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Letter to the Editor

# Natural frequencies of thick square plates made of orthotropic, trigonal, monoclinic, hexagonal and triclinic materials

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## 1. Introduction

Exact natural frequencies of thick orthotropic simply supported rectangular plates were obtained by Srinivas and Rao [1]. Batra and Aimmancee [2] have recently pointed out that some in-plane distortional modes of vibration are missing in their solutions and in solutions of other investigators based on the same method (e.g., see Refs. [3–6]). Because of the current interest in nanomaterials which are anisotropic and exhibit less symmetries than an orthotropic material, we provide here the first 10 natural frequencies of orthotropic, trigonal, monoclinic, hexagonal and triclinic thick square plates. These frequencies have been computed by the finite element (FE) code IDEAS with a uniform  $40 \times 40 \times 4$  mesh of 20-node brick elements with four elements in the thickness direction and the consistent mass matrix. Thus computed frequencies are upper bounds for the corresponding analytical frequencies. It was shown in Ref. [7] that, for isotropic materials, this FE mesh gives very accurate values of the frequencies. For a simply supported square orthotropic plate, the presently computed frequencies are found to compare very well with those listed in Table 5 of Ref. [1]. Liew et al. [8], amongst others, have used the three-dimensional elasticity equations and the Ritz method to find natural frequencies of a simply supported thick rectangular isotropic plate.

## 2. Results

A schematic sketch of the problem studied, dimensions of the plate, and the location of the rectangular Cartesian co-ordinate axes used to describe deformations of the plate are shown in

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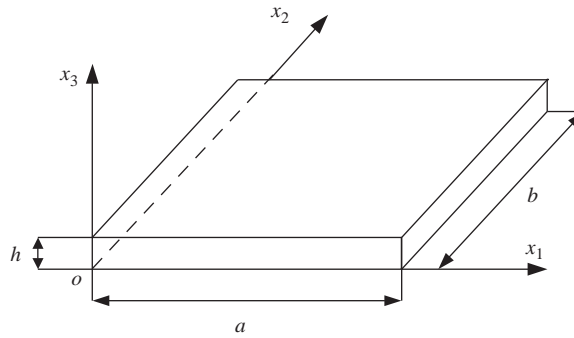


Fig. 1. Schematic sketch of the problem studied.

Fig. 1. Displacements of a point along the  $x_1$ -,  $x_2$ - and  $x_3$ -axis are denoted by  $u_1$ ,  $u_2$  and  $u_3$ , respectively. The components  $\sigma_{ij}$  of the stress tensor are related to those of the infinitesimal strain tensor  $e_{ij}$  by

$$\boldsymbol{\sigma} = \mathbf{D}\mathbf{e}, \tag{1}$$

where

$$\boldsymbol{\sigma} = \{\sigma_{11} \ \sigma_{22} \ \sigma_{33} \ \sigma_{23} \ \sigma_{31} \ \sigma_{12}\}^T, \quad \mathbf{e} = \{e_{11} \ e_{22} \ e_{33} \ 2e_{23} \ 2e_{31} \ 2e_{12}\}, \tag{2}$$

and  $\mathbf{D}$  is the  $6 \times 6$  matrix of elastic constants.

The following boundary conditions are prescribed at a simply supported ( $S$ ) edge, a clamped ( $C$ ) edge, and a free ( $F$ ) edge:

$$\begin{aligned} S: \quad & u_2 = u_3 = 0, \ \sigma_{11} = 0 \quad \text{on } x_1 = 0, a, \\ & u_1 = u_3 = 0, \ \sigma_{22} = 0 \quad \text{on } x_2 = 0, b; \\ C: \quad & u_1 = u_2 = u_3 = 0 \quad \text{on } x_1 = 0, a; \ x_2 = 0, b; \\ F: \quad & \sigma_{11} = \sigma_{12} = \sigma_{13} = 0 \quad \text{on } x_1 = 0, a, \\ & \sigma_{21} = \sigma_{22} = \sigma_{23} = 0 \quad \text{on } x_2 = 0, b. \end{aligned} \tag{3}$$

Of these only boundary conditions at a free edge can be easily realized in a laboratory. Boundary conditions (3)<sub>3</sub> at a clamped edge model well edges welded to rigid supports. Boundary conditions (3)<sub>1,2</sub> at a simply supported edge are difficult to simulate in a laboratory. The usual way of supporting a simply supported edge on rollers or sharp knife edges does not provide boundary conditions (3)<sub>1,2</sub>. However, these have often been used to model simply supported edges; e.g., see Refs. [1–6]. Batra and Geng [10,11] have used rollers to simulate simply supported edges.

