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ADIABATIC SHEAR BANDING IN DYNAMIC PLANE STRAIN COMPRESSION OF A VISCOPLASTIC MATERIAL

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Abstract—Dynamic plane strain thermomechanical deformations of a thermally softening viscoplastic body subjected to compressive loads on the top and bottom faces are studied with the objective of exploring the effect of (a) modeling the material inhomogeneity by introducing a temperature perturbation or assuming the existence of a weak material within the block, (b) introducing two defects symmetrically placed on the vertical axis of the block. The effect of setting the thermal conductivity equal to zero is also studied in the latter case. It is found that, irrespective of the way the material inhomogeneity is modeled, a shear band initiates from the site of the defect and propagates in the direction of maximum shearing stress. The value of the average strain at the instant of the initiation of the band depends upon the strength of the material defect introduced. Once the shear band reaches the boundaries of the block it is reflected back, the angle of reflection being nearly equal to the angle of incidence.

I. INTRODUCTION

Adiabatic shear banding refers to the localization of the deformation into thin narrow bands of intense plastic deformation that usually form during high-rate plastic deformation. These bands often precede shear fractures. The experimental work in this area is due to ZENER and HOLLOMON [1944], COSTIN *et al.* [1980], MOSS [1981], LINDHOLM and JOHNSON [1983], HARTLEY, DUFFY and HAWLEY [1987], and MARCHAND and DUFFY [1988]. HARTLEY *et al.* [1987], and MARCHAND and DUFFY [1988], have given a detailed history of the temperature and strain fields within a band formed in a thin steel tube deformed in simple torsion.

During the last ten years, there have been numerous studies aimed at analyzing the initiation and growth of shear bands in the one-dimensional simple shearing problem. For example, CLIFTON [1980] and BAI [1981] analyzed the growth of infinitesimal periodic perturbations superimposed on a body deformed by a finite amount in simple shear. BURNS [1985] used a dual asymptotic expansion to account for the time dependence of the homogeneous solution in the analysis of the growth of superimposed periodic perturbations. Other works include those of MERZER [1983], WU and FREUND [1984], CLIFTON *et al.* [1984], COLEMAN and HODGDON [1985], WRIGHT and BATRA [1985], WRIGHT and WALTER [1987], BATRA [1987a,1987b], ZBIB and AIFANTIS [1988], and BATRA and KIM [1990]. We note that ROGERS [1979,1983] and TIMOTHY [1987] have reviewed various aspects of shear banding, and ANAND *et al.* [1988] have generalized one-dimensional stability analysis of CLIFTON [1980] to three-dimensional problems.

Recently LEMONDS and NEEDLEMAN [1986a,1986b], ANAND *et al.* [1988], NEEDLEMAN [1989], BATRA and LIU [1989], and SHUTTLE and SMITH [1988] have studied the initiation and growth of shear bands in plane strain deformations of a softening material. Except for Needleman, and Batra and Liu, these works neglected the effect of inertia forces. Batra and Liu studied the coupled thermomechanical deformations of a thermally softening viscoplastic solid and modeled the material inhomogeneity by introducing a temperature bump at the center of the block whose boundaries were taken to be perfectly insulated. Two different loadings, namely, those corresponding to simple shearing and simple compression of the block, were considered. Here, we examine the effect of (a) modeling the material inhomogeneity in two different ways, namely, introducing a temperature perturbation and assuming the existence of a weak material, (b) introducing two defects placed symmetrically on the vertical axis of the block, (c) varying the reduction in the flow stress of the weak material, and (d) two different sets of initial conditions.

II. FORMULATION OF THE PROBLEM

We use an updated Lagrangian description (e.g., see BATHE [1982]) to analyze the plane strain thermomechanical deformations of the viscoplastic body. That is, in order to solve for the deformations of the body at time $(t + \Delta t)$, the configuration at time t is taken as the reference configuration. However, it is not assumed that the deformations of the body from time t to time $(t + \Delta t)$ are infinitesimal. With respect to a fixed set of rectangular Cartesian coordinate axes, we denote the position of a material particle in the configuration at time t by X_α and in the configuration at time $(t + \Delta t)$ by x_i. In terms of the referential description the governing equations are

$$(\rho J)^{\cdot} = 0, \tag{2.1}$$

$$\rho_o \dot{v}_i = T_{i\alpha,\alpha}, \tag{2.2}$$

$$\rho_0 \dot{e} = -Q_{\alpha,\alpha} + T_{i\alpha} v_{i,\alpha}, \tag{2.3}$$

supplemented by appropriate constitutive relations, and initial and boundary conditions. Equations (2.1), (2.2), and (2.3) express, respectively, the balance of mass, the balance of linear momentum, and the balance of internal energy. Here ρ is the mass density of a material particle in the current configuration at time $t + \Delta t$, ρ_o its mass density in the reference configuration; a superimposed dot indicates a material time derivative; $J = \rho_o/\rho$ equals the determinant of the deformation gradient $F_{i\alpha} \equiv x_{i,\alpha}$; v_i is the velocity of a material particle in the x_i-direction, $T_{i\alpha}$ is the first Piola-Kirchoff stress tensor; a comma followed by $\alpha(i)$ implies partial differentiation with respect to $X_\alpha(x_i)$; a repeated index signifies summation over the range of the index; e is the internal energy per unit mass; and Q_α is the heat flux. We assume that plane strain deformations occur in the $X_1 - X_2$ plane, so that $x_3 = X_3$ and the indices i and α range over 1,2.

We note that even when the applied overall strain-rate is kept fixed, different material particles undergo deformations at varying strain-rates. During the course of a loading process in which a shear band forms, the temperature of a material particle may also increase considerably. A constitutive relation that can model the material response over changes in plastic strain-rate and temperature of several orders of magnitude is needed to properly analyze the shear band problem. HARTLEY *et al.* [1987], and MARCHAND and DUFFY [1988], have proposed a power law that seems to describe adequately the simple shearing deformations of the steels tested. However, a constitutive relation applicable to more general deformations is not readily available in the open literature.

