Pergamon

International Journal of Plasticity, Vol. 12, No. 1, pp. 29–33, 1996

0749-6419/96 $15.00+.00

0749-6419(95)00042-9

RESEARCH NOTE

A COMPARISON OF 1-D AND 3-D SIMULATIONS OF THE TWISTING OF A THERMOVISCOPLASTIC TUBE

R.C. Batra,[1] C. Adulla[2] and T.W. Wright[3]

[1]Virginia Polytechnic Institute and State University, Blacksburg, VA 24061–0219, U.S.A.
[2]443 Tobin Dr., Apt. #205, Inkster, MI 48141, U.S.A.
[3]U.S. Army Research Laboratory, Aberdeen Proving Ground, MD 21005–5006, U.S.A.

(*Received in final revised form* 20 *April* 1995)

Several investigators (e.g. Wright & Walter [1987], Shawki & Clifton [1989] and Batra & Kim [1992]) have modeled the torsional deformations of a thin tubular steel specimen by using a 1-D (one-dimensional) model in which the material is assumed to undergo simple shearing deformations. Here we address the question of how good the predictions of the average shear strain at which a shear band initiates from the 1-D analysis are as compared to the 3-D analysis.

We assume that the material of the tube exhibits strain-hardening, strain-rate hardening and thermal softening. The governing equations obtained by substituting the constitutive relations into the equations expressing the balance of mass, linear momentum and internal energy are highly coupled and nonlinear and are not easily amenable to analysis. These equations subjected to suitable initial and boundary conditions are solved numerically by the finite element method using the large scale explicit finite element code DYNA3D (Whirley & Hallquist, 1991). It is assumed that all of the plastic work is converted into heat. The thermomechanical response of the 4340 steel, of which the tube is made, is modeled by the Johnson–Cook (Johnson & Cook [1983]) relation, viz.

$$\sigma_m = \left(792.2 + 509.5(\epsilon_p)^{0.26}\right)\left(1 + 0.014\ln(\dot{\epsilon}_p/\dot{\epsilon}_0)\right)\left(1 - T^{1.03}\right)\ \text{MPa} \tag{1}$$

where σ_m is the effective stress, ϵ_p the effective plastic strain, $\dot{\epsilon}_p$ the effective plastic strain-rate, $\dot{\epsilon}_0$ the reference strain-rate of $1/s$ and T the homologous temperature, defined as $(\theta - \theta_0)/(\theta_m - \theta_0)$, of the material particle. Here θ equals the present temperature of a material particle, θ_0 the ambient temperature, and $\theta_{\rm m}$ the melting temperature of the material. The values of material parameters used in (1) are taken from Rajendran's report [1992].

With the inner radius of the tube kept fixed, its outer radius is assumed to vary so as to result in a sinusoidal variation in the tube thickness w:

$$\frac{w(x_3)}{0.76\ \text{mm}} = 1 + 0.04\left(\cos\frac{2\pi x_3}{2.5} - 1\right), \quad 0 \le x_3 \le 2.5\ \text{mm}.$$

Thus the minimum thickness of 0.7 mm occurs at the mid-section $x_3 = 1.25$ mm of the tube; the longitudinal section of the tube is shown in Fig. 1. The tube is twisted by applying an equal and opposite tangential velocity at its two end faces so as to induce an average shear strain-rate of 5000/s; the end surfaces are constrained from moving in the axial direction. The lateral surfaces of the tube are taken to be traction free, and all boundary surfaces are presumed to be thermally insulated.

Figure 2 depicts the 1-D and the 3-D finite element models used. In the 3-D model there are 100 elements in the circumferential direction but only five elements in the x_2-direction are used in the 1-D model; actually one element in the x_2-direction should be sufficient for the 1-D model. When computing results by DYNA3D for the 1-D model, all bounding surfaces except the top and the bottom ones are taken to be traction free, displacements u_1 and u_3 at all nodes are constrained to be zero, and $-v_2(x_1,x_2,0,t) = v_2(x_1,x_2,L,t) = \omega r_m$ are prescribed where ω equals the angular speed at which the tube is twisted, r_m the mean radius of an end surface of the tube, v_2 the velocity of a point in the x_2-direction and L the axial length of the tube. The constraints $u_1 = u_3 = 0$ at all nodes and hence everywhere in the body imply that the thickness of the specimen (i.e. dimension in x_1-direction) and its height (i.e. dimension in x_3-direction) do not change. In the simple shearing problem one also assumes that u_2 is a function of x_3 and t and all components of the stress tensor except σ_{32} vanish; here it was found to be the case. Thus the state of deformations and stresses for the 1-D problem studied herein matches with that assumed in the simple shearing problem. All bounding surfaces are taken to be thermally insulated as in the 3-D problem. The computed temperature field for the l-D problem was found to be independent of x_1 and x_2.

