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ELASTO-COMPOSITION WAVES IN INHOMOGENEOUS SOLIDS

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The formulation of the properties of compositional waves (1) opened the now classical field of spinodal decomposition of multicomponent solids. Similarly, there is an increasing awareness that the linear and nonlinear elastic waves are instrumental in the martensitic transformations (2,3). Specifically, it is presently suggested that tweed represents an incommensurate periodic strain ordered state wedged between the high temperature (austenite) and low temperature martensitic phase (4). It is also known that compositional adjustments play an important role in the formation of lower bainite which has surface relief patterns similar to those of martensite. There are thus many reasons why one might wish to explore the characteristics of coupled elasto-compositional waves. In this note we will give a first account of these kinds of waves. We will concentrate on the attentuation of the compositional wave as it has been recently observed in In-Tℓ alloys (5).

The derivation of the coupled elastic and compositional wave equations in a one-dimensional inhomogeneous solid can start with the series expansion of the free energy around a stress free equilibrium configuration,

$$f(c,c',c'',\ldots,e,e',e'',\ldots) - f_0 = \tfrac{1}{2}\bar{f}_{cc}c^2 + \tfrac{1}{2}\bar{f}_{c'c'}c'^2 + \tfrac{1}{2}\bar{f}_{c''c''}c''^2 + \tfrac{1}{2}\bar{f}_{ee}e^2 + \tfrac{1}{2}\bar{f}_{e'e'}e'^2 + \tfrac{1}{2}\bar{f}_{e''e''}e''^2 + \bar{f}_{ec}ec + \tfrac{1}{2}\bar{f}_{e'e'c}e'^2c + \bar{f}_{e''c}e''c. \tag{1}$$

In this expression c, c' and c" denote the deviation from the average composition, the spatial gradient and curvature of this quantity. Similarly, the quantities e, e' and e" denote the strain, its gradient and curvature. The expansion coefficients of the free energy are the partial derivatives with respect to the appropriate parameter evaluated at the equilibrium configuration. For example, the coefficient $\bar{f}_{c'c'}$ denotes the second partial derivative of the free energy with respect to the composition gradient. Mixed terms of the free energy involving one parameter and its derivatives, e.g., $\bar{f}_{ee''}ee''$, mixed terms involving derivatives of different parameters, e.g., $\bar{f}_{e'c'}e'c'$ and terms vanishing because of the mirror symmetry of the problem, e.g., $\bar{f}_{ec'}ec'$ have been omitted from eqn. (1). Since there is presently no compelling need (6) to include the term $\bar{f}_{e''e''}(e'')^2$ in the theory of premartensitic topological solitons, this term will be neglected as well.

The one-dimensional wave equation of the strain is derived from (7)

$$\rho\ddot{e} = S_1'' - S_2''' + S_3'''' \tag{2}$$

where S_1 is the shear stress, S_2 the dipolar stress, and S_3 represents the curvature stress. These stresses can be derived from the free energy as

$$S_1 = f_e\ ,\quad S_2 = f_{e'}\ ,\quad S_3 = f_{e''}\ . \tag{3}$$

The desired wave equation resulting from the combination of equations (1) through (3) is

$$\rho\ddot{e} = \bar{f}_{ee}e'' + \bar{f}_{ec}c'' + \bar{f}_{e''c}c^{iv} - \bar{f}_{e'e'}e^{iv} - \bar{f}_{e'e'c}(e'c)'''\ . \tag{4}$$

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In this paper we are not concerned with spontaneous compositional fluctuations. It will, therefore, be sufficient to consider a simple extension of Fick's law to describe the coupling of the composition to the strain waves. The constitutive relation

$$J = Dc' + \lambda e' \tag{5}$$

satisfies this requirement. The consequences of the logical extension of this constitutive relation which includes all features of the inhomogeneous state embodied in equation (1) will be described later (9). The quantity λ in equation (5) represents a mechanical transport coefficient A more general form of the constitutive relation (5) can be found in Ref. 8. Combining equation (5) with

$$\dot{c} = J' \tag{6}$$

yields the desired equation describing the temporal and spatial evolution of the composition

$$\dot{c} = Dc'' + \lambda e'' \; . \tag{7}$$

The coupled equations governing the variations of strain and composition can now be stated in terms of the following normalized coordinates and parameters.

$$\begin{gathered} \tau = \omega^* t, \quad \xi = x k_0, \quad \omega^* = c_1^2/D, \quad k_0 = c_1/D, \quad c^2{}_1 = \bar{f}_{ee}/\rho, \\ a_1 = (\bar{f}_{e'e'}/\bar{f}_{ee})k_0^2 \; , \quad a_2 = \bar{f}_{ec}/\bar{f}_{ee} \; , \quad a_3 = (\bar{f}_{e''c}/\bar{f}_{ee})k_0^2 \; , \\ a_4 = (\bar{f}_{e'e'c}/\bar{f}_{ee})k_0^2 \; , \quad a_5 = (\lambda/D)k_0^2 \end{gathered} \tag{8}$$

as

$$\begin{aligned} e_{\tau\tau} &= e_{\xi\xi} + a_2 c_{\xi\xi} + \phi(e,c) \; , \\ c_\tau &= c_{\xi\xi} + a_5\, e_{\xi\xi} \; , \end{aligned} \tag{9}$$

where

$$\phi(e,c) = -a_1\, e_{\xi\xi\xi\xi} + a_3 c_{\xi\xi\xi\xi} - a_4(c\, e_{\xi\xi\xi\xi} + 3c_\xi e_{\xi\xi\xi} + 3c_{\xi\xi}e_{\xi\xi} + c_{\xi\xi\xi}e_\xi) \; ,$$

and the subscripts in equation (9) denote partial derivatives with respect to the normalized coordinates ξ and τ.