Here we assume that the following constitutive relations describe adequately the material response:

$$\sigma_{ij} = -p(\rho)\delta_{ij} + 2\mu D_{ij},$$

$$T_{i\alpha} \equiv (\rho_o/\rho)X_{\alpha,j}\sigma_{ij}, \tag{2.4}$$

$$2\mu = \left[\sigma_o/(\sqrt{3}I)\right](1 - \nu\theta)(1 + bI)^m,$$

$$2D_{ij} = v_{i,j} + v_{j,i}, \tag{2.5}$$

$$2I^2 = \tilde{D}_{ij}\tilde{D}_{ji}, \quad \tilde{D}_{ij} = D_{ij} - (1/3)D_{kk}\delta_{ij}, \tag{2.6}$$

$$p(\rho) = B(\rho/\rho_r - 1), \tag{2.7}$$

$$Q_\alpha = -k(\rho_0/\rho)X_{\alpha,i}\theta,i, \tag{2.8}$$

$$\rho_{o\dot{e}} = \rho_o c\dot{\theta} + \rho_o\dot{\rho}p(\rho)/\rho^2. \tag{2.9}$$

Here, σ_{ij} is the Cauchy stress tensor, σ_o is the yield stress in a quasi-static simple tension or compression test, ν is the coefficient of thermal softening, $\tilde{D}_{ij}$ is the deviatoric strain-rate tensor, D_{ij} is the strain-rate tensor, δ_{ij} is the Kronecker delta, B may be interpreted as the bulk modulus, ρ_r is the mass density in the stress free reference configurations, c is the specific heat, k is the thermal conductivity, and parameters b and m describe the strain-rate sensitivity of the material. The material parameters b, m, B, k, and c are taken to be independent of the temperature. Equation (2.8) is the Fourier law of heat conduction, and eqn $(2.4)_1$ may be interpreted as a constitutive relation for a non-Newtonian fluid whose viscosity μ depends upon the temperature and the strain-rate. Alternatively, defining s_{ij} by

$$s_{ij} = \sigma_{ij} + [p - (2/3)\mu D_{kk}]\delta_{ij} \tag{2.10}$$

$$= 2\mu\tilde{D}_{ij}, \tag{2.11}$$

we can write eqns (2.4) and (2.5) as

$$[(1/2)(s_{ij}s_{ji})]^{1/2} = \left[\sigma_o/\sqrt{3}\right](1 - \nu\theta)(1 + bI)^m \tag{2.12}$$

which can be viewed as a generalized von Mises yield surface when the flow stress (given by the right-hand side of (2.12)) at a material particle depends upon its strain-rate and temperature. That the flow stress decreases linearly with the temperature rise has been observed by BELL [1968], LINDHOLM and JOHNSON [1983], and LIN and WAGONER [1986]. The range of temperatures examined by these investigators is not as large as that expected to occur in the shear band problem. However, constitutive relations akin to eqn (2.4) have been used by ZIENKIEWICZ *et al.* [1981] for analyzing the extrusion problem, by BATRA [1988] in studying the steady-state penetration of a viscoplastic target by a rigid cylindrical penetrator, and by BATRA and LIU [1989] for studying the shear band problem.

In terms of the nondimensional variables

$$
\begin{gathered}
\bar{\sigma} = \sigma/\sigma_o, \quad \bar{p} = p/\sigma_o, \quad \bar{\mathbf{s}} = \mathbf{s}/\sigma_o, \quad \bar{\mathbf{v}} = \mathbf{v}/v_o, \\
\bar{t} = tv_o/H, \quad \bar{\mathbf{T}} = \mathbf{T}/\sigma_o, \\
\bar{\mathbf{x}} = \mathbf{x}/H, \quad \bar{\theta} = \theta/\theta_o, \quad \bar{b} = b(v_o/H), \quad \bar{\nu} = \nu\theta_o, \\
\bar{\rho} = \rho/\rho_r, \quad \tilde{\rho} = \rho_o/\rho_r, \quad \bar{\mathbf{X}} = \mathbf{X}/H, \\
\delta = \rho_r v_o^2/\sigma_o, \quad \beta = k/(\rho_r c v_o H), \\
\theta_o = \sigma_o/(\rho_r c), \quad \bar{B} = B/\sigma_o,
\end{gathered} \tag{2.13}
$$

the governing equations can be written as

$$\dot{\rho} + \rho v_{i,i} = 0, \tag{2.14}$$

$$\delta\rho\dot{v}_i = T_{i\alpha,\alpha}, \tag{2.15}$$

$$\tilde{\rho}\dot{\theta} = \beta\theta_{,ii} + (\tilde{\rho}/\rho_r)[1/(\sqrt{3}I)](1 + bI)^m(1 - \nu\theta)\tilde{D}_{ij}\tilde{D}_{ij}, \tag{2.16}$$

$$\sigma_{ij} = -B(\rho - 1)\delta_{ij} + [1/\sqrt{3}I)](1 + bI)^m(1 - \nu\theta)D_{ij}, \tag{2.17}$$

where we have dropped the superimposed bars. In eqns (2.13) $2H$ is the height of the block and v_o the imposed velocity on the top and bottom surfaces. In eqns (2.14)–(2.16) all of the differentiations are with respect to nondimensional variables. We note that in eqn (2.16) all, rather than 90–95% as stated by TAYLOR and QUINNEY [1934], of the plastic working is assumed to be converted into heat.

For the viscoplastic block being deformed in simple compression we study only those deformations that remain symmetric about the horizontal and vertical planes passing through the center of the block. Thus we analyze the deformations of the material in the first quadrant. With the origin of the coordinate axes situated at the center of the undeformed block (cf. Fig. 1), we can write the pertinent boundary conditions as follows

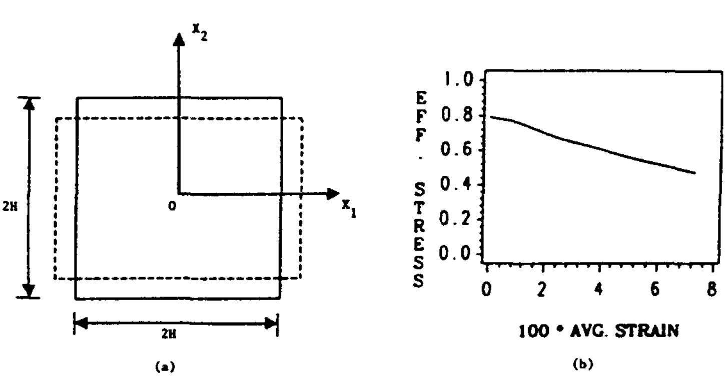

Fig. 1. (a) The problem studied, (b) Stress-strain curve in simple compression for the material studied.

$$\begin{aligned}
v_1 = 0, \quad T_{21} = 0, \quad Q_1 = 0 \text{ at } x_1 = X_1 = 0, \\
v_2 = 0, \quad T_{12} = 0, \quad Q_2 = 0 \text{ at } x_2 = X_2 = 0, \\
T_{i\alpha}N_\alpha = 0, \quad Q_\alpha N_\alpha = 0 \text{ on the right face}, \\
v_2 = -U(t), \quad T_{12} = 0, \quad Q_2 = 0 \text{ on the top surface.}
\end{aligned} \tag{2.18}$$

That is, boundary conditions resulting from the assumed symmetry of deformations are applied to the left and bottom faces, the right face of the block is taken to be traction free, and a prescribed normal velocity and zero tangential tractions are applied on the top face. Note that the initially flat top surface is assumed to stay flat throughout the deformations of the block. All four sides of the block are assumed to be perfectly insulated.