The results for the 3-D problem could not be compared with those for the simple shearing problem obtained by, say, Batra and Kim [1992] since assumptions made in the development of DYNA3D and the code developed by Batra and Kim are different.

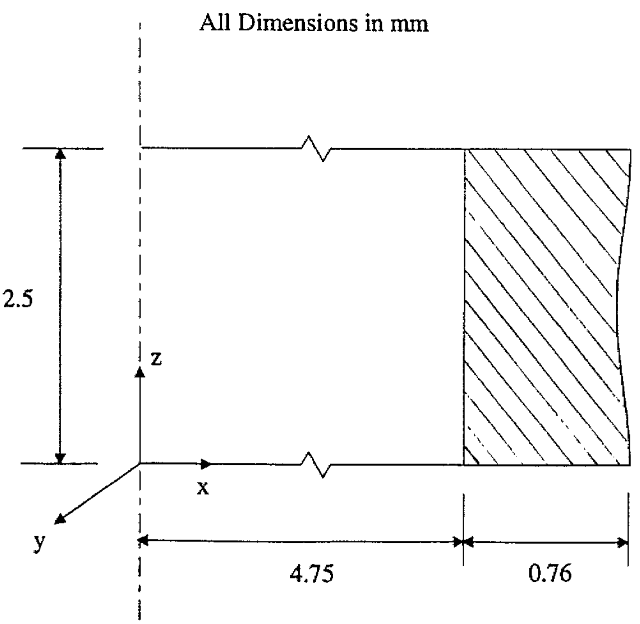
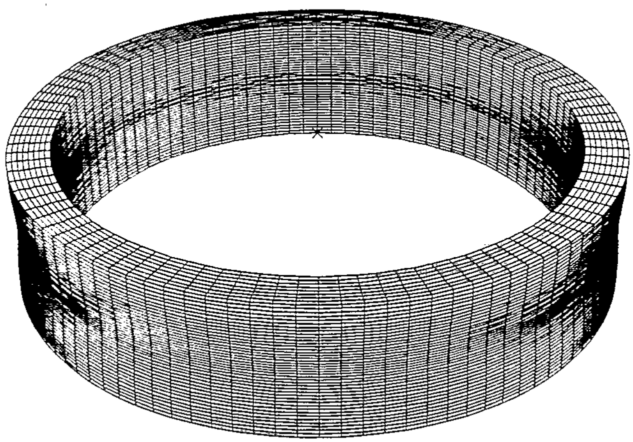
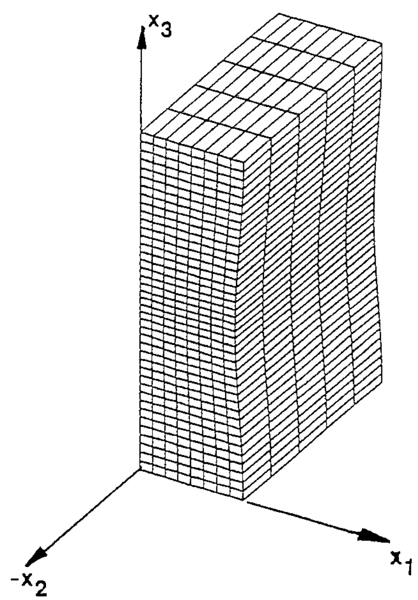


Fig. 1. Tube geometry.

For example, DYNA3D employs 1-point integration rule, considers displacements and temperatures at nodes as primary variables, employs explicit integration and a lumped mass matrix but Batra and Kim regard velocity, temperature, shear stress, and an internal variable (here the effective plastic strain-rate) at node points as unknowns and use the consistent mass matrix and an implicit integration method. The quantitative differences, if any, between the two sets of results could be attributed to different assumptions in the two codes.



