THE INITIATION AND GROWTH OF, AND THE INTERACTION AMONG, ADIABATIC SHEAR BANDS IN SIMPLE AND DIPOLAR MATERIALS

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Abstract — The problem of simple shearing of a block of simple (nonpolar) and dipolar thermoviscoplastic materials is studied with the objectives of exploring the initiation and growth of, and the interaction among, adiabatic shear bands. A shear band is assumed to have formed if the addition of a perturbation to the homogeneous fields just before the peak stress is reached results in a localization of the shear strain. The effect of adding perturbations of different sizes and of the same size but at different locations in the slab is investigated. It is shown that in simple materials, two shear bands coalesce if the distance between them is small but grow independently, although at a slower rate, if the distance between them is large. However, for dipolar materials, the two bands coalesce even when the distance between them at the time of their initiation is 20 times the material characteristic length.

I. INTRODUCTION

Adiabatic shear is the name given to a localization phenomenon that occurs during high-rate plastic deformation, such as machining, explosive forming, shock impact loading, ballistic penetration, fragmentation, ore crushing, impact tooling failure, and metal shaping and forming processes. The localization of shear strain has been observed mostly in steels, but also in nonferrous metals and polymers. Practical interest in the phenomenon derives from the fact that progressive shearing on an intense shear band provides an undesirable mode of material resistance to imposed deformation, and the bands are often precursors to shear fractures.

Shear band formation is generally enhanced at high strain rates because the lack of time for heat diffusion allows nonuniform straining to cause nonuniform heating. Nonuniform temperatures enhance plastic flow in the hotter regions and reduce plastic flow in the colder regions. Furthermore, heat generation is greatest in the regions of highest strain rate. Thus, the strain rates in the hot, high-strain-rate regions tend to become larger, while those in the cold, low-strain-rate regions tend to become more nonuniform and may localize into a narrow region referred to as a shear band. Whether or not this thermoplastic instability mechanism leads to shear bands seems to depend upon strain hardening, strain-rate hardening, thermal softening, thermal diffusivity, and the strength of the initial inhomogeneity.

ZENER and HOLLOMAN [1944] recognized the destabilizing effect of thermal softening in reducing the slope of the stress-strain curve in nearly adiabatic deformations. The dynamic torsional experiments of CULVER [1973] on mild steel, titanium, and 6061-T6 aluminum indicate that the localization began near the peak in the stress-strain curve for each material tested. This observation seems to be borne out by the experimental work of COSTIN *et al.* [1980]. They found that in dynamic torsion tests on short specimens of 1018 CRS and 1020 HRS, the shear bands appeared in the 1018 CRS specimens

and not in 1020 HRS specimens. Extensive experiments, conducted at different temperatures and strain rates, revealed that the shear stress–shear strain curve for HRS was increasing monotonically, whereas the curve for CRS had a peak in it.

JOHNSON et al. [1983] and LINDHOLM and JOHNSON [1983] reported dynamic torsion test data on six ductile metals and six additional materials of much less ductility. They proposed a constitutive relation that accounts for strain and strain-rate hardening and thermal softening. Their analysis indicates that high ductility tends to produce a relatively wide shear band when compared to other materials. In their experimental setup, the frictional force at the grips provided a constraining axial force. Thus, an axial load or stress component developed with increasing torsional deformation. However, this axial load component was not measured.

STAKER [1981] made use of an instability analysis to model the appearance of adiabatic shear bands in the controlled explosive expansion of steel cylinders. He argued that because of the short times involved in explosive loading the deformation could be considered adiabatic and there was no need to consider the thermophysical properties of materials. RECHT [1964] included heat conduction and thermal softening in the interpretation of shear bands formed during high-speed machining. In his investigation of instability in the shear zone ahead of a cutting tool during orthogonal machining, he used a thermal model incorporating uniform, constant-rate heat generation at a plane in an infinite medium. He showed that the critical strain rate for catastrophic shear in mild steel is 1400 times greater than that for titanium. Based on these data it can be shown that the difference in the thermophysical properties of St and Ti contributes a factor of 6, the ratio of yield stresses a factor of 4, and the difference in mechanical behavior a factor of 58. This last factor is directly proportional to the material's capacity to strain-harden and inversely proportional to its tendency to thermally soften.

In a departure from the notion of a criterion based on a stress maximum, CLIFTON [1980] and BAI [1981] examined the growth of infinitesimal periodic nonuniformities in an otherwise uniform simple shearing deformation. They included strain hardening, thermal softening, strain rate sensitivity, and heat conduction. Bai also included the effect of inertia forces. Both linearized the field equations about the unperturbed time-dependent homogeneous deformation state and found that the magnitude of the imperfections may grow or decay in time, depending on the material parameters, the average rate of strain, and the fixed spatial wavelength of the initial imperfection. BURNS [1983] used a dual asymptotic expansion to include the time dependence of the homogeneous solution in the analysis of the growth of an initially small perturbation. His work suggests that initiation of an unstable shear band, followed by exponential growth, occurs after a critical shear strain corresponding to the peak stress in the homogeneous deformation for the same overall strain rate is reached. On the other hand, SHAWKI et al. [1983], by using both numerical and perturbation techniques, concluded that exponential growth is not a sufficient condition for judging whether or not a shear band forms, as the corresponding homogeneous deformation may also grow extremely rapidly once the peak stress has been reached and growth is not restricted to a narrow band.

ERLICH et al. [1980] noted that according to simple wave theory applied to one-dimensional plastic wave propagation, the strain level at which the shear tangent modulus becomes zero propagates at zero speed. They postulated a criterion of adiabatic shear band formation based on this "wave trapping" idea. This idea was applied by OLSON et al. [1981] in a numerical finite element simulation of plastic shear wave propagation under adiabatic conditions. The numerical solution did indeed exhibit a concentration

of shear strain in a layer of finite elements that was similar in nature to observed adiabatic shear bands.

In studying the growth of shear bands in the center of a finite slab after initiation at a small imperfection, MERZER [1983] concluded that the final width of the band depends on the thermal diffusivity and the overall strain rate. WU and FREUND [1984], in studying the formation of shear bands at a moving boundary, concluded that thermal diffusivity has little influence on the final shape of the band. The detailed geometry and constitutive equations considered in these two papers are quite different, so perhaps it is not surprising to find apparently contradictory results. In fact, in both papers there are two natural length scales: one arising from the rate effect in the constitutive equation and one arising from heat conductivity. In the latter paper, these scales have been arbitrarily set equal to each other, whereas in the former paper the relative effect of heat conductivity has been examined parametrically for at least one specific type of constitutive equation.