We consider two different sets of initial conditions. First we take

$$\rho(\mathbf{X},0) = 1.0, \quad v_1(\mathbf{X},0) = 0, \quad v_2(\mathbf{X},0) = 0, \quad \theta(\mathbf{X},0) = 0, \tag{2.19}$$

and model a material inhomogeneity/flaw by assuming that

$$\mu = [1 - \epsilon(1 - r^2)^9 \exp(-5r^2)]\{[(1 + bI)^m/(\sqrt{3}I)](1 - \nu\theta)\} \tag{2.20}$$

$$r^2 = (X_1 - X_1^0)^2 + (X_2 - X_2^0)^2. \tag{2.21}$$

That is, the material around the point $\mathbf{X}^0$ is weaker than the surrounding material. In this case we took

$$\begin{aligned}
U(t) &= t/0.005, \quad 0 \le t \le 0.005 \\
&= 1 \qquad\qquad t \ge 0.005.
\end{aligned} \tag{2.22}$$

Another set of initial conditions studied involved perturbing the steady state solution corresponding to

$$v_1 = 0.37x_1, \quad v_2 = -x_2 \tag{2.23}$$

for an average applied strain-rate of 5,000 $\sec^{-1}$ by superposing on it a temperature perturbation given by

$$\Delta\theta = \epsilon(1 - r^2)^9 \exp(-5r^2). \tag{2.24}$$

The velocity field (2.23) and the temperature distribution (2.24) were taken as the initial conditions, and $U(t)$ was set equal to 1.0 for $t \ge 0$. We note that the value of ϵ in eqns (2.20) and (2.24) models, in some sense, the strength of the defect.

We refer the reader to Batra and Liu [1989] for details of seeking an approximate solution of the problem numerically.

III. COMPUTATION AND DISCUSSION OF RESULTS

In order to compute numerical results we assigned following values to various material and geometric parameters.

$$
\begin{gathered}
b = 10{,}000 \text{ sec}, \quad \nu = 0.0222°\mathrm{C}^{-1}, \quad \sigma_o = 333 \text{ MPa}, \\
k = 49.22 W\ \mathrm{m}^{-1}\ °\mathrm{C}^{-1}, \quad c = 473 J\ \mathrm{kg}^{-1}\ °\mathrm{C}^{-1}, \\
\rho_o = 7{,}800 \text{ kg m}^{-3}, \quad B = 128 \text{ GPa}, \quad H = 5 \text{ mm}, \\
v_o = 25\ m \text{ sec}^{-1}, \quad m = 0.025.
\end{gathered} \tag{3.1}
$$

Except for the value of the thermal softening coefficient ν, these values are for a typical hard steel. We assigned a rather large value to ν to reduce the CPU time required to solve the problem. For the values given in (3.1), $\theta_o = 89.6°\mathrm{C}$, the nondimensional melting temperature equals 0.5027, and the average applied strain-rate equals 5,000 sec^{-1}. Figure 1b depicts the effective stress s_e, defined as the left-hand side of eqn (2.12), versus the average strain. The presumed high value of the thermal softening coefficient results in material softening due to the heating of the material overcoming the material hardening due to strain-rate effects right from the beginning.

III.1. *Results with initial temperature perturbation*

Figure 2a depicts the isotherms for the initial temperature distribution (2.24) with $\epsilon = 0.2$ centered around the point (0.0, 0.375). In this case the initial velocity field is assumed to be given by (2.23) and $U(t) = 1.0$ for $t \geq 0$. The peak temperature $\theta_{\max}$ of 0.2 occurs at the center of perturbation. The isotherms look elliptical because of the different scales along the horizontal and vertical axes. Since the boundaries of the block are taken to be perfectly insulated the heat generated due to plastic working raises the temperature of every material point. The isotherms at five different values of the average strain are plotted in Figs. 2b through 2e. These suggest that material points along lines passing through the center of perturbation and inclined at $\pm 45°$ with the horizontal axis are heated more than other particles. Also contours of successively higher temperatures seem to originate from (0.0, 0.375) and propagate in the direction of maximum shearing stress. They get arrested temporarily at the boundaries of the block and when the material at the boundary where these contours meet it gets heated up, these start propagating into the material as if the incident contours were reflected back into the body, the angle of reflection being almost equal to the angle of incidence. This phenomenon becomes more evident from the plots in Fig. 3 of the contours of the second invariant I of the deviatoric strain-rate tensor. In Figs. 3a through 3f the contours of I are plotted at successively higher values of the average strain γ_{avg}. In each case the peak value $I_{\max}$ of I occurs at the point (0.0, 0.375) where the temperature is maximum. At an average strain of 0.04, $I_{\max} = 11.44$ implying thereby that the material surrounding it is deforming at a strain-rate greater than 50,000 sec^{-1}. For $\gamma_{\text{avg}} = 0.04$, $\theta_{\max} = 0.341$ occurs at (0.0, 0.375) and equals 68.2% of the presumed melting temperature of the material. We note that when the temperature perturbation was introduced at (0.0, 0.0) (BATRA & LIU [1989]), $I_{\max}$ and $\theta_{\max}$ at $\gamma_{\text{avg}} = 0.04$ equalled 8.73 and 0.313, respectively. For the problem being currently analyzed, the contours