A solution of equation (9) for the case $\phi = 0$ can be found by using the ansatz

$$\begin{aligned} e &= e_0 \exp\,(i\mu\tau + \gamma\xi) \; , \\ c &= c_0 \exp\,(i\mu\tau + \gamma\xi) \; , \end{aligned} \tag{10}$$

where

$$\mu = \omega/\omega^* \; , \quad \gamma = k/k_0 \; .$$

This approach leads to the dispersion relation

$$(1 - \eta)\gamma^4 + (\mu^2 - i\mu)\gamma^2 - i\mu^3 = 0 \tag{11}$$

where $\eta = a_2\, a_5$. The solution of (11) is

$$\gamma_{1,2}^2 = \mu\,\frac{-(\mu - i) \pm [(\mu - i)^2 + 4i(1 - \eta)\mu]^{\frac{1}{2}}}{2(1 - \eta)} \; . \tag{12}$$

The subscripts 1 and 2 in Eq. (12) refer to the characteristic wave vector of the elastic and compositional variations, respectively. The attenuation of the compositional field is thus given by $\mathrm{Re}(\gamma_2)$. In the limits of high and low frequencies this attenuation is

$$\begin{aligned} \mathrm{Re}(\gamma_2)|_\infty &= \frac{\eta}{2(1 - \eta)} \; , \quad \mu \gg 1 \; , \\ \mathrm{Re}(\gamma_2)|_0 &= \frac{\eta}{2}\,\mu^2 \; , \quad \mu \ll 1. \end{aligned} \tag{13}$$

The contributions of the interaction terms $\bar{f}_{e''c}$ and $\bar{f}_{e'e'c}$ to the attenuation can be found by treating the term $\phi(e,c)$ as a perturbation. The solutions of the perturbed equations (9) will be characterized by a frequency $\mu' = \mu + \delta\mu$ and a wave vector $\gamma' = \gamma + \delta\gamma$. Substitution of the ansatz

$$c = c_0 \exp(i\mu'\tau + \gamma'\xi) \ , \qquad e = e_0 \exp(i\mu'\tau + \gamma'\xi) \ , \tag{14}$$

into equations (9) yields, after calculations very similar to the ones sketched above, the necessary correction to the attentuation of the compositional field in an inhomogeneous solid.

$$\mathrm{Re}(\delta\gamma_2)|_0 = (-e_0\,\bar{f}_{e'e'} + c_0\,\bar{f}_{e''c} - 8e_0c_0\bar{f}_{e'e'c})\mu^4\,\beta \ , \quad \mu << 1 \tag{15}$$

where

$$\beta = \frac{3}{4}\,\frac{\eta}{(e_0 + a_2c_0)} - \frac{(c_0 + a_5e_0)e_0}{C_0(e_0 + a_2c_0)^2} \ . \tag{16}$$

The low frequency total attenuation of the compositional "wave" is given by the sum

$$\mathrm{Re}(\gamma_2)|_0 + \mathrm{Re}(\delta\gamma_2)|_0 = \frac{\eta}{2}\,\mu^2 + (-e_0\bar{f}_{e'e'} + c_0\bar{f}_{e''c} - 8e_0c_0\bar{f}_{e'e'c})\mu^4\beta \tag{17}$$

The attenuation given by equation (17) is proportional to the damping caused by the interaction of the composition and inhomogeneous strain fields. It can be seen from equation (17) that it includes the well known Gorsky damping (10), $\frac{1}{2}\eta\mu^2$ characteristic of long wave length interactions of these two fields. The recent experiments on premartensitic damping of In-24 at % Tℓ (5) represent the first experimental evidence for short wavelength interactions. Both, the linear damping proportional to $\bar{f}_{e''c}$ and the nonlinear damping proportional to $\bar{f}_{e'e'c}$ have been observed in this alloy. The experimental observations indicate that the nonlinear damping in this alloy is negative, i.e. the quantity $\bar{f}_{e'e'c}$ is positive. Since impurities in metals are usually attracted to incipient boundaries it is interesting to speculate that $\beta > 0$ and the negative sign of the nonlinear damping in In-24 at % Tℓ stems from the contribution of the couple stress in equation (2).

In summary, the damping due to the short wave length interaction of one-dimensional compositional and strain fields has been calculated. It is shown that this damping is caused by the interaction of the composition with the curvature and gradient of the strain field. Experimental support for this concept has also been noted.

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