Recently, WRIGHT and BATRA [1985a, 1985b, and 1986] described the results of computations that simulate the formation of a single shear band from a local temperature inhomogeneity in simple and dipolar materials. A general theory of thermoviscoplasticity, obtained by modifying the dipolar theory of GREEN et al. (1968) to include rate effects, was used. Wright and Batra's calculations for simple materials, as well as the experimental observations of Moss [1981], indicated that peak strain gradients reached 0.2 per μm or higher. Therefore, they considered worthwhile to investigate the dipolar effects. Their computations show increasing inhomogeneity in the strain-, temperature-, and strain-rate fields, with the central amplitudes growing at an accelerating rate. The inclusion of dipolar effects has a stiffening effect in the sense that the rate of growth of central amplitudes of the strain, temperature, and strain-rate fields is lower as compared to that for simple materials. For dipolar materials there are at least three length scales: one is from viscous stress effects, the second is from thermal conductivity, and the third is the material characteristic length. WRIGHT and BATRA [1986] studied the case when all three length scales are equal to one another.

This paper describes the results of some numerical experiments conducted with the objective of analyzing the interaction between two shear bands. It also examines the effect of the amplitude and distribution of the initial inhomogeneity on the initiation and growth of a shear band. It is shown that a narrower inhomogeneity results in a rapid growth of the band as compared to a wider one having the same central amplitude. A stronger inhomogeneity results in the formation of a shear band even before the peak in the homogeneous stress-strain curve is reached. The two shear bands that would grow independently in simple materials seem to coalesce in dipolar materials even when the material characteristic length is $\frac{1}{20}$ of the distance between them at the time of their initiation.

We note that there is no experimental evidence available on the interaction between two or more shear bands in simple or dipolar materials.

II. FORMULATION OF THE SIMPLE SHEARING PROBLEM

We study the simple shearing deformations of a dipolar viscoplastic material and assume that all of the variables have been nondimensionalized. Thus, the body occupies the infinite slab bounded by the planes $y = \pm 1$. Referring the reader to WRIGHT and BATRA [1986] for details, we note that the governing equations are

$$\dot{v} = \frac{1}{\rho}(s - l\sigma_{,y})_{,y},\tag{1}$$

$$\dot{\theta} = k\theta_{,yy} + \Lambda(s^2 + \sigma^2),\tag{2}$$

$$\dot{s} = \mu(v_{,y} - \Lambda s),\tag{3}$$

$$\dot{\sigma} = l\mu\left(v_{,yy} - \frac{\Lambda}{l}\sigma\right),\tag{4}$$

$$\dot{\psi} = \Lambda(s^2 + \sigma^2)\bigg/\left(1 + \frac{\psi}{\psi_0}\right)^n,\tag{5}$$

$$\Lambda = \max\left[0, \left\{\left(\frac{(s^2 + \sigma^2)^{1/2}}{\left(1 + \dfrac{\psi}{\psi_0}\right)^n(1 - a\theta)}\right)^{1/m} - 1\right\}\bigg/\{b(s^2 + \sigma^2)^{1/2}\}\right]\tag{6}$$

with boundary conditions

$$v(\pm 1, t) = \pm 1, \quad \theta_{,y}(\pm 1, t) = 0, \quad \sigma(\pm 1, t) = 0\tag{7}$$

and a suitable set of initial conditions. Equations (1) and (2) express the balance of linear momentum and internal energy, respectively. Herein v is the velocity of a material particle, l is a material characteristic length, θ is the temperature change of the material particle from that in the reference configuration, and s and σ may be interpreted as the shear stress and the dipolar shear stress, respectively. A superimposed dot indicates material time differentiation, and a comma followed by y signifies partial differentiation with respect to y. The nondimensional variables are related to their dimensional (barred) counterparts as follows:

$$y = \bar{Y}/H, \quad t = \bar{t}\dot{\gamma}_0, \quad \dot{\gamma} = v_{,y}, \quad \dot{d} = v_{,yy}, \quad \dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_p, \quad \dot{d} = \dot{d}_e + \dot{d}_p,$$

$$v = \bar{v}/H\dot{\gamma}_0, \quad s = \bar{s}/\kappa_0, \quad \sigma = \bar{\sigma}/\bar{l}\kappa_0, \quad \theta = \bar{\theta}\bar{\rho}\bar{c}/\kappa_0,\tag{8}$$

$$\gamma = \bar{\gamma}, \quad d = \bar{d}H, \quad \dot{\gamma}_p = \dot{\bar{\gamma}}_p/\dot{\gamma}_0, \quad \dot{d}_p = \dot{\bar{d}}_p H/\dot{\gamma}_0, \quad \Lambda = \bar{\Lambda}\kappa_0/\dot{\gamma}_0.$$

Besides m, n, and ψ_0, there are six other nondimensional parameters, which are related to their dimensional (barred) counterparts as follows:

$$a = \bar{a}\kappa_0/\bar{\rho}\bar{c}, \quad b = \bar{b}\dot{\gamma}_0, \quad k = \bar{k}/\bar{\rho}\bar{c}\dot{\gamma}_0 H^2, \quad l = \bar{l}/H,$$

$$\mu = \bar{\mu}/\kappa_0, \quad \rho = \bar{\rho}H^2\dot{\gamma}_0^2/\kappa_0.\tag{9}$$

In eqns (8) and (9), $\dot{\gamma}_0 = \bar{v}(H, \bar{t})/H$ is the average applied strain rate between the boundaries $\bar{Y} = \pm H$, and κ_0 is the yield stress in a reference quasistatic test.

The constitutive relations (3)–(6) give one possible model of viscoplastic materials.

Equation (6) implies that the plastic parts, Λs and $\Lambda \sigma / l$, of the strain rate and the dipolar strain rate vanish when

$$(s^2 + \sigma^2)^{1/2} \le \left(1 + \frac{\psi}{\psi_0}\right)^n (1 - a\theta). \tag{10}$$

The material parameters ψ_0 and n describe the strain hardening of the material, a the thermal softening, and b and m the strain-rate sensitivity of the material.