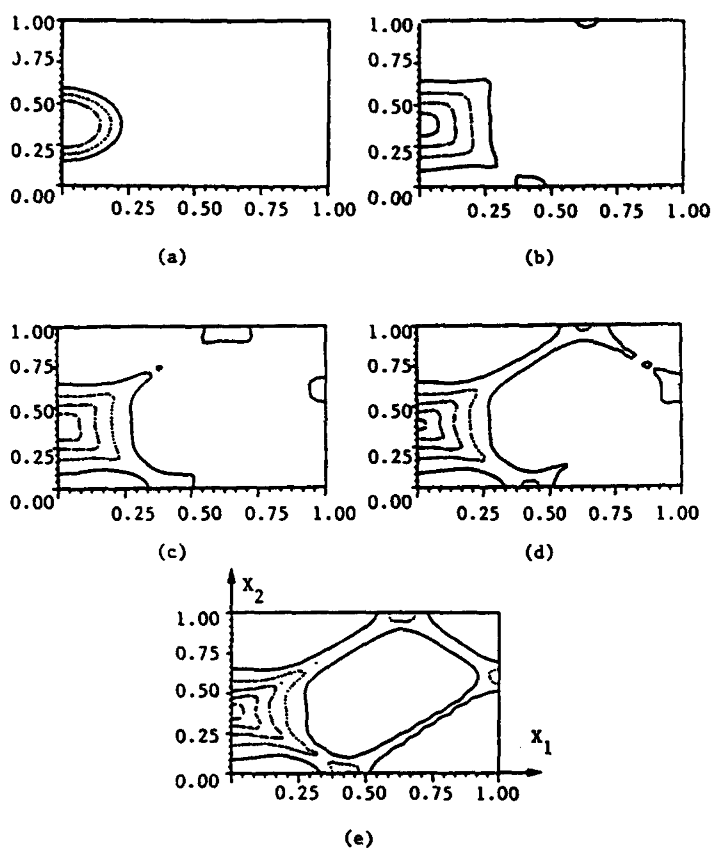

Fig. 2. Isotherms plotted in the reference configuration at different values of the average strain when the material defect is modeled by introducing a temperature perturbation.

(a) $\gamma_{avg} = 0.0$, $\theta_{max} = 0.2$; ___ 0.10, ... 0.125, _ . _ . _ . 0.150.

(b) $\gamma_{avg} = 0.03$, $\theta_{max} = 0.28$; ___ 0.10, ... 0.15, _ . _ . _ . 0.20, _ _ _ _ 0.25.

(c) $\gamma_{avg} = 0.035$, $\theta_{max} = 0.304$; ___ 0.10, ... 0.15, _ . _ . _ . 0.20, _ _ _ _ 0.25.

(d) $\gamma_{avg} = 0.0375$, $\theta_{max} = 0.3213$; ___ 0.10, ... 0.15, _ . _ . _ . 0.20, ---- 0.25, _ _ _ 0.30.

(e) $\gamma_{avg} = 0.04$, $\theta_{max} = 0.341$; ___ 0.10, ... 0.15, _ . _ . _ . 0.20, ---- 0.25, _ _ _ 0.30.

of I originate at (0.0, 0.375) and then fan out along the direction of maximum shearing. There appear to be sources of energy building up at (0.0, 0.375) and three other points on the boundary where the parallelogram through (0.0, 0.375) with adjacent sides making angles of $\pm 45°$ with the horizontal axis intersect it. When there is sufficient energy built up at these points contours of successively higher values of I originate from these points and propagate along the direction of maximum shearing stress. Also as the deformation of the block progresses, these contours become narrower implying thereby that severe deformations are localizing into thin bands.

Figures 4a through 4c depict the velocity field in the X_1 and X_2 direction for $\gamma_{avg} = 0.0$, 0.035 and 0.040. The velocity field at $\gamma_{avg} = 0.0$ is a graphical representation of eqns (2.23) and corresponds to a homogeneous deformation of the block. Once the deformation localizes the velocity field within the material adjoining the sides of the parallelogram stated above varies sharply, and it varies almost linearly within the

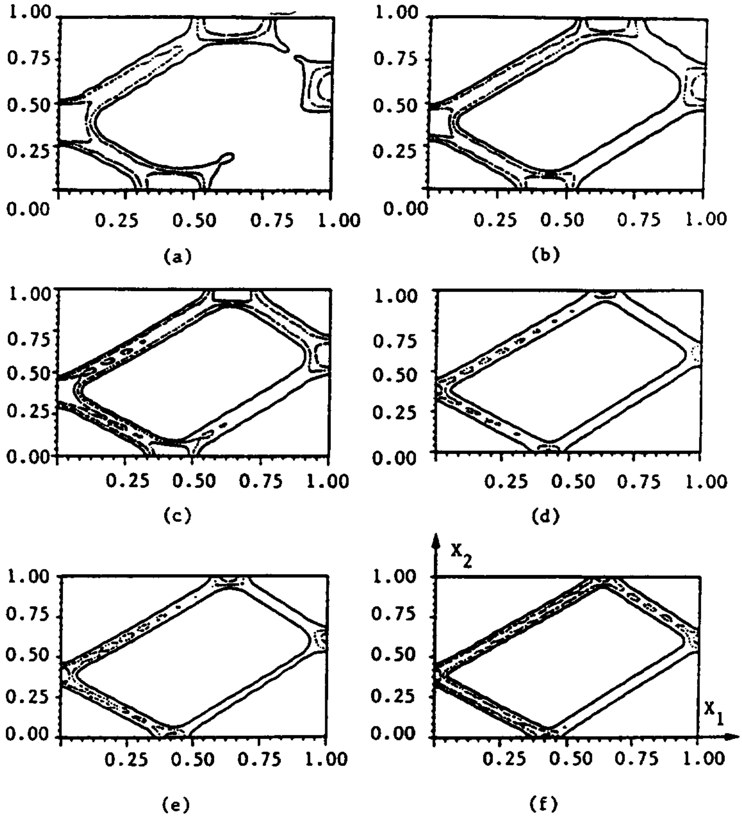

Fig. 3. Contours of the second invariant I of the deviatoric strain-rate tensor at different values of the average strain when the material defect is modeled by a temperature perturbation.
(a) $\gamma_{avg} = 0.02$, $I_{max} = 3.5$; ___ 1.50, ... 1.75, _ . _ . _ . 2.0.
(b) $\gamma_{avg} = 0.025$, $I_{max} = 4.51$; ___ 1.50, ... 2.0, _ . _ . _ . 2.5.
(c) $\gamma_{avg} = 0.030$, $I_{max} = 4.83$; ___ 1.50, ... 2.0, _ . _ . _ . 2.5.
(d) $\gamma_{avg} = 0.035$, $I_{max} = 7.91$; ___ 2.50, ... 3.75, _ . _ . _ . 5.0.
(e) $\gamma_{avg} = 0.0375$, $I_{max} = 10.51$; ___ 2.50, ... 5.0, _ . _ . _ . 7.5.
(f) $\gamma_{avg} = 0.040$, $I_{max} = 11.44$; ___ 2.50, ... 5.0, _ . _ . _ . 7.5.

remainder of the material. This contrast between the velocity field in separate regions becomes sharper (e.g. see Fig. 4c) as the deformation becomes more localized.

