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Natural frequencies of thick plates made of orthotropic, monoclinic, and hexagonal materials by a meshless method

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Abstract

We use the first-order shear deformation theory and a meshless method based on radial basis functions in a pseudospectral framework for predicting the free vibration behavior of thick orthotropic, monoclinic and hexagonal plates. The shape parameter is obtained by an optimization procedure. The three translational and two rotational degrees of freedom of a point of the laminate are independently approximated. Through numerical experiments, the capability and efficiency of the radial basis functions—pseudospectral method for eigenvalue problems are demonstrated, and the numerical accuracy and convergence are examined.

1. Introduction

We analyze free vibrations of rectangular plates comprised of three anisotropic materials with a meshless (collocation) technique based on radial basis functions in a pseudospectral framework that uses an optimal shape parameter [1,2]. The first 10 frequencies of thick plates comprised of orthotropic, monoclinic and hexagonal materials are compared with those from Batra et al. [3] who computed frequencies by the finite element method using the computer code IDEAS and a uniform $40 \times 40 \times 4$ mesh of 20-node brick elements with four elements in the thickness direction and the consistent mass matrix.

We use Wendland compact support radial basis functions in the form

$$\phi_\varepsilon(r) = (1 - \varepsilon r)_+^8 (32(\varepsilon r)^3 + 25(\varepsilon r)^2 + 8\varepsilon r + 1), \tag{1}$$

where ε is an (optimal) shape parameter (e.g., see Refs. [1,2]) and r is the Euclidean distance between two distinct points. Exact natural frequencies of thick orthotropic simply supported rectangular plates were obtained by Srinivas and Rao [4]. Batra and Aimmanee [5,6] have pointed out that some inplane distortional

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modes of vibration are missing in their solutions and in solutions of other investigators based on the same method (e.g., see Refs. [7–10]). Because of the current interest in nanomaterials which are anisotropic and exhibit less symmetries than an orthotropic material, we provide here the first 10 natural frequencies of orthotropic, monoclinic and hexagonal thick square plates.

Unlike the previous work of Ferreira and Batra [11], where multiquadrics and user-defined shape parameter were used, the present optimized radial basis functions with compact support—pseudospectral method produces accurate natural frequencies for orthotropic, monoclinic and hexagonal plates. A multiquadratic basis function is given by

$$g_c(r) = (r^2 + c^2)^{1/2} \tag{2}$$

where c is a constant. Whereas ϕ_ε given by Eq. (1) is a polynomial of degree in r on its support, g_c defined by Eq. (2) is not a polynomial in r and is essentially proportional to r. A numerical technique employing higher-order polynomials as basis functions is expected to improve the accuracy of the numerical solution. Another reason for using basis functions given by Eq. (1) is that the numerical algorithm is stable, gives good eigenmodes and converges rapidly with an increase in the number of collocation points. On the other hand, an algorithm using basis functions given by Eq. (2) may be unstable and yield poor eigenmodes. Eigen-solutions computed with