We presume that the initial values of θ, s, and ψ are symmetric and the initial values of v and σ are antisymmetric in y and seek solutions of eqns (1) through (6) with the same symmetry. Thus, the problem is to be studied over the spatial domain $[0,1]$ and the boundary conditions become

$$v(0,t) = 0, \quad \theta,_y(0,t) = 0, \quad \sigma(0,t) = 0, \tag{11}$$

$$v(1,t) = 1, \quad \theta,_y(1,t) = 0, \quad \sigma(1,t) = 0. \tag{12}$$

For the initial conditions we take

$$v(y,0) = y, \quad \sigma(y,0) = 0, \quad \psi(y,0) = \tilde{\psi}_0,$$

$$\theta(y,0) = \tilde{\theta}_0 + \tilde{\theta}(y), \tag{13}$$

$$s(y,0) = s_0 = \left(1 + \frac{\psi_0}{\psi_0}\right)^n (1 + b\Lambda\tilde{s}_0)^m (1 - a\theta(y,0)).$$

The values of $\tilde{\theta}_0$, $\tilde{s}_0$, and $\tilde{\psi}_0$ are such that during homogeneous deformations of the block the shear stress $\tilde{s}_0$ and the strain corresponding to $\tilde{\psi}_0$ lie on the shear stress–shear strain curve for the material. Λ in eqn (13)$_5$ is given by eqn (6), with $\theta = \tilde{\theta}_0$, $s = \tilde{s}_0$, $\psi = \tilde{\psi}_0$, and $\sigma = 0$. The function $\tilde{\theta}$ describes the aberration in the initial temperature distribution and will result in nonhomogeneous deformations of the body.

III. NUMERICAL INTEGRATION OF GOVERNING EQUATIONS

With the auxiliary variables

$$u = v,_y, \quad g = \theta,_y, \quad p = \sigma,_y, \tag{14}$$

we rewrite eqns (1)–(4) as

$$\dot{v} = \frac{1}{\rho} (s - lp),_y, \tag{15}$$

$$\dot{\theta} = kg,_y + \Lambda(s^2 + \sigma^2), \tag{16}$$

$$\dot{s} = \mu(u - \Lambda s), \tag{17}$$

$$\dot{\sigma} = l\mu \left(u,_y - \frac{\Lambda}{l}\sigma\right). \tag{18}$$

Thus, only first-order spatial derivatives of the unknowns v, θ, s, σ, u, g, and p appear in the governing equations. Let H^1 denote the space of functions defined on $[0,1]$ the square of whose first-order derivative is integrable over $[0,1]$. We approximate the unknown functions v, θ, s, etc. by linear combination of the finite element basis functions $\{\phi_i(y), i = 1, 2, \ldots, N\}$ in an N-dimensional subspace of H^1. For example,

$$v(y,t) = v_i(t)\phi_i(y). \tag{19}$$

Throughout the article, a repeated index implies summation over the range of the index. Using Galerkin's method (e.g., see BECKER *et al.* [1981]) we thus reduce eqns (14) through (18) to the following set of equations:

$$M_{ij}u_i = -Q_{ij}v_i, \quad M_{ij}g_i = -Q_{ij}\theta_i, \quad M_{ij}p_i = -Q_{ij}\sigma_i, \tag{20}$$

$$M_{ij}\dot{v}_i = -\frac{1}{\rho}A_{ij}s_i + \frac{l}{\rho}Q_{ij}p_i, \quad M_{ij}\dot{\theta}_i = -kQ_{ij}g_i + \Lambda_i P_{ij}, \tag{21}$$

$$M_{ij}\dot{s}_i = \mu M_{ij}u_i - \mu \Lambda_i s_k R_{ijk}, \quad M_{ij}\dot{\sigma}_i = -\mu l\tilde{Q}_{ij}u_i - \mu \Lambda_i \sigma_k R_{ijk}, \tag{22}$$

where

$$M_{ij} = \int_0^1 \phi_i\phi_j \, dy = M_{ji}, \quad Q_{ij} = \int_0^1 \phi_i\phi_{j,y} \, dy, \tag{23}$$

$$\tilde{Q}_{ij} = Q_{ij} - (\phi_i\phi_j)\Big|_0^1, \tag{24}$$

$$R_{ijk} = \int_0^1 \phi_i\phi_j\phi_k \, dy = R_{ikj} = R_{kij}, \tag{25}$$

$$P_{ij} = \int_0^1 \phi_i\phi_j(s^2 + \sigma^2) \, dy = P_{ji}. \tag{26}$$

We note that because of the nonlinear dependence of P_{ij} and Λ upon s, σ, ψ, and θ, the coupled set of ordinary differential eqns (20)–(22) is not that easy to integrate. The matrices M_{ij}, Q_{ij}, $\tilde{Q}_{ij}$, R_{ijk}, and P_{ij} have been evaluated by using the linear basis functions. Also, $v_i(t)$ denotes the velocity of node i at time t.

We use the Crank-Nicolson method to integrate eqns (20)–(22), with respect to time t. In it, eqns (20)–(22), assumed to hold at time $(t + \Delta t/2)$, are used to predict the values of v, θ, s, σ, g, p, u, and ψ at time $(t + \Delta t)$ from a knowledge of their values at time t. This is accomplished by approximating $\dot{\theta}_i(t + \Delta t/2)$ by $(\theta_i(t + \Delta t) - \theta_i(t))/\Delta t$, $\theta_i(t + \Delta t/2)$ by $(\theta_i(t + \Delta t) + \theta_i(t))/2$, and so forth and by first evaluating the non-linear terms on the right-hand side of eqns (20)–(22) at time t. The resulting system of linear algebraic equations is solved for $v_i(t + \Delta t)$, etc., the right-hand side in eqns (20)–(22) is now evaluated at time $(t + \Delta t/2)$, and the system of equations is solved again for $v_i(t + \Delta t)$ etc. This iterative process is continued until at each nodal point,

$$\left|\frac{\Delta v}{v}\right| + \left|\frac{\Delta \theta}{\theta}\right| + \left|\frac{\Delta s}{s}\right| + \left|\frac{\Delta \psi}{\psi}\right| + |\Delta \sigma| + |\Delta g| + |\Delta p| + |\Delta u| \le \epsilon \tag{27}$$

where the subscript i has been dropped from v_i and elsewhere, Δv denotes the difference between the newly found value of v and that used to compute the right-hand side in eqns (20)–(22), and ϵ is a preassigned small number. The initial conditions (13) were used to find $v_i(0)$ and so forth.