The variation of the effective stress s_e, defined as being equal to the left-hand side of eqn (2.12), within the block at $\gamma_{avg} = 0.0$, 0.035, 0.0375 and 0.040 is plotted in Figs. 5a through 5d. Initially the effective stress is lower within the material surrounding the center of temperature perturbation because it is computed from the prescribed velocity and temperature fields. Since s_e satisfies eqn (2.12), the initially higher temperature around (0.0, 0.375) reduces the flow stress needed there to deform the material plastically. Even though the values of both the temperature and I are higher within the band as compared to those in the surrounding material, the effect of thermal softening exceeds the material hardening due to strain-rate effects, and the effective stress within the band is lower than that in the rest of the material. The plots of s_e and the velocity field suggest that the band first forms along the shorter side of the parallelogram that passes through the center of the temperature bump. Also the magnitude of the deformation within the band along the four sides of the parallelogram is not the same.

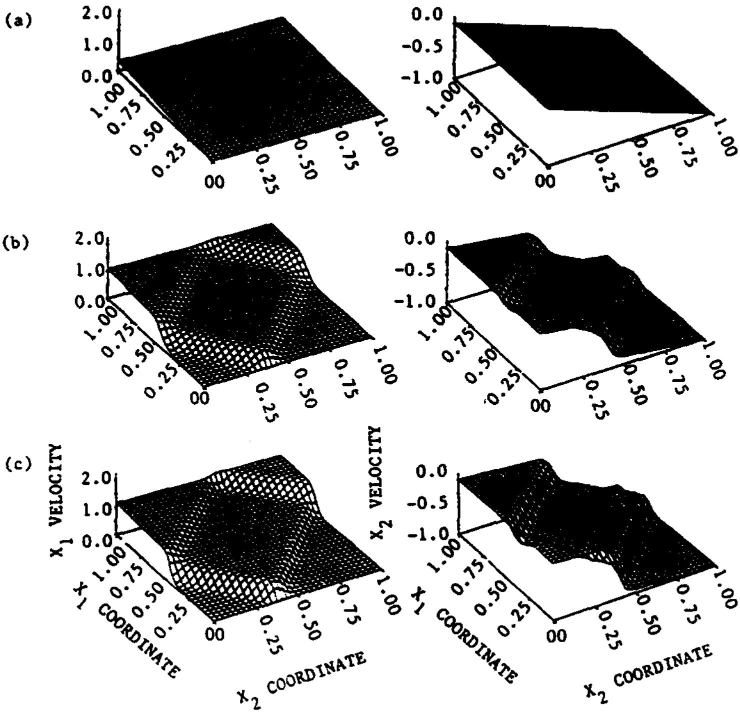

Fig. 4. Velocity field within the block at different values of the average strain with the material defect modeled by a temperature perturabation, (a) $\gamma_{avg} = 0.0$, (b) $\gamma_{avg} = 0.035$, (c) $\gamma_{avg} = 0.040$.

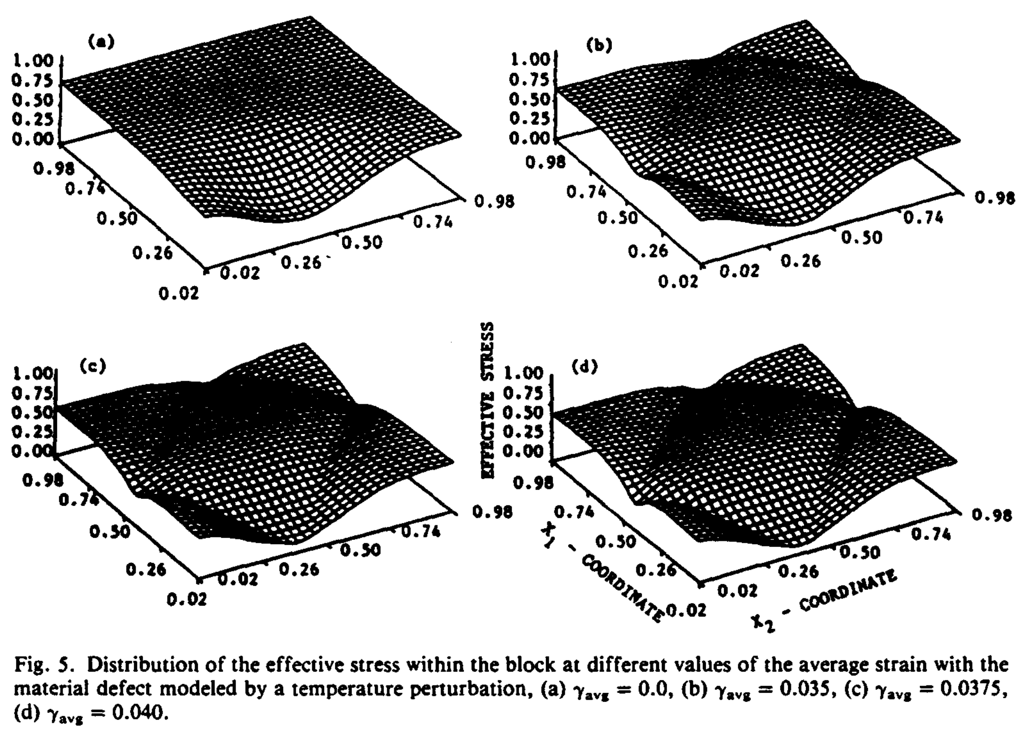

Fig. 5. Distribution of the effective stress within the block at different values of the average strain with the material defect modeled by a temperature perturbation, (a) $\gamma_{avg} = 0.0$, (b) $\gamma_{avg} = 0.035$, (c) $\gamma_{avg} = 0.0375$, (d) $\gamma_{avg} = 0.040$.

III.2 *Results with material inhomogeneity modeled by a weak material*

We now assume that the initial velocity field is given by (2.23), $\theta(X,0) = 0$, and the material parameter μ is represented by eqn (2.20) with $\epsilon = 0.1$. That is, the material surrounding the point (0.0, 0.375) is weaker than the rest of the material. In Fig. 6 we have plotted the contours of I and θ at different values of γ_{avg}. A comparison of these results with those in Figs. 2 and 3 reveals that the pattern of the shear band development is identical to that when the material defect was modeled by a temperature perturbation. In this case it takes a little longer for the shear band to form and the maximum value 10.76 computed for I is comparable to that (11.44) obtained for the temperature perturbation. However, the maximum temperature rise of 0.141 computed with the temperature perturbation is lower than the maximum temperature change of 0.2435 obtained in this case. This is to be expected since with the material defect modeled by a weak material a band forms at a higher value of the average strain.