Henceforth, we assume that the material principal axes are aligned along the rectangular Cartesian co-ordinate axes.

### 2.1. Orthotropic materials

We assume that the plate material is Aragonite for which [1]

$$[D] = \begin{bmatrix} 160 & 37.3 & 1.72 & 0 & 0 & 0 \\ & 86.87 & 15.72 & 0 & 0 & 0 \\ & & 84.81 & 0 & 0 & 0 \\ & \text{sym.} & & 25.58 & 0 & 0 \\ & & & & 42.68 & 0 \\ & & & & & 42.06 \end{bmatrix} \text{ GPa.} \quad (4)$$

Results presented in Tables 1–4 are in terms of the non-dimensional frequency  $\bar{\omega}$ :

$$\bar{\omega} = \omega h \sqrt{\frac{\rho}{D_{11}}}, \quad (5)$$

where  $\omega$ ,  $h$  and  $\rho$  are, respectively, the dimensional frequency, the plate thickness, and the mass density of the plate material. The aspect ratio,  $\bar{h}$ , of a square plate is defined as

$$\bar{h} = h/a. \quad (6)$$

An asterisk on the value of a frequency signifies that the corresponding mode of vibration is flexural. We have also listed in parentheses frequencies of a simply supported plate from Table 5 of Ref. [1]. It is clear that the presently computed frequencies match very well with their exact values.

For an SSSS square orthotropic plate of aspect ratio  $\bar{h} \geq 0.2$ , the second frequency is that of an in-plane pure distortional mode of vibration with null transverse displacements, and is given by Ref. [2].

$$\bar{\omega}_2 = \pi \bar{h} \sqrt{\frac{D_{66}}{D_{11}}} = 0.32215 \quad (7)$$

for  $\bar{h} = 0.2$ . The 9th frequency, which equals  $2\bar{\omega}_2$ , is also that of an in-plane mode of vibration. The in-plane modes of vibration are also admissible in an SCSC plate, and the 2nd and the 8th frequencies listed in Table 2 for some values of the aspect ratios are for the in-plane modes of vibration. These in-plane modes of vibration are inadmissible in a plate with either all four edges clamped or with two opposite edges clamped and the other two edges traction free.

### 2.2. Triclinic materials

The matrix of elastic constants of a triclinic material can be obtained from that of a transversely isotropic material by appropriate rotations about the  $x_1$ - and the rotated  $x_2$ -axis; e.g., see Ref. [9].

Table 1  
For different aspect ratios, first 10 non-dimensional natural frequencies of an SSSS orthotropic square plate

No.	$\bar{h} = 0.1$	$\bar{h} = 0.2$	$\bar{h} = 0.3$	$\bar{h} = 0.4$	$\bar{h} = 0.5$
1	0.0477* (0.0474)	0.1721* (0.1694)	0.3407* (0.3320)	0.5304* (0.5134)	0.7295* (0.7034)
2	0.1021* (0.1033)	0.3221 [0.3222]	0.4832 [0.4833]	0.6443 [0.6444]	0.8054 [0.8055]
3	0.1227* (0.1188)	0.3221 [0.3222]	0.4832 [0.4833]	0.6443 [0.6444]	0.8054 [0.8055]
4	0.1611 [0.1611]	0.3372* (0.3476)	0.6198*	0.8666 (0.8667)	1.0823 (1.0824)
5	0.1611 [0.1611]	0.4012* (0.3707)	0.6504 (0.6504)	0.9158*	1.2144*
6	0.1721* [0.1694]	0.4338 (0.4338)	0.7318*	1.0756*	1.4214*
7	0.1828* (0.1888)	0.5304* (0.5134)	0.9324*	1.2886	1.4924
8	0.2169 (0.2170)	0.5508*	0.9566*	1.2886	1.6107 [1.6110]
9	0.2327*	0.6443 [0.6444]	0.9664 [0.9666]	1.3409*	1.6107 [1.6110]
10	0.2459* (0.2475)	0.6443 [0.6444]	0.9664 [0.9666]	1.3668*	1.7119

Exact frequencies from Ref. [1] are listed in parentheses, and those from Ref. [2] in square brackets.