IV. COMPUTATION AND DISCUSSION OF RESULTS

In order to compute numerical results, the following values of various nondimensional parameters that correspond to a typical hard steel and the average applied strain rate $\dot{\gamma}_0 = 500 \ \text{sec}^{-1}$ were chosen.

$$\rho = 3.928 \times 10^{-5}, \quad k = 3.978 \times 10^{-3}, \quad a = 0.4973, \quad \mu = 240.3,$$

$$n = 0.09, \quad \psi_0 = 0.017, \quad b = 5 \times 10^6, \quad m = 0.025.$$

For homogeneous deformations of the block, the peak (marked as point P in Fig. 1) in the shear stress–shear strain curve occurs at a strain of 0.093. The uniform temperature $\theta_0 = 0.1003$ in the block when $\gamma = 0.0692$, corresponding to the point I in Fig. 1, was perturbed by adding a smooth temperature bump

$$\tilde{\theta}(y) = \Delta\theta_0(1 - |y^2 - y_0^2|)^p e^{-\alpha|y^2 - y_0^2|}, \tag{28}$$
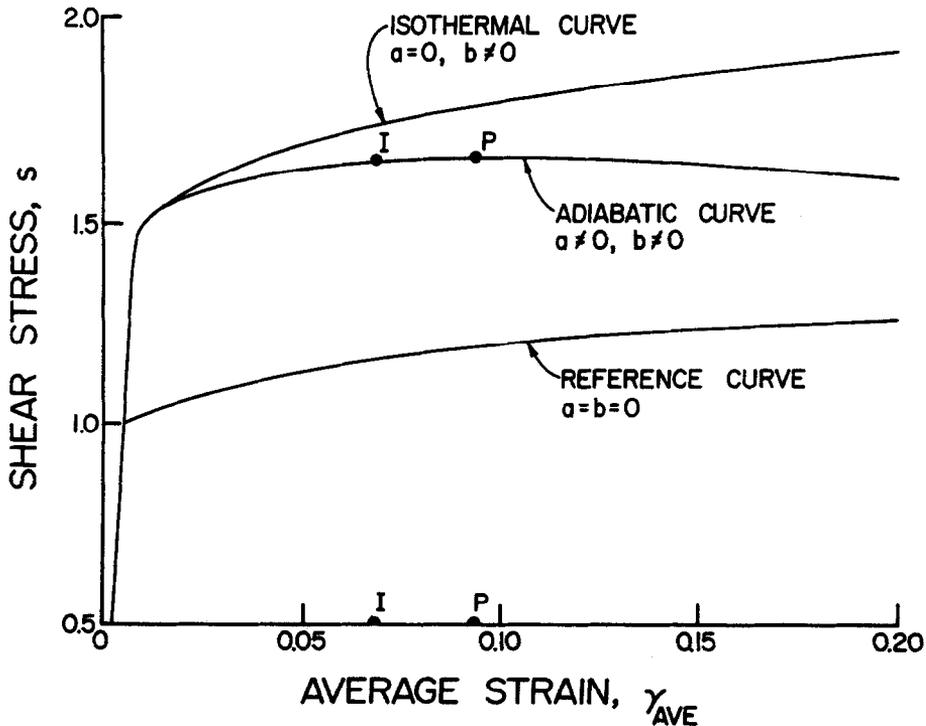
Fig. 1. Average shear stress–average shear strain curve for a typical steel.