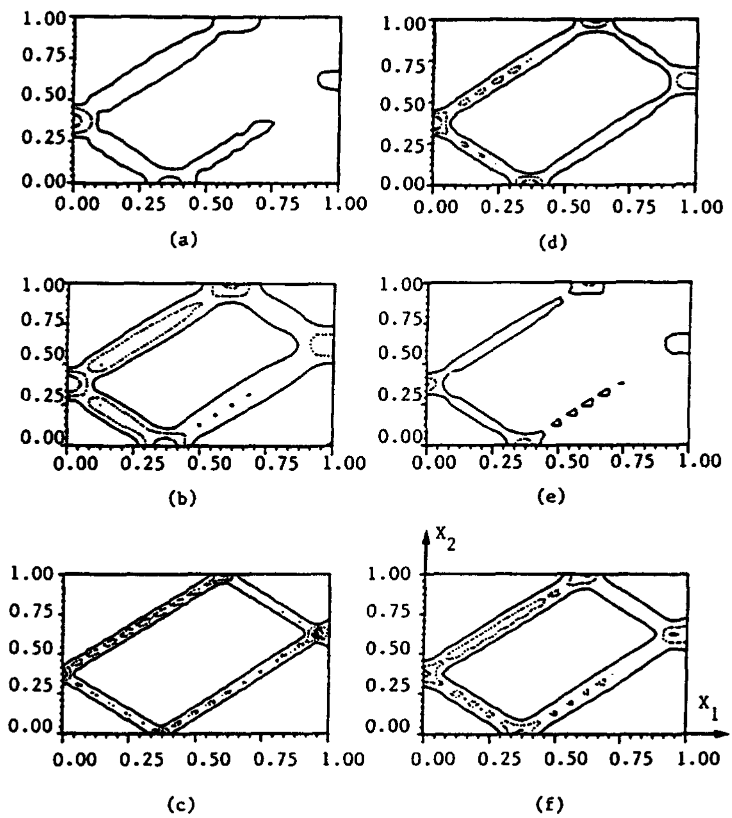

Fig. 6. Contours of the second invariant I of the deviatoric strain-rate tensor and the temperature change θ at different values of the average strain when the material defect is modeled by lowering the flow stress of the material at (0.0, 0.375) by 10%.

(a) $\gamma_{avg} = 0.02$, $I_{max} = 1.938$; ___ 1.2, ... 1.5, _._._._. 1.8.
(b) $\gamma_{avg} = 0.035$, $I_{max} = 3.297$; ___ 1.5, ... 2.0, _._._._. 2.5.
(c) $\gamma_{avg} = 0.0545$, $I_{max} = 10.757$; ___ 2.5, ... 5.0, _._._._. 7.5, _ _ _ _ 10.0.
(d) $\gamma_{avg} = 0.035$, $\theta_{max} = 0.0943$; ___ 0.06, ... 0.07, _._._._. 0.08.
(e) $\gamma_{avg} = 0.045$, $\theta_{max} = 0.154$; ___ 0.10, ... 0.125, _._._._. 0.150.
(f) $\gamma_{avg} = 0.0545$, $\theta_{max} = 0.2435$; ___ 0.10, ... 0.15, _._._._. 0.20.

The plots of the velocity field and the effective stress look similar to those shown in Figs. 4 and 5, and are therefore omitted.

The determination of the equivalent amplitudes of the temperature perturbation and the weakness in the material parameter μ in the sense that the two will result in the formation of the shear band at the same value of the average strain is laborious and has not been attempted here.

III.3 *Effect of the reduction in the strength of the weak material*

For the one-dimensional problem Batra [1988] found that the temperature perturbation with the higher amplitude hastened the initiation of the shear band. Here we examine the effect of introducing near the center of the block a weak material with flow stress reduced by either 5% or 10%. In each case the initial velocity field given by eqn (2.23) was assumed. Figures 7 and 8 show, respectively, the contours of I and θ for the two cases at various values of the average strain. As expected, the existence of a stronger defect enhances the initiation and development of the shear band. In each case the band forms essentially along the main diagonal, the slight offset is possibly due to the singular nature of the deformations near the top right corner. When the reduction in the flow stress of the material near the center is small the singularity in the deformations near the top right corner may cause a shear band to initiate from this point. With the 5% reduction in the flow stress, I_{max} at $\gamma_{avg} = 0.06$ equals 5.32; and it equals 17.79 for the same value of γ_{avg} but with a 10% reduction in the flow stress. The higher value of I increases the temperature of the material within the band faster which, in turn, reduces the effective stress required to deform the material plastically. The cumulative effect builds upon itself and enhances the growth of the shear band. Whereas a shear band has practically formed at $\gamma_{avg} = 0.06$ for the 10% reduction in the flow stress, it forms at $\gamma_{avg} = 0.0825$ when the flow stress for the material near the center is reduced by 5%. The maximum temperature computed in the two cases equals 0.343 and 0.398, respectively. We note that, except for the delay in the formation of the shear band with the 5% reduction in strength, the results for I and θ, as well as those for the velocity field and the effective stress field, are similar in the two cases.

A comparison of these results with those reported by Batra and Liu [1989] who introduced the temperature perturbation (2.24) with $\epsilon = 0.2$ at the center of the specimen reveals that the results agree qualitatively with each other. With the temperature perturbation the maximum values of I and the temperature rise $\Delta\theta$ at $\gamma_{avg} = 0.059$ were computed to be 20.7 and 0.249, respectively. At $\gamma_{avg} = 0.06$ and with a 10% reduction in the flow stress at the center of the block, I_{max} and $\Delta\theta_{max}$ equal 17.79 and 0.308, respectively. And these equal 5.324 and 0.167, respectively, with a 5% reduction in the flow stress.

III.4 *Results with zero initial conditions*

The results presented above were obtained by perturbing a steady state solution. We now examine the effect of initial conditions, if any, on the initiation and growth of a shear band. Figure 9 depicts the contours of the second invariant I of the deviatoric strain-rate tensor and the temperature rise when zero initial conditions (i.e., those given by eqn (2.19)), and the boundary velocity field $U(t)$ defined by (2.22) were applied. Also, in this case the thermal conductivity was set equal to zero. The material defect