Table 2  
For different aspect ratios, first 10 non-dimensional natural frequencies of an SCSC orthotropic square plate

No.	$\bar{h} = 0.1$	$\bar{h} = 0.2$	$\bar{h} = 0.3$	$\bar{h} = 0.4$	$\bar{h} = 0.5$
1	0.0614*	0.2041*	0.3800*	0.5699*	0.7666*
2	0.1281*	0.3221	0.4832	0.6443	0.8054
3	0.1283*	0.3823*	0.6639*	0.9531*	1.2452*
4	0.1611	0.4096*	0.7394*	1.0089	1.2610
5	0.1869*	0.5045	0.7568	1.0821*	1.4272*
6	0.2138*	0.5493*	0.9486*	1.2886	1.6107
7	0.2351*	0.5904*	0.9664	1.3108	1.6383
8	0.2522	0.6443	0.9831	1.3551*	1.7623*
9	0.2667*	0.6554	0.9889*	1.3923*	1.7967*
10	0.2831*	0.6922*	1.0923	1.4559	1.8099*

Table 3  
For different aspect ratios, first 10 non-dimensional natural frequencies of a CCCC orthotropic square plate

No.	$\bar{h} = 0.1$	$\bar{h} = 0.2$	$\bar{h} = 0.3$	$\bar{h} = 0.4$	$\bar{h} = 0.5$
1	0.0804*	0.2563*	0.4593*	0.6674*	0.8755*
2	0.1379*	0.4053*	0.6943*	0.9850*	1.2749*
3	0.1650*	0.4770*	0.8097*	1.0886	1.3606
4	0.2120*	0.5442	0.8164	1.1441*	1.4788*
5	0.2193*	0.5921*	0.9930*	1.3631	1.7040
6	0.2721	0.6011*	1.0015*	1.3965*	1.7937
7	0.2775*	0.6814	1.0222	1.4058*	1.8010*
8	0.2830*	0.7178	1.0765	1.4351	1.8121*
9	0.3145*	0.7469*	1.2354*	1.6683*	1.8740*
10	0.3175*	0.7561*	1.2466*	1.7282*	2.2113

Table 4  
For different aspect ratios, first 10 non-dimensional natural frequencies of a CFCF orthotropic square plate

No.	$\bar{h} = 0.1$	$\bar{h} = 0.2$	$\bar{h} = 0.3$	$\bar{h} = 0.4$	$\bar{h} = 0.5$
1	0.0585*	0.1930*	0.3525*	0.5172*	0.6821*
2	0.0656*	0.2117*	0.3814*	0.5545*	0.7260*
3	0.0948*	0.2974	0.4462	0.5949	0.7436
4	0.1472*	0.2983*	0.5363*	0.7863*	1.0418*
5	0.1487	0.4306*	0.7324*	1.0220	1.2756
6	0.1540*	0.4541*	0.7673	1.0346*	1.3362*
7	0.1560*	0.4661*	0.7743*	1.0982*	1.3506*
8	0.1868*	0.5118	0.8081*	1.1195*	1.4237*
9	0.2392*	0.5350*	0.8574	1.1432	1.4290
10	0.2394*	0.5716	0.8777	1.1701	1.4625

Here we take  $\mathbf{D}$  to be

$$[D] = \begin{bmatrix} 98.84 & 53.92 & 50.78 & -0.10 & 1.05 & 0.03 \\ & 99.19 & 50.87 & -0.18 & 0.55 & 0.03 \\ & & 87.23 & -0.18 & 1.03 & 0.02 \\ & sym. & & 21.14 & 0.07 & 0.25 \\ & & & & 21.10 & -0.04 \\ & & & & & 22.55 \end{bmatrix} \text{ GPa}, \quad (8)$$

and set  $\rho = 7750 \text{ kg/m}^3$ .