and the resulting initial-boundary-value problem was solved by using the aforementioned method. The domain was divided into either 20 uniform subdomains (usually called finite elements) or 20 nonuniform subdomains, with nodes at 0, 0.0025, 0.01, 0.0225, 0.04, 0.0625, 0.09, 0.1225, 0.160, 0.2025, 0.2500, 0.3025, 0.360, 0.4225, 0.490, 0.5625, 0.640, 0.7225, 0.81, 0.9025, and 1.0. The two temperature perturbations for $\Delta\theta_0 = 0.1$, $p = 9$, $\alpha = 5$ and for $\Delta\theta_0 = 0.1$, $p = 2$, $\alpha = 1$ are shown in Fig. 2. For nonpolar materials, Fig. 3 depicts the growth of the central plastic strain rate in time for the two subdivisions of the domain. Numerical experiments with different values of Δt indicated that $\Delta t = 5 \times 10^{-6}$ gave accurate results. All of the results presented herein are for this value of Δt and $\epsilon = 0.01$. Unless otherwise noted, the nonuniform grid has been used. WRIGHT and BATRA [1986] gave a heuristic reasoning to explain that the pla-
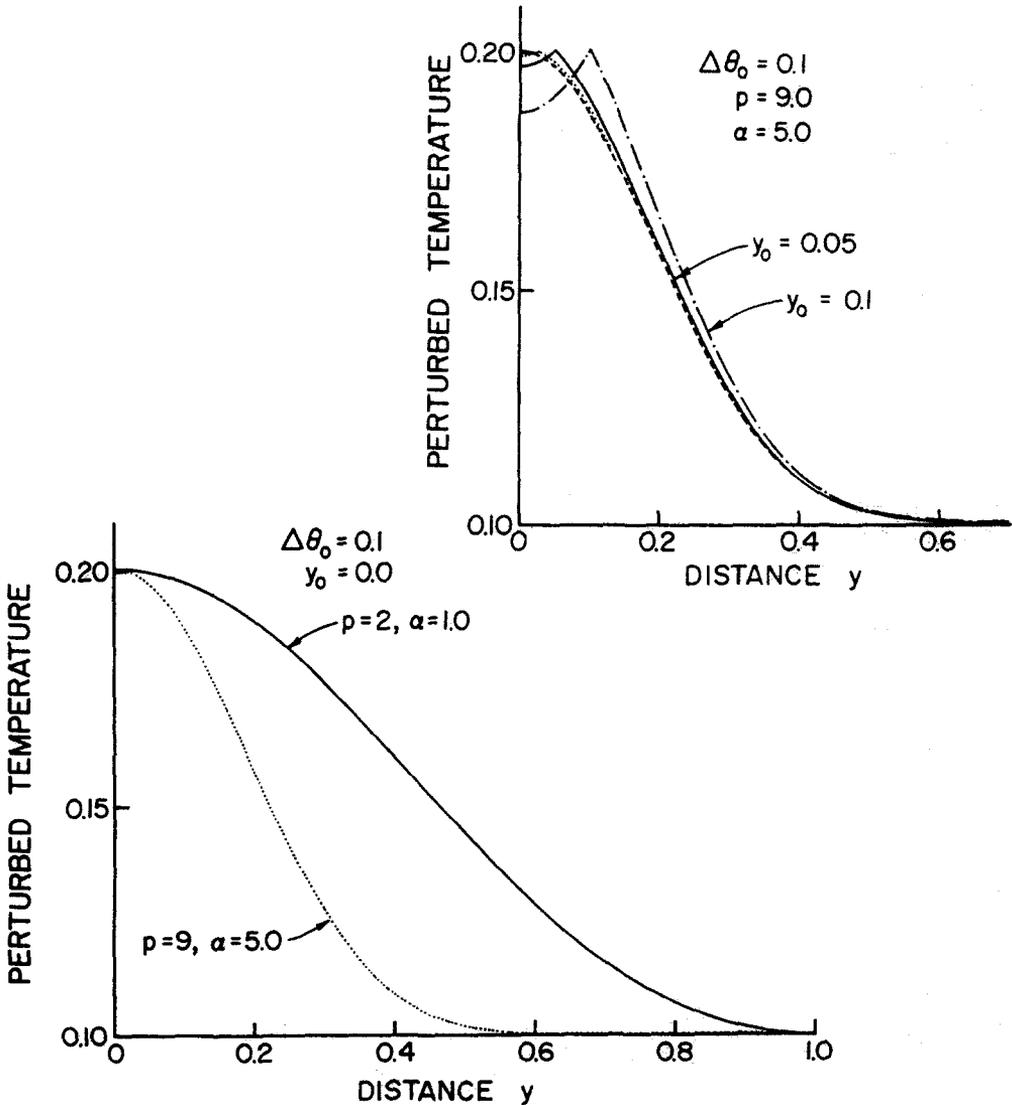
Fig. 2. Temperature perturbations.
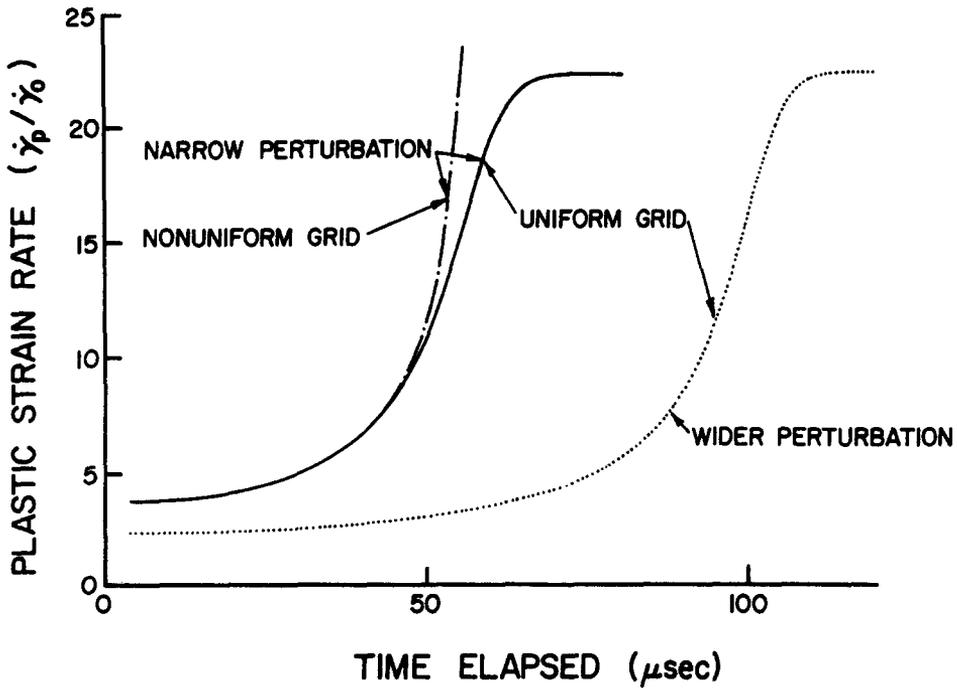
Fig. 3. Central plastic strain rate vs. elapsed time.

teau was formed because of the coarse mesh. The plot in Fig. 2 clearly proves that. Also, for the wider perturbation the rate of growth of the central plastic strain rate is quite a bit slower. Whereas for the narrower perturbation the shear band is formed when the average strain in the block corresponds to the point just past the peak in the shear stress–shear strain curve, the broader perturbation results in the formation of the band well past the peak in the average shear stress–average shear strain curve, which occurs at $t = 47.6$ μsec after the temperature field was perturbed. That a band is indeed formed for the wider perturbation should be clear from the plot of Fig. 4. It is apparent that the rapidly deforming region progressively narrows down to the one close to the center. The width of the rapidly deforming region is narrower for the finer mesh. Whether we have obtained the final band width or not is not quite clear yet, since we have not experimented with other fine meshes. Figure 5 depicts how the central shear stress changes in time. It decreases at a very slow rate and essentially follows the average shear stress–shear strain curve. It stays nearly uniform throughout the specimen. Numerical instabilities developed for values of time t a little beyond the one up to which results are plotted here. These can be due to the improper size of the time increment and/or to the mesh size, among other factors. Efforts are now under way to overcome these and to extend calculations for longer values of time.