Three-D Model



One-D Model

Fig. 2. 3-D and 1-D Finite Element Models used.

As is vivid from the results plotted in Fig. 3, the computed average shear strain at the initiation of the shear band, defined as the one at which the load (torque for the 3-D problem and the shearing force for the 1-D problem) required to deform the tube begins to drop sharply, is higher for the 1-D model as compared to that for the 3-D model. We note that, in the interpretation of their experimental results, Marchand and Duffy [1988] also regard the shear band to initiate when the torque required to deform the tube suddenly drops. In order to investigate the effect of the inner radius of the tube, results were also computed by keeping the tube thickness fixed but increasing the inner radius to first twice and then four times the initial value of 4.75 mm. The value of the tangential velocity applied at the ends was adjusted so as to obtain the average shear strain-rate of 5000/s, and the finite element mesh used was coarse to save on the computational resources. The coarse mesh for an inner radius of 4.75 mm is shown in Fig. 2; the fine mesh has approximately twice the number of elements in the axial and radial directions and 50% more elements in the circumferential direction. The computed results, plotted in Fig. 3, show that (i) the value of the average shear strain at which the torque begins to drop is essentially the same for the coarse and the fine meshes employed herein for the 3-D model, and (ii) the average shear strain at the initiation of the shear band increases with an increase in the inner radius of the tube and apparently approaches that for the 1-D model as the inner radius of the tube becomes infinity. This is to be expected since with a decrease in the value of wall thickness/inner radius of the tube, the effect of curvature of the tube diminishes and its deformations closely resemble those observed in simple shearing. This suggests that the inner radius of the tube affects the average shear strain at which a shear band initiates. We should add that in the 3-D analysis of the problem, the shear stress on the plane of shearing was found to be several orders of magnitude higher than other components of the stress tensor.

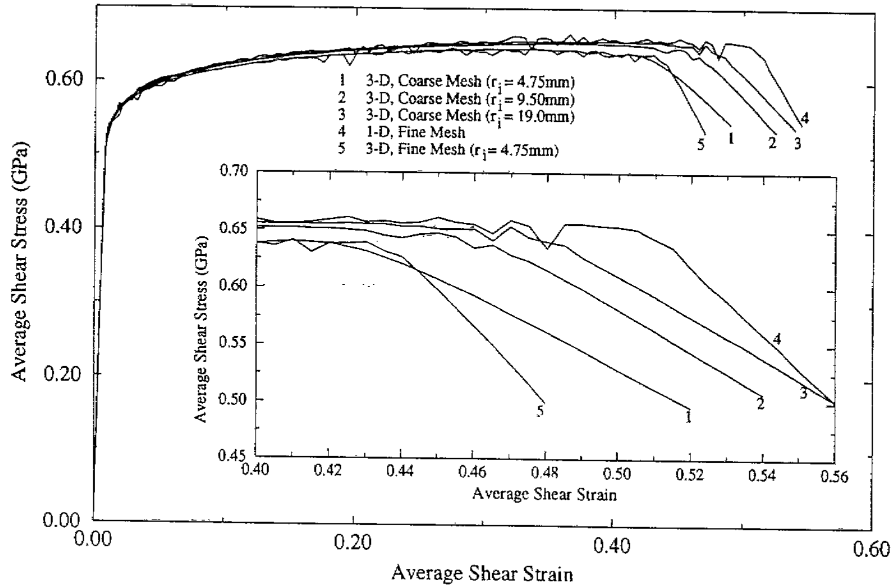


Fig. 3. The average shear stress at the end forces required to deform the tube vs the average shear strain for five different cases.

We note that results for the 1-D and the 3-D model agree with each other qualitatively but the average shear strain at the initiation of the shear band with the 1-D model is found to be higher than that for the 3-D model; the difference between the two decreases with an increase in the inner radius of the tube. Thus the analysis of the simple shearing problem is adequate to obtain qualitative nature of results but one should study the 3-D problem to get quantitative results.

Acknowledgements—This work was supported by the U.S. National Science Foundation grant CMS9411383 and the U.S. Army Research Office grant DAAH.4-95-1-0043 to the Virginia Polytechnic Institute and State University. Some of the computations were performed on the NSF sponsored supercomputer center at the Cornell University, Ithaca, NY.

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