The first 10 non-dimensional frequencies of free vibration of a square triclinic plate for different edge conditions are listed in Tables 5–8. For each set of boundary conditions a triclinic plate has several non-flexural, not necessarily in-plane distortional, modes of vibration.

Table 5

For different aspect ratios, first 10 non-dimensional natural frequencies of an SSSS trilinear square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0157	0.0314	0.0471	0.0628	0.0785
2	0.0157	0.0314	0.0471	0.0629	0.0786
3	0.0314	0.0628	0.0942	0.1256	0.1570
4	0.0314	0.0628	0.0942	0.1257	0.1571
5	0.0383*	0.0941	0.1412	0.1883	0.2354
6	0.0471	0.0942	0.1413	0.1884	0.2355
7	0.0471	0.1254	0.1882	0.2509	0.3136
8	0.0627	0.1255	0.1882	0.2509	0.3136
9	0.0627	0.1425*	0.2350	0.3133	0.3915
10	0.0783	0.1567	0.2350	0.3133	0.3915

Table 6

For different aspect ratios, first 10 non-dimensional natural frequencies of a CCCC trilinear square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0742*	0.2451*	0.4484*	0.6588*	0.8691*
2	0.1420*	0.4249*	0.6456	0.8599	1.0731
3	0.1421*	0.4251*	0.7284	0.9729	1.2175
4	0.1914*	0.4312	0.7297	0.9746	1.2197
5	0.2152	0.4847	0.7316*	0.9964	1.2483
6	0.2381*	0.4856	0.7322*	1.0023	1.2573
7	0.2395*	0.4950	0.7454	1.0185	1.2762
8	0.2423*	0.4962	0.7480	1.0205	1.2786
9	0.2423	0.5043	0.7606	1.0369*	1.3085
10	0.2467	0.5053	0.7621	1.0380*	1.3125

Table 7

For different aspect ratios, first 10 non-dimensional natural frequencies of an SCSC trilinear square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0157	0.0314	0.0472	0.0629	0.0787
2	0.0314	0.0628	0.0942	0.1257	0.1571
3	0.0471	0.0942	0.1413	0.1884	0.2355
4	0.0591*	0.1255	0.1882	0.2509	0.3136
5	0.0627	0.1567	0.2350	0.3133	0.3915
6	0.0783	0.1878	0.2817	0.3755	0.4692
7	0.0939	0.1997*	0.3281	0.4373	0.5464
8	0.1084*	0.2188	0.3735*	0.4989	0.6232
9	0.1094	0.2497	0.3744	0.5598*	0.6996
10	0.1248	0.2803	0.4203	0.5601	0.7524*

Table 8  
For different aspect ratios, first 10 non-dimensional natural frequencies of a CFCF triclinc square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0156	0.0312	0.0468	0.0624	0.0780
2	0.0312	0.0623	0.0935	0.1247	0.1558
3	0.0468	0.0937	0.1405	0.1874	0.2342
4	0.0484*	0.1250	0.1875	0.2500	0.3124
5	0.0490*	0.1562	0.2343	0.3124	0.3904
6	0.0625	0.1606*	0.2810	0.3746	0.4681
7	0.0745*	0.1657*	0.2906*	0.4147	0.5234*
8	0.0781	0.1874	0.3074*	0.4366*	0.5455
9	0.0937	0.2184	0.3276	0.4551*	0.6032*
10	0.1092	0.2493	0.3739	0.4983	0.6224

### 2.3. Monoclinic materials

For a monoclinic material, we take

$$[D] = \begin{bmatrix} 86.74 & -8.25 & 27.15 & -3.66 & 0 & 0 \\ & 129.77 & -7.42 & 5.7 & 0 & 0 \\ & & 102.83 & 9.92 & 0 & 0 \\ & sym. & & 38.61 & 0 & 0 \\ & & & & 68.81 & 2.53 \\ & & & & & 29.01 \end{bmatrix} \text{ GPa,} \tag{9}$$

and  $\rho = 2649 \text{ kg/m}^3$ . Computed natural frequencies for different edge conditions are listed in Tables 9–12.