Figures 6 and 7 depict the effect of the amplitude $\Delta\theta_0$ of the perturbation upon the growth of the central plastic strain rate for simple and dipolar materials with $l = 0.01$. Obviously, for larger amplitudes of the perturbation, the shear band is formed well before the peak in the average shear stress–average shear strain curve is reached. A comparison of the results presented in Figs. 3, 6, and 7 clearly brings out the stiffening caused by the inclusion of the dipolar stresses.
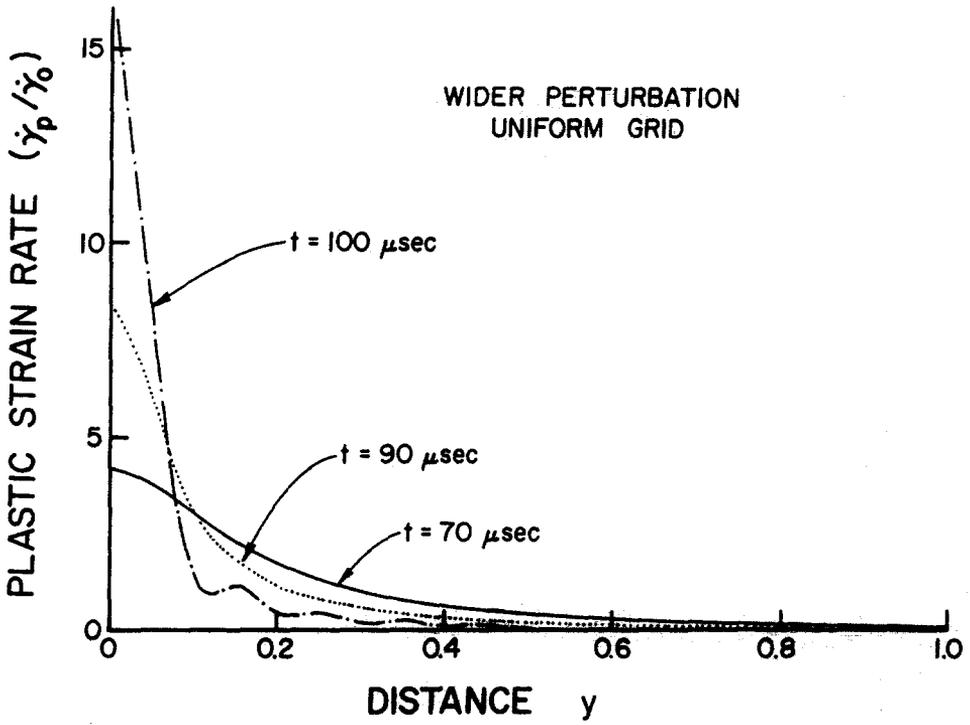
Fig. 4. Plastic strain rate distribution in a finite block of material at various times as a shear band forms.
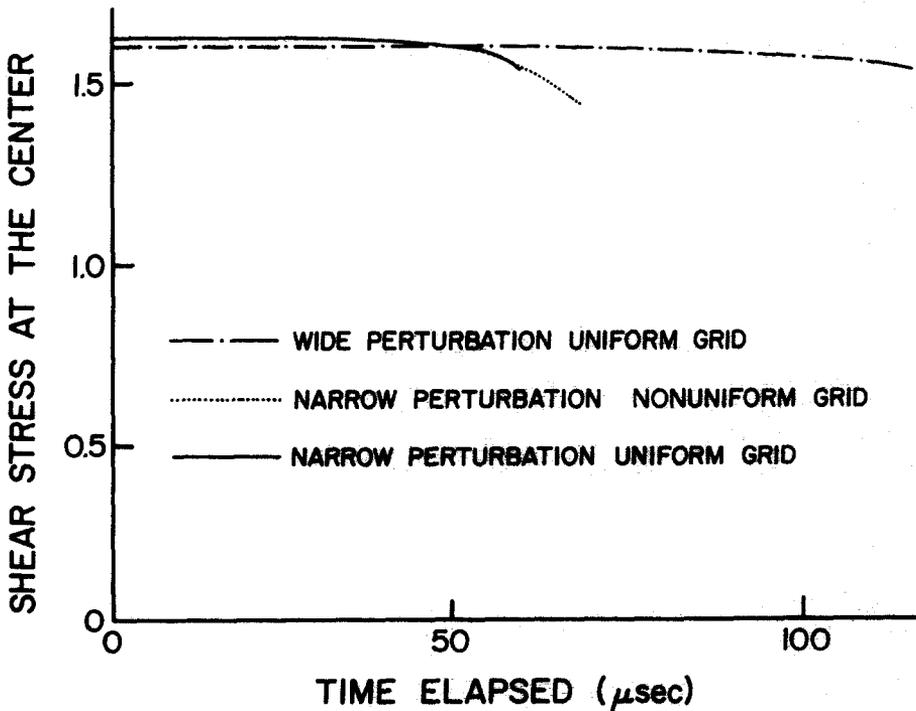
Fig. 5. Central shear stress vs. elapsed time.
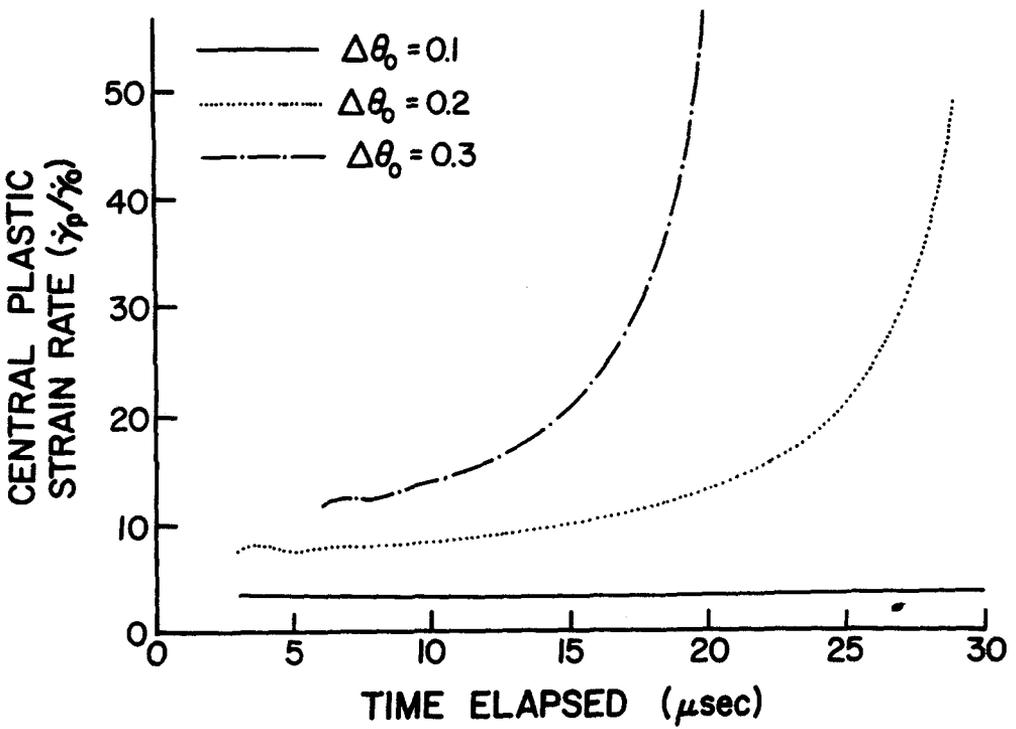
Fig. 6. Central plastic strain rate vs. elapsed time for nonpolar materials.
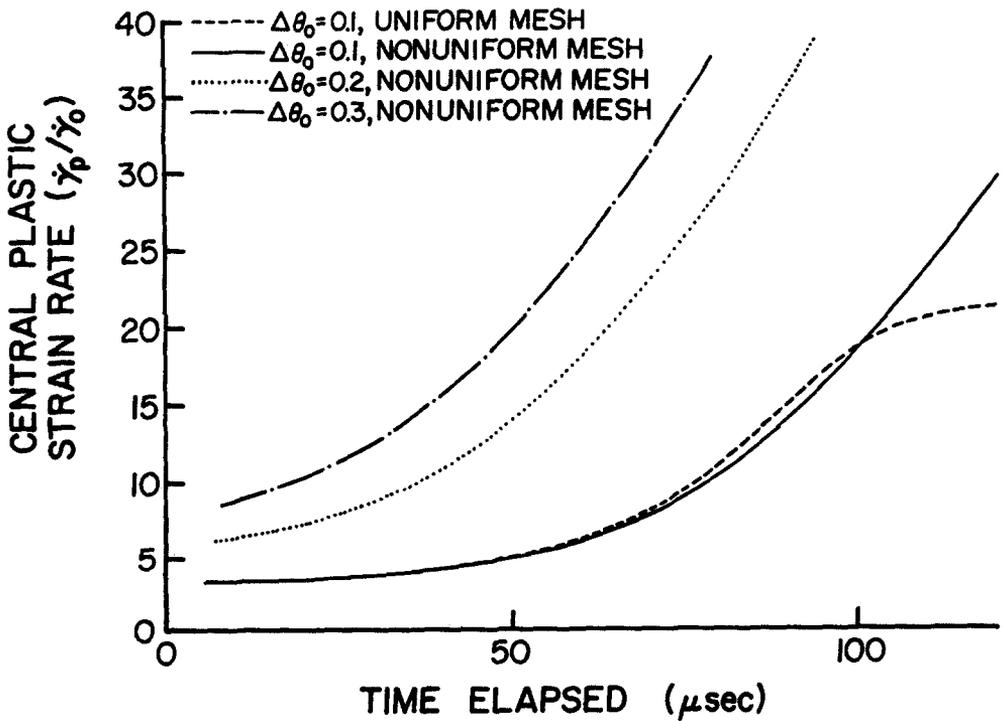
Fig. 7. Central plastic strain rate vs. elapsed time for dipolar materials ($l = 0.01$).

