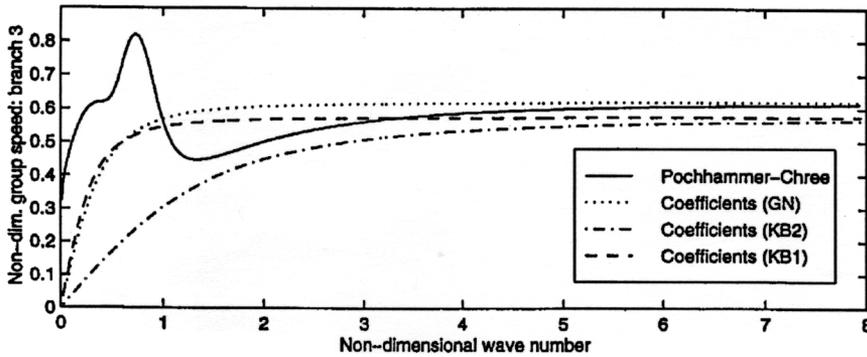


Fig. 11. Group speed  $\hat{c}_g$  versus wave number  $\hat{k}$ : third mode.

The Cosserat solution with coefficients KB1 has these correct limits, except that the high frequency limit of the second mode is closer to the dilatational wave speed  $c_D$  rather than to  $c_S$ . The GN coefficients predict a limit of  $c_D$ . Coefficients KB2 pull this limit closer to  $c_S$ ; this was the main reason KB2 was used in [1]. However, an examination of the first mode curves in Figures 3, 6, and 9 suggest that using KB2 compromises the first mode intermediate frequency behavior significantly. Moreover, the natural frequencies of all three modes of finite-length rods of small length to radius ratio are approximated well when KB1 is used, and approximated poorly when KB2 is used. Based on the overall behavior of the three modes in an infinite rod and on the results of [2], the Cosserat theory using coefficients KB1 yields good results for the study of both wave propagation in infinite circular rods and for vibrations of finite-length rods.

On the basis of the frequency spectra in Figures 3 through 11, we conclude that it is extremely fruitful to analyze wave propagation in elastic rods using the theory of Cosserat curves with coefficients KB1.

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## REFERENCES

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