### 2.4. Trigonal materials

The elastic constants for a trigonal material are taken to be

$$[D] = \begin{bmatrix} 86.74 & 6.99 & 11.91 & -17.91 & 0 & 0 \\ & 86.74 & 11.91 & 17.91 & 0 & 0 \\ & & 107.2 & 0 & 0 & 0 \\ & sym. & & 57.94 & 0 & 0 \\ & & & & 57.94 & -17.91 \\ & & & & & 39.88 \end{bmatrix} \text{ GPa,} \tag{10}$$

and  $\rho = 2649 \text{ kg/m}^3$ . Natural frequencies are listed in Tables 13–16.

Table 9

For different aspect ratios, first 10 non-dimensional natural frequencies of an SSSS monoclinic square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0527*	0.1972*	0.4058*	0.6545*	0.9036
2	0.1241*	0.3627	0.5439	0.7251	0.9064
3	0.1424*	0.3628	0.5441	0.7253	0.9299*
4	0.1814	0.4441*	0.8745*	1.2999*	1.6280
5	0.1814	0.4780*	0.8887*	1.3494*	1.7819*
6	0.1971*	0.6539*	0.9979	1.3587*	1.7939*
7	0.2423*	0.6662	1.0855	1.4467	1.8064
8	0.2782*	0.7245	1.0865	1.4472	1.8810*
9	0.3004*	0.7249	1.2129*	1.7281	2.1418
10	0.3211*	0.8124*	1.3003	1.8056*	2.2511

Table 10

For different aspect ratios, first 10 non-dimensional natural frequencies of a CCCC monoclinic square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0993*	0.3382*	0.6405*	0.9694*	1.3091*
2	0.1835*	0.6012*	1.0465*	1.4010	1.7518
3	0.2005*	0.6061*	1.0501	1.5028*	1.9593*
4	0.2633*	0.6994	1.1119*	1.6295*	2.0888
5	0.3133*	0.8000*	1.2602	1.6766	2.1274*
6	0.3393*	0.8408	1.3865	1.8479	2.3086
7	0.03492	0.09244	1.4035*	2.0116*	2.6005*
8	0.03746*	0.9336*	1.5608*	2.1029	2.6146
9	0.3875*	0.9751*	1.5828	2.1942*	2.8229*
10	0.4210	1.0557*	1.7128*	2.4169*	2.8245*

Table 11

For different aspect ratios, first 10 non-dimensional natural frequencies of an SCSC monoclinic square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0830*	0.2779*	0.5185*	0.7253	0.9064
2	0.1397*	0.3628	0.5441	0.7796*	1.0536*
3	0.1814	0.4780*	0.9150	1.3798	1.7197*
4	0.1924*	0.5706*	0.9836*	1.4014*	1.8064
5	0.2345*	0.6948	1.0406*	1.4099*	1.8363*
6	0.2504*	0.7131*	1.0865	1.4472	1.9126*
7	0.3246*	0.7249	1.2285	1.6341	2.0353
8	0.3343*	0.8197	1.2669*	1.8527*	2.3649
9	0.3478	0.8273*	1.4279	1.8995	2.4506*
10	0.3626	0.9132*	1.5261*	2.0471	2.5429*



Table 12

For different aspect ratios, first 10 non-dimensional natural frequencies of a CFCF monoclinic square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0598*	0.2206*	0.4451*	0.6583	0.8241
2	0.0698*	0.2513*	0.4938*	0.7069*	0.9800*
3	0.1178*	0.3303	0.4963	0.7588*	1.0225*
4	0.1580*	0.4052*	0.7708*	1.1655*	1.5061
5	0.1651	0.5366*	0.9017	1.2037	1.5155
6	0.1706*	0.5723*	0.9103	1.2135	1.5666*
7	0.2145*	0.6002	1.0029*	1.4775*	1.8820
8	0.2221*	0.6086	1.0574*	1.5151*	1.9357*
9	0.2945*	0.6940*	1.1339	1.5523*	1.9461*
10	0.2994	0.6999*	1.1957*	1.5876*	2.0238*