In order to understand the interaction among shear bands, we introduced a perturbation in the temperature centered at different points along the thickness of the slab. Because of the symmetry of θ about the horizontal axis, this amounts to introducing two aberrations symmetrically placed about the center line $y = 0$. The hypothesis here is that if the resulting nonhomogeneous fields, such as the plastic strain rate, temperature, and the plastic strain, eventually peak out at the center of the slab, then the two bands have coalesced; otherwise they grow independently. Perturbations in the temperature centered at $y = 0.025$, $y = 0.05$, and $y = 0.1$ but $\Delta\theta_0 = 0.1$ were introduced and the ensuing initial-boundary-value problems were solved. Figures 8 and 9 show the distribution of the plastic strain rate through the thickness of the slab for simple and dipolar materials, respectively. For simple materials (Fig. 8), the two bands initially centered at $y = \pm0.025$ coalesce but the ones centered at $\pm.05$ and $\pm.1$ grow independently of each other. The rate of growth of the peak strain rate (not the strain rate at $y = 0$) is considerably less as compared to that when either only one band appears at $y = 0$ or two bands initially centered at $y = \pm.025$ merge and grow as a single band. For dipolar materials, the bands coalesce in all three cases. Recall that the material characteristic length is $\frac{1}{20}$ of the distance between the bands originating at $y = \pm0.1$.

