Correction to the paper

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Values of constants c_2 and c_3 in Table 1 should be interchanged (i.e., $c_2 = 0.126$ and $c_3 =$ 0.325. Authors apologize for any inconvenience caused to users of the Table.

Capacitance estimate for electrostatically actuated narrow microbeams

R.C. Batra, M. Porfiri and D. Spinello

Abstract: A novel estimate for the line-to-ground capacitance that accurately predicts the pull-in instability parameters for narrow electrostatically actuated microbeams is proposed. Parameters in the proposed formula are obtained by least square fitting data from a fully converged numerical solution with the method of moments. For a narrow microbeam, it is shown that the new formula significantly improves upon classical formulas that neglect fringing field effects due to the finite thickness of the microbeam cross-section.

1 Introduction

Electrostatically actuated microbeams are extensively used as microelectromechanical systems (MEMS) [1]. An electrostatically actuated microbeam is an elastic beam suspended above a ground plate, both made of conductive materials, and a dielectric medium filling the gap, g, between them.

The distributed electrostatic force, F_e, acting on the microbeam is given by

$$F_e = -\frac{1}{2}V^2\frac{\partial C}{\partial g} \tag{1}$$

where V is the applied voltage and C the capacitance of the two-conductor system composed of the beam cross-section and the ground plate. For a rectangular cross-section, the capacitance C is a function of the beam height h, the beam width b, the gap g and the dielectric constant ε of the medium between the beam and the plate. The gap g varies with the point x of the microbeam span. Fig. 1 depicts a typical cantilever MEMS.

The applied voltage V has an upper limit beyond which the electrostatic force overwhelms the elastic restoring force in the deformable beam, the beam spontaneously deflects towards the ground plate, and the device collapses. This upper limit is called the pull-in voltage and its accurate characterisation represents a focal point of research in the MEMS community [1]. A comprehensive review of models and applications of MEMS is given in Marques *et al.* [2].

It is pointed out in Pamidighantam *et al.* [3] that for relatively narrow microbeams undergoing large deflections, the effects of fringing fields on the electrostatic force are not negligible because of the nonzero thickness and finite width of beams. The problem of accurately estimating the capacitance of a rectangular conductor facing a ground plane has also been a main research focus in the VLSI community for the last two decades [4]. For very narrow microbeams, classical results from VLSI are not applicable. Here, we deduce an accurate estimate of the capacitance and show its effectiveness in accurately predicting the pull-in behaviour of a narrow microbeam.

2 Proposed capacitance approximation

We use the method of moments (MoM) [5] to determine the capacitance C for different values of parameters b, h and g in the range $0.2 \le h/b \le 2$ and $0.4 \le h/g \le 5$. The data from fully converged numerical solutions are then least-squares fitted by the following function

$$C = \varepsilon\left(\frac{b}{g} + c_0 + c_1\left(\frac{b}{g}\right)^{c_2} + c_3\left(\frac{h}{g}\right)^{c_4} + c_5\left(\frac{h}{b}\right)^{c_6} + c_7\left(\frac{h^2}{bg}\right)^{c_8}\right) \tag{2}$$

where c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7 and c_8 are constants. The first term on the right hand side of (2) describes the parallel-plate approximation and the remaining terms, and the effects of fringing fields due to finite width and nonzero thickness. Numerical values of these constants are given in Table 1.

Within the above given range of variations for h/g and h/b, the maximum deviation in the capacitance computed from (2) and the fully converged MoM numerical solution is $<0.2\%$, e.g. confer Table 2.

Fig. 2 is a schematic sketch of different fringing field approximations, namely, Mejis–Fokkema formula [6], Palmer's formula [7] and the classical parallel plate approximation, considered in the MEMS literature to estimate the electrostatic force acting on a microbeam [8–10]. Palmer's formula

$$C = \varepsilon\frac{b}{g}\left(1 + \frac{2g}{\pi b}\left(1 + \log\left(\frac{\pi b}{g}\right)\right)\right) \tag{3}$$

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R.C. Batra and D. Spinello are with the Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

M. Porfiri is with the Department of Mechanical, Aerospace and Manufacturing Engineering, Polytechnic University, Six MetroTech Center, Brooklyn, NY 11201, USA

E-mail: dspinell@vt.edu

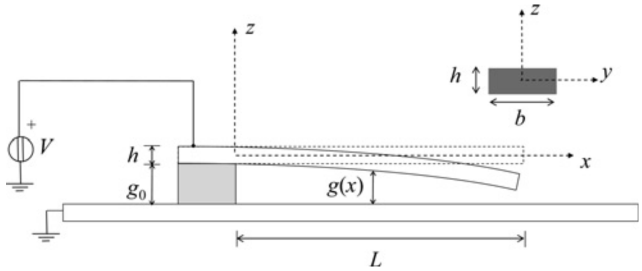

Fig. 1 *Geometry of a cantilever microbeam*

neglects fringing fields from the lateral surfaces of the rectangular conductor. Mejis–Fokkema formula

$$C = \varepsilon\left(\frac{b}{g} + 0.77 + 1.06\left(\frac{b}{g}\right)^{0.25} + 1.06\left(\frac{h}{g}\right)^{0.5}\right) \quad (4)$$

considers all fringing fields and has a maximum deviation of 2% for $b/g \geq 1, 0.1 \leq h/g \leq 4$ and of 6% for $b/g \geq 0.3$ and $h/g < 10$ [6]. In Table 2, we also include results from these classical formulas. We note that the Mejis–Fokkema formula yields errors >1% when the two conductors are relatively close ($h/g = 5$) to each other and much larger errors for very narrow conductors ($h/b = 2$). Results included in Table 2 imply that Palmer's formula and the parallel plate approximation are not suitable for narrow conductors.