Table 13

For different aspect ratios, first 10 non-dimensional natural frequencies of an SSSS trigonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0535*	0.2016*	0.4163*	0.6676*	0.9210*
2	0.1296*	0.3986	0.6022	0.8100	1.0174
3	0.1297*	0.4004	0.6042	0.8142	1.0511*
4	0.1984	0.4557*	0.8219	1.1025	1.3741
5	0.1990	0.4562*	0.8748*	1.3163*	1.7433*
6	0.2018*	0.5607	0.9035*	1.3715*	1.8550*
7	0.2471*	0.6773*	1.2040	1.6078*	2.0053
8	0.2473*	0.7932*	1.2161*	1.6563	2.1255*
9	0.2795	0.7994	1.2558*	1.7133	2.1435
10	0.3131*	0.7996	1.2841	1.7346	2.1626

Table 14

For different aspect ratios, first 10 non-dimensional natural frequencies of a CCCC trigonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0944*	0.3322*	0.6430*	0.9847*	1.3372*
2	0.1841*	0.5987*	1.0756	1.4380	1.8007
3	0.1843*	0.6001*	1.0756	1.4416	1.8087
4	0.2622*	0.7122	1.0848*	1.5789*	2.0575*
5	0.3100*	0.7143	1.0863*	1.5798*	2.0621*
6	0.3138*	0.8190*	1.3823	1.8557	2.3282
7	0.3558	0.9126	1.4486*	2.0794*	2.6007
8	0.3566	0.9343*	1.5837	2.1068	2.6891*
9	0.3791*	0.9466*	1.5908*	2.2255*	2.8286
10	0.3811*	1.0483*	1.6312*	2.3040*	2.8382*

Table 15  
For different aspect ratios, first 10 non-dimensional natural frequencies of an SCSC trigonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0766*	0.2731*	0.5342*	0.8100	1.0174
2	0.1419*	0.4004	0.6042	0.8247*	1.1281*
3	0.1747*	0.4902*	0.9363*	1.3760	1.7262
4	0.1990	0.5682*	1.0043*	1.3835	1.7387
5	0.2333*	0.6818	1.0306	1.4220*	1.9199*
6	0.2545*	0.7040	1.0849*	1.5404*	2.0053
7	0.3063*	0.7444*	1.2040	1.6078	2.0089*
8	0.3355*	0.7996	1.3354*	1.8891	2.3666
9	0.3393	0.8238*	1.4126	1.9353*	2.4422
10	0.3502	0.9139*	1.4681	1.9641	2.5194*

Note: The data with asterisk are the bending frequencies.

Table 16  
For different aspect ratios, first 10 non-dimensional natural frequencies of a CFCF trigonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0595*	0.2137*	0.4125*	0.6132*	0.8005*
2	0.0715*	0.2519*	0.4849*	0.7335*	0.9795*
3	0.1141*	0.3601*	0.5579*	0.7797*	1.0253*
4	0.1558*	0.3883*	0.7241*	1.0792*	1.4338*
5	0.1720*	0.5030*	0.8692*	1.2007*	1.5112
6	0.1787	0.5586*	0.9046	1.2079	1.5169*
7	0.1999*	0.6019	0.9916	1.3196	1.6441
8	0.2190*	0.6433*	1.0021*	1.4197*	1.8099*
9	0.2875*	0.6537*	1.0243*	1.4366*	1.8480*
10	0.2981*	0.6659*	1.1680*	1.6144*	2.0318*

### 2.5. Hexagonal materials

The beryllium crystal belongs to the close-packed hexagonal system. It has an axis of symmetry such that a rotation of the crystal through 60° about that axis brings the space lattice into coincidence with its original configuration. The mass density of beryllium equals 1850 kg/m<sup>3</sup>, and the elastic constants are

$$[D] = \begin{bmatrix} 298.2 & 27.7 & 11.0 & 0 & 0 & 0 \\ & 298.2 & 11.0 & 0 & 0 & 0 \\ & & 340.8 & 0 & 0 & 0 \\ & sym. & & 165.5 & 0 & 0 \\ & & & & 165.5 & 0 \\ & & & & & 135.3 \end{bmatrix} \text{ GPa.} \quad (11)$$

The computed frequencies are listed in Tables 17–20.