The distribution of the dipolar stress across the slab is plotted in Fig. 10. Because of the boundary conditions $\sigma(0,t) = \sigma(1,t) = 0$ and the fact that the dipolar stress for the elastic problem is proportional to the curvature, the maximum value of σ cannot occur at the center of the slab. Note that σ is very small as compared to 1.0, whereas s is gen-
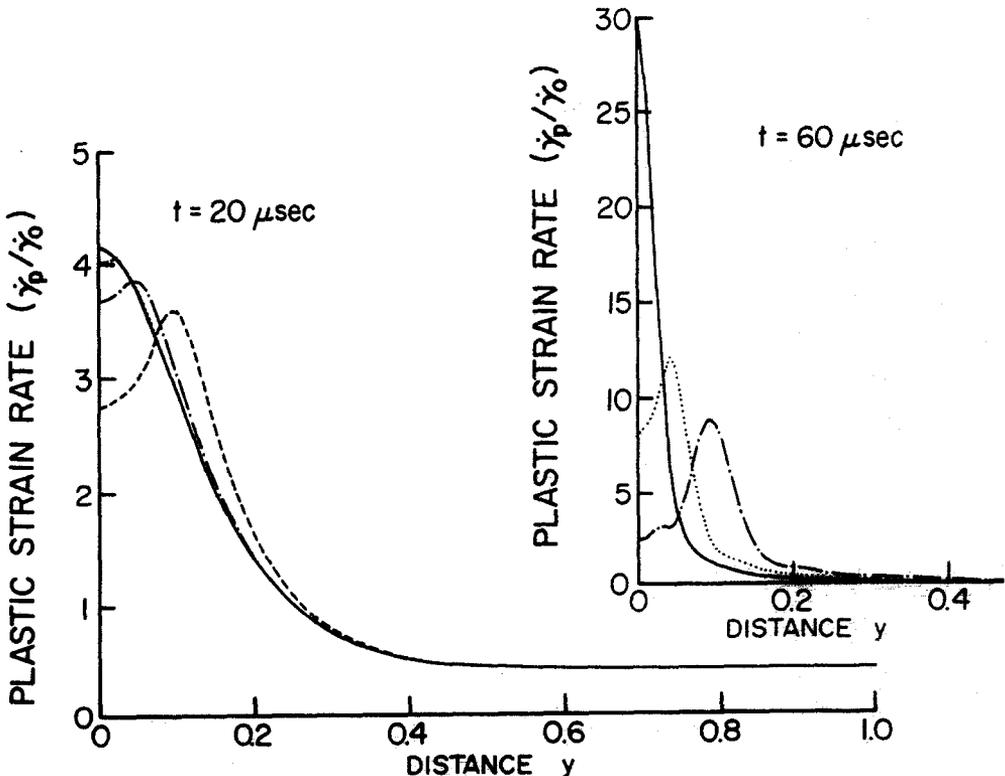
Fig. 8. Plastic strain rate distribution for four perturbations centered at different points (nonpolar materials).
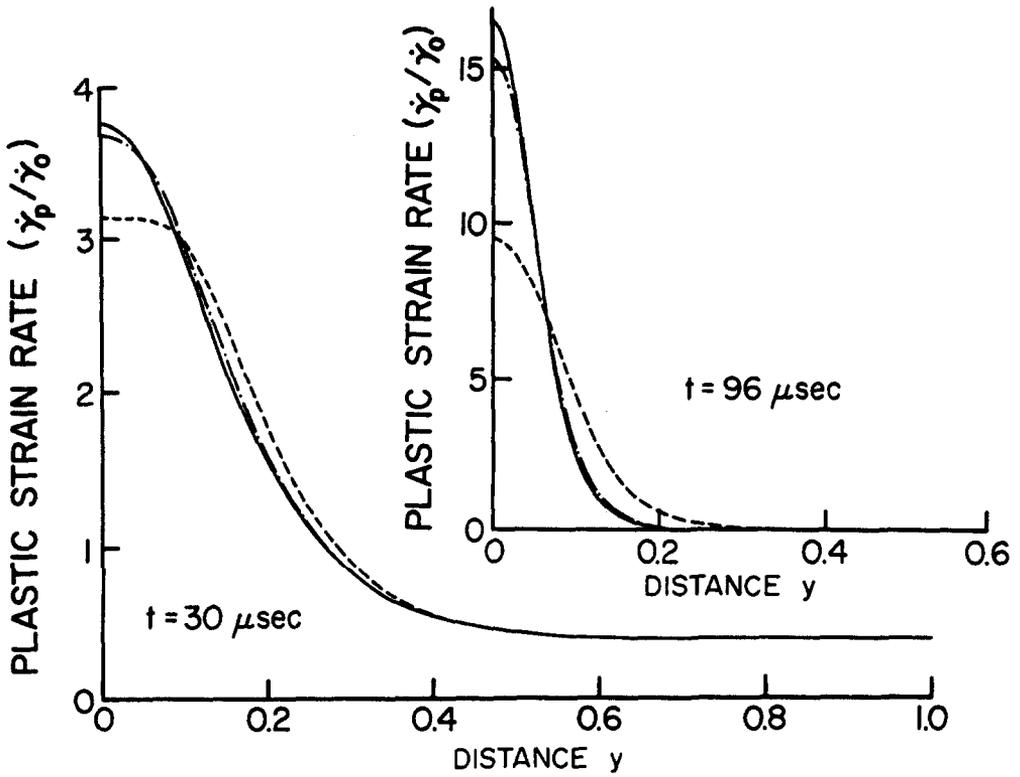
Fig. 9. Plastic strain rate distribution for four perturbations centered at different locations (dipolar materials, $l = 0.01$).
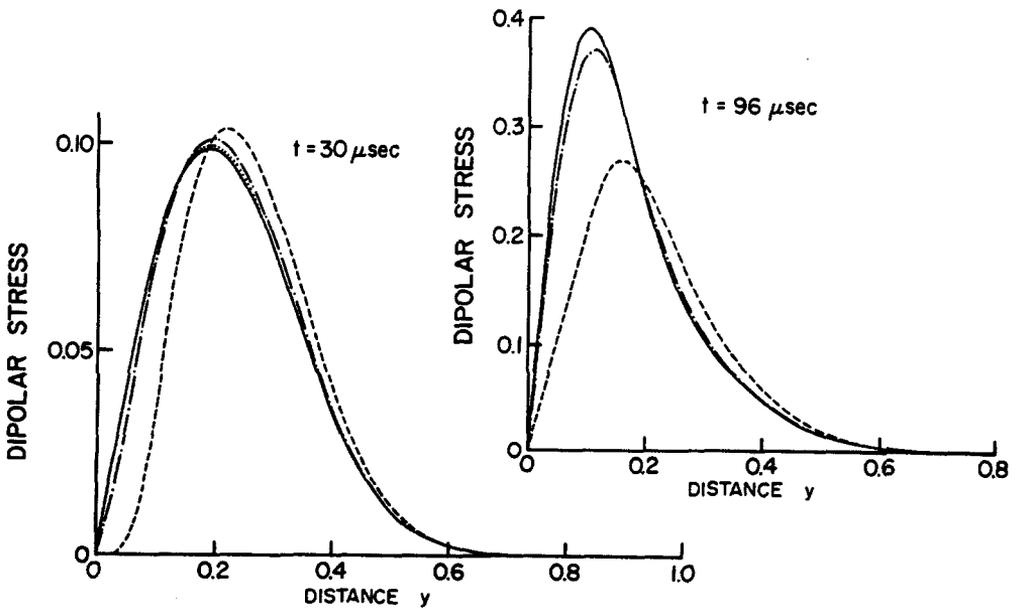
Fig. 10. Dipolar stress distribution for four perturbations centered at different locations ($l = 0.01$).

erally greater than or equal to 1.0. At a point where $\dot{\gamma}_p \neq 0$, $\dot{d}_p \neq 0$, the dipolar stress has negligible contribution to the effective stress $(s^2 + \sigma^2)^{1/2}$ that determines whether $\Lambda > 0$ or $\Lambda = 0$ at that point.

V. CONCLUSION

It is shown that the constitutive model proposed by WRIGHT and BATRA [1986] does predict adiabatic shear bands in a block undergoing simple shearing. The rate at which a band grows depends upon the strength of the inhomogeneity, herein modeled as a perturbation in the otherwise uniform temperature within the block. Perturbations of larger amplitude result in the formation of a shear band even before the peak in the shear stress–shear strain curve is reached. Also, a wider perturbation results in the shear strain localization at a lower rate as compared to the narrow perturbation, both being of the same amplitude. The inclusion of dipolar effects results in a very stiffening effect in the sense that the formation of bands is delayed considerably as compared to that in simple materials. In addition, two bands that would grow independently in a simple material coalesce when dipolar effects are included. Of course, the minimum distance between two shear bands to grow independently of each other will possibly depend upon, among other factors, the value of the material characteristic length l.

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