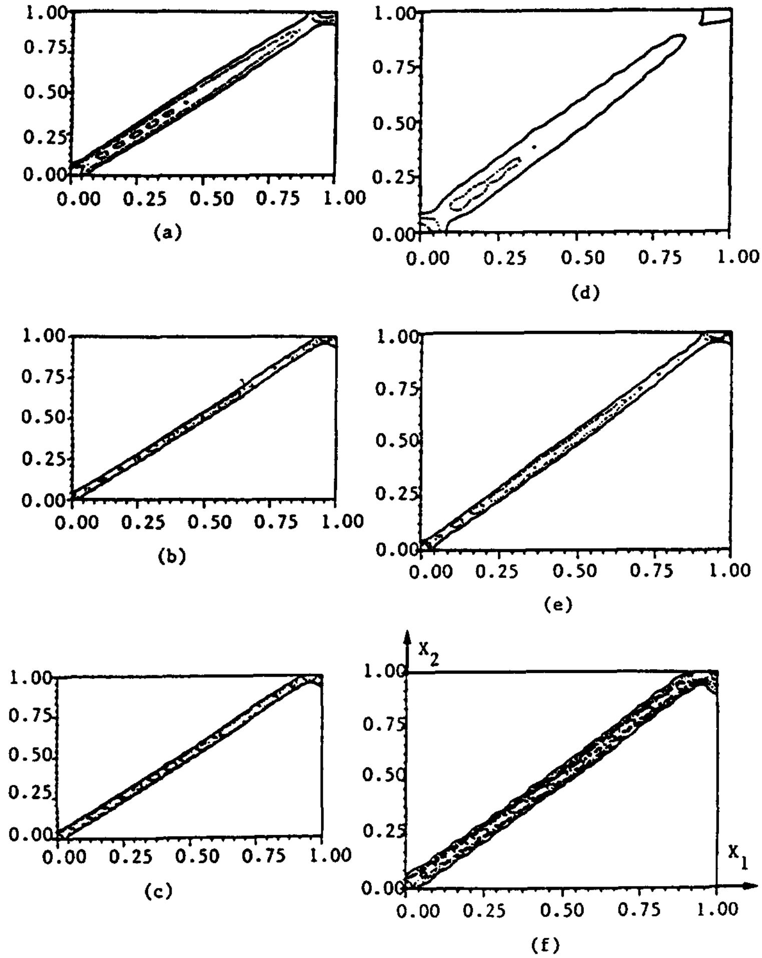

Fig. 7. Contours of the second invariant I of the deviatoric strain-rate tensor at different values of the average strain when the material defect is modeled by reducing the flow stress of the material near the center of the block by either 10% (Figs. 7a–7c) or 5% (Figs. 7d–7f).

(a) $\gamma_{avg} = 0.04$, $I_{max} = 4.21$; ___ 1.5, ... 2.0, _._._. 2.5.
(b) $\gamma_{avg} = 0.06$, $I_{max} = 17.79$; ___ 5.0, ... 7.5, _._._. 10.0.
(c) $\gamma_{avg} = 0.064$, $I_{max} = 22.08$; ___ 5.0, ... 7.5, _._._. 10.0.
(d) $\gamma_{avg} = 0.04$, $I_{max} = 2.0$; ___ 1.25, ... 1.50, _._._. 1.75.
(e) $\gamma_{avg} = 0.06$, $I_{max} = 5.324$; ___ 2.5, ... 3.0, _._._. 3.5.
(f) $\gamma_{avg} = 0.0825$, $I_{max} = 19.04$; ___ 2.5, ... 5.0, _._._. 7.5, _ _ _ _ 10.0.

was modeled by eqn (2.20) with $\epsilon = 0.1$ and $\mathbf{X}^0 = (0.0, 0.375)$, viz, the flow stress of the material surrounding the point (0.0, 0.375), was presumed to be lower than that of the remaining material. A comparison of these results with those shown in Fig. 6 shows that the precise values of initial conditions do not affect the qualitative nature of results. However, quantitatively the results are affected by the choice of initial data. As expected, the values of I_{max} computed for the same value of γ_{avg} is higher when the steady state solution is taken as the initial data as compared to that computed with zero initial conditions. One reason for this difference is that, in both cases, γ_{avg} is

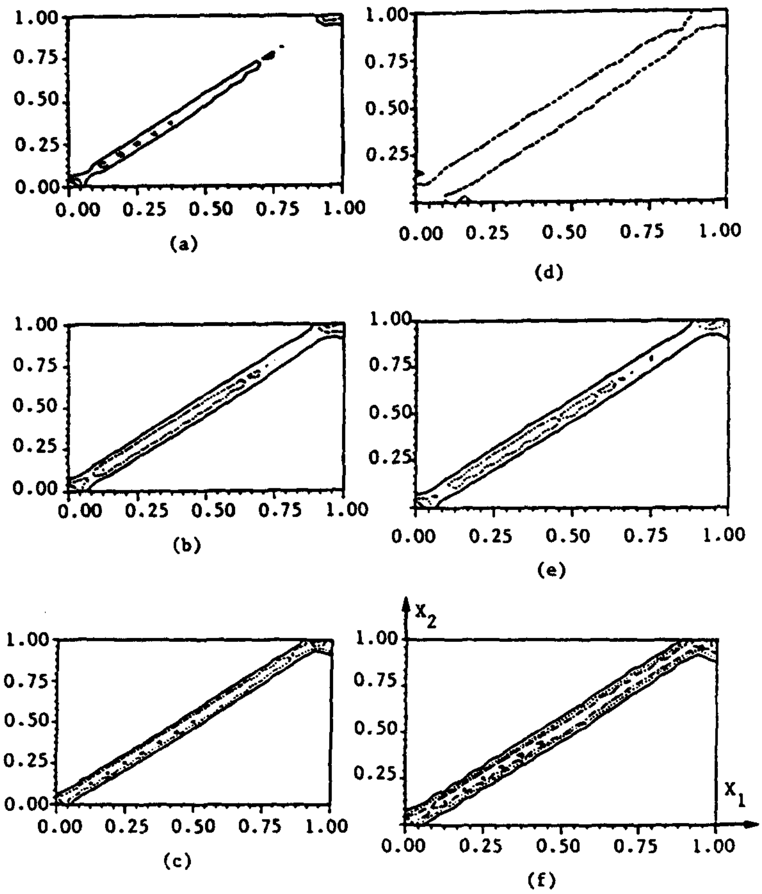

Fig. 8. Contours of the temperature rise θ at different values of the average strain when the material defect is modeled by reducing the flow stress of the material near the center of the block by either 10% (Figs. 8a–8c) or 5% (Figs. 8d–8f).

(a) $\gamma_{avg} = 0.045$, $\theta_{max} = 0.149$; ___ 0.10, ... 0.1125, _._._. 0.1250.
(b) $\gamma_{avg} = 0.05$, $\theta_{max} = 0.192$; ___ 0.10, ... 0.125, _._._. 0.150.
(c) $\gamma_{avg} = 0.063$, $\theta_{max} = 0.343$; ___ 0.15, ... 0.20, _._._. 0.25, _ _ _ _ 0.30.
(d) $\gamma_{avg} = 0.045$, $\theta_{max} = 0.099$; _ _ _ 0.08.
(e) $\gamma_{avg} = 0.065$, $\theta_{max} = 0.2113$; ___ 0.125, ... 0.150, _._._. 0.175.
(f) $\gamma_{avg} = 0.0825$, $\theta_{max} = 0.3984$; ___ 0.15, ... 0.20, _._._. 0.25, _ _ _ _ 0.30.

taken to be zero at time $t = 0$. The difference is reduced somewhat because of neglecting the heat transfer due to conduction. Setting $k = 0$ should result in a slightly higher temperature locally than would be obtained if k were positive. The higher temperature softens the material more which, in turn, results in higher values of I. What effect the thermal conductivity has on the computed results has not yet been ascertained. For the one-dimensional simple shearing problem, BATRA [1987b] used a constitutive relation similar to eqn (2.4) and found that the thermal conductivity had very little effect on the initiation of the shear band. However, MERZER [1983] used BODNER and PARTOM's [1975] constitutive relation and found that the thermal conductivity significantly affects the width of the shear band.