Table 17

For different aspect ratios, first 10 non-dimensional natural frequencies of an SSSS hexagonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0555*	0.2076*	0.4264*	0.6857*	0.9681*
2	0.1340*	0.4230	0.6343	0.8453	1.0558
3	0.1340*	0.4230	0.6343	0.8453	1.0558
4	0.2076*	0.4662*	0.8940*	1.1935	1.4898
5	0.2116	0.4662*	0.8940*	1.3599*	1.8379*
6	0.2116	0.5979	0.8961	1.3599*	1.8379*
7	0.2543*	0.6855*	1.2624*	1.6834	2.0983
8	0.2543*	0.8165*	1.2654	1.6834	2.0983
9	0.2991	0.8165*	1.2654	1.7656	2.2003
10	0.3214*	0.8449	1.3273	1.8659*	2.3407

Table 18

For different aspect ratios, first 10 non-dimensional natural frequencies of a CCCC hexagonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0968*	0.3325*	0.6305*	0.9510*	1.2778*
2	0.1878*	0.5960*	1.0639*	1.4856	1.8530
3	0.1878*	0.5960*	1.0639*	1.4856	1.8530
4	0.2660*	0.7449	1.1160	1.5370*	1.9980*
5	0.3157*	0.7449	1.1160	1.5370*	1.9980*
6	0.3183*	0.8081*	1.4089*	1.9088	2.3772
7	0.3727	0.9280*	1.4357	2.0105*	2.5981*
8	0.3727	0.9405*	1.5847*	2.1974	2.7334
9	0.3840*	0.9591	1.6125*	2.2343*	2.8658*
10	0.3840*	1.1045*	1.6542	2.2810*	2.9352*

Table 19

For different aspect ratios, first 10 non-dimensional natural frequencies of an SCSC hexagonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0789*	0.2768*	0.5343*	0.8178*	1.0558
2	0.1459*	0.4230	0.6343	0.8453	1.1128*
3	0.1788*	0.4941*	0.9298*	1.3984*	1.7787
4	0.2116	0.5717*	1.0261*	1.4258*	1.8310
5	0.2386*	0.7148	1.0710	1.4679*	1.8767*
6	0.2607*	0.7360	1.1027	1.4911*	1.9513*
7	0.3118*	0.7475*	1.2654*	1.6834	2.0983
8	0.3412*	0.8285*	1.3326*	1.9334*	2.4603
9	0.3576	0.8449	1.4865	1.9760	2.5297*
10	0.3664*	0.9221*	1.4883*	2.1022	2.5356

Table 20

For different aspect ratios, first 10 non-dimensional natural frequencies of a CFCF hexagonal square plate

No.	$h/a = 0.1$	$h/a = 0.2$	$h/a = 0.3$	$h/a = 0.4$	$h/a = 0.5$
1	0.0615*	0.2191*	0.4264*	0.6538*	0.8877*
2	0.0741*	0.2566*	0.4868*	0.7301*	0.9477
3	0.1184*	0.3796	0.5692	0.7586	0.9714*
4	0.1602*	0.3941*	0.7311*	1.0862*	1.4431*
5	0.1766*	0.5163*	0.9257*	1.2484	1.5582
6	0.1898	0.5628*	0.9374	1.3368*	1.6766
7	0.2063*	0.6254	1.0057*	1.3437	1.7288
8	0.2250*	0.6570*	1.0092	0.3860	1.7338*
9	0.2939*	0.6735	1.0411	1.4546*	1.8961*
10	0.3062*	0.6949	1.1720*	1.6607	2.0698

### 3. Conclusions

For square orthotropic, triclinic, trigonal, monoclinic and hexagonal plates, we have listed the first 10 frequencies for different edge conditions and aspect ratios. The converged frequencies were computed with the 3-dimensional finite element code IDEAS, and are thus approximate solutions of the 3-dimensional linear elasticity equations for free vibrations of a plate. However, for cases where analytical results are available, computed frequencies match very well with the analytical frequencies. For each set of boundary conditions and material symmetry, there are non-flexural modes of vibration. These results can be used for comparison with those obtained from plate theories.

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