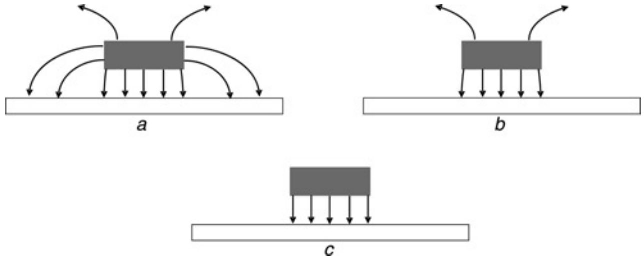

Fig. 2 *Sketch of fringing fields considered in different models*

a Present work and Mejis–Fokkema
b Palmer
c Parallel plate

3 Application to pull-in extraction

The governing equations for the gap $g(x)$ [9] are

$$-EI\frac{d^4g(x)}{dx^4} = F_e(g(x))$$

$$g(0) = g_0, \ \frac{dg(x)}{dx}\bigg|_{x=0} = 0\,\frac{d^2g(x)}{dx^2}\bigg|_{x=L} = 0, \ \frac{d^3g(x)}{dx^3}\bigg|_{x=L} = 0 \quad (5)$$

where the electrostatic distributed force F_e is given by (1), E is Young's modulus of the material of the microbeam, I is

Table 1: Numerical values of constants in (2)

c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
−5.40	4.60	0.325	0.126	−0.554	−0.00388	0.891	3.47	0.118

Table 2: Comparison between the capacitance per unit length, *C*, computed using approximation (2) or other available formulas and that from the MoM

Geometry		MoM	% Deviation			
h/b	h/g	C/ε	Eq. (2)	Mejis–Fokkema	Palmer	Parallel plate
0.2	0.5	5.37	−0.12	0.41	17	53
0.2	1	8.42	0.0091	0.11	12	41
0.2	2.5	16.8	0.012	−0.76	8.0	26
0.2	5	30.1	−0.0098	−1.4	5.6	17
0.4	0.5	3.92	−0.024	0.65	30	68
0.4	1	5.69	−0.010	0.46	22	56
0.4	2.5	10.3	0.015	−0.72	15	39
0.4	5	17.3	−0.013	−1.9	11	28
1	0.5	2.96	0.1	1.7	52	83
1	1	3.96	0.0092	1.8	40	75
1	2.5	6.29	−0.0044	0.16	29	60
1	5	9.53	−0.0020	−2.1	22	48
1.4	0.5	2.76	0.078	2.2	61	87
1.4	1	3.61	0.0019	2.6	48	80
1.4	2.5	5.50	−0.0088	0.83	36	68
1.4	5	8.02	−0.0089	−1.9	28	55
2	0.5	2.60	0.041	2.9	72	90
2	1	3.34	−0.028	3.5	57	85
2	2.5	4.90	0.0014	1.8	44	75
2	5	6.88	−0.0018	−1.4	35	64

Deviations are measured as $(C^{MoM} - C)/C^{MoM}$, where C^{MoM} is the fully converged MoM solution

Table 3: Comparison of the pull-in voltages of the cantilever microbeam obtained from different models

	Pull-in voltage, V
COVENTORWARE	1.20
Parallel plate	2.17
Palmer	1.25
Mejis–Fokkema	1.25
Present work	1.21

the moment of inertia of the cross-section ($I = 1/12\, bh^3$) and L is the beam length. The approximation of the line-to-ground capacitance C determines the electrostatic force in (5) and influences the numerical value of the pull-in voltage. The determination of the pull-in parameters is performed using the displacement pull-in extraction method presented in Degani-Bochobza *et al.* [11].

We consider the problem analysed in Pamidighantam *et al.* [3]. The sample geometry is: $L = 300\ \mu\text{m}$, $b = 0.5\ \mu\text{m}$, $h = 1\ \mu\text{m}$ and $g_0 = 2.5\ \mu\text{m}$. The material parameters are: $E = 77$ GPa and the Poisson ratio, $\nu = 0.33$. The commercial MEMS software COVENTORWARE is used in Pamidighantam *et al.* [3] for determining the pull-in voltage.

In Table 3, we compare the results obtained by solving (5) with formulas (2)–(4) and the parallel plate approximation, with those of Pamidighantam *et al.* [3]. The rather large difference between results from the parallel plate approximation and the COVENTORWARE suggests that fringing field effects are not negligible in finding the pull-in voltage. The consideration of fringing field corrections (3) and (4) significantly improves the pull-in voltage prediction, but the 4% error in the pull-in voltage is not acceptable in many applications. The present estimate of capacitance reduces the error in the pull-in voltage to $<1\%$.

4 Conclusions

We propose a new estimate of the line-to-ground capacitance of a rectangular conductor facing a ground plane, which takes into account fringing fields emanating from all surfaces of the conductor. Values obtained from the proposed estimate match those from the MoM to within 0.2% in the range $0.2 \le h/b \le 2$ and $0.4 \le h/g \le 5$. The proposed formula is suitable for studying narrow microbeams, where the effect of fringing fields is not negligible. It is shown that the electrostatic distributed force acting on the microbeam computed from the present estimate of the capacitance gives values of the pull-in voltage that have considerably less error than those obtained with the electrostatic force derived from classical formulas.

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