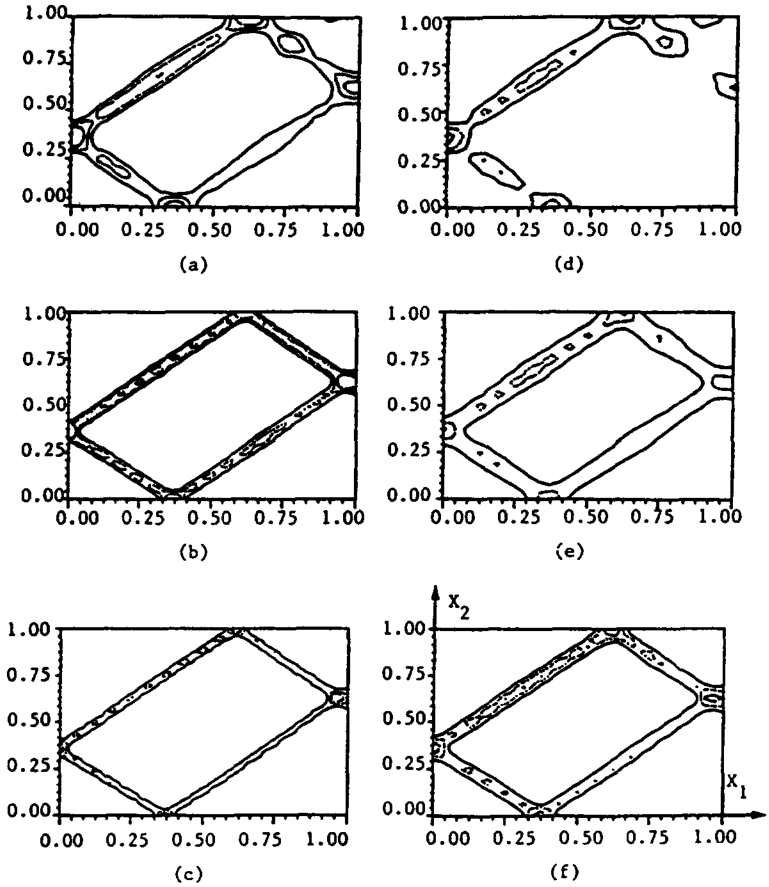

Fig. 9. Contours of the second invariant I of the deviatoric strain-rate tensor and the temperature change θ at different values of the average strain when the material defect is modeled by reducing the flow stress of the material by 10%, taking zero initial conditions and setting the thermal conductivity $k = 0$.

(a) $\gamma_{avg} = 0.035$, $I_{max} = 2.39$; ___ 1.25, ... 1.50, _._._. 1.75.
(b) $\gamma_{avg} = 0.055$, $I_{max} = 9.68$; ___ 2.50, ... 3.75, _._._. 5.00.
(c) $\gamma_{avg} = 0.065$, $I_{max} = 12.47$; ___ 5.0, ... 7.5, _._._. 10.0.
(d) $\gamma_{avg} = 0.04$, $\theta_{max} = 0.1$; ___ 0.075, ... 0.085, _._._. 0.095.
(e) $\gamma_{avg} = 0.055$, $\theta_{max} = 0.211$; ___ 0.10, ... 0.15, _._._. 0.20.
(f) $\gamma_{avg} = 0.065$, $\theta_{max} = 0.310$; ___ 0.15, ... 0.20, _._._. 0.25.

III.5 *Material damage as a softening mechanism*

The results presented thus far have considered the material softening caused by the rise in its temperature. Another possible softening mechanism is the nucleation, coalescence and growth of voids and/or cracks in the body. One way to model this is to introduce an internal parameter ϕ whose rate of evolution $\dot{\phi}$ is a function of the history of deformation and/or plastic working. If we assume that ϕ is a function of the plastic working and the material softening caused by ϕ can be adequately represented by lowering the flow stress by $(1 - \hat{v}\phi)$ where $\hat{v}$ is a material parameter, then we may write

$$\dot{\phi} = \lambda(1 - \hat{v}\phi)\tilde{D}_{ij}\tilde{D}_{ij}(1 + bI)^m/(\rho_r\sqrt{3}I), \tag{4.2}$$

$$\sigma_{ij} = -\beta(\rho - 1)\delta_{ij} + (1/\sqrt{3}I)(1 + bI)^m(1 - \hat{v}\phi)D_{ij}, \tag{4.3}$$

where λ is a constant. In this case the results of section 4.4 may be thought of as representing the dynamic development of an adiabatic shear band in plane strain compression of a viscoplastic block when the material softening mechanism is the internal damage caused by plastic working.

IV. CONCLUSIONS

The development of a shear band in plane strain compression of a block made of a thermally softening viscoplastic material being deformed at an overall strain-rate of 5,000 $\sec^{-1}$ has been studied. The results computed when the material defect is modeled by perturbing the steady-state solution for a homogeneous body (a) with a superimposed temperature bump, and (b) with the introduction of a weaker material agree with each other qualitatively. The qualitative nature of the results remains unchanged even when zero initial conditions are assumed and the transient problem solved.

When the material defect is on the vertical axis of symmetry and away from the center of the block, a shear band initiates from the site of the defect, it propagates along the direction of maximum shearing and is reflected back from the boundaries, the angle of reflection being nearly equal to the angle of incidence. The shear stress within the band is considerably lower than that in the surrounding material. The eventual development of the band along the sides of the parallelogram divides the block into five regions. The velocity field within each region varies linearly and sharp gradients in the velocity field occur at the sides of the parallelogram.

We add that the conclusions drawn above are strictly valid for the constitutive model used herein. However, similar results were obtained by NEEDLEMAN [1989], LEMONDS and NEEDLEMAN [1986a,1986b] and ANAND *et al.* [1988] who used different constitutive relations and the latter two papers ignored the effect of inertia forces. Possibly sharper results could be obtained by using a finer mesh and/or a different space of trial solutions and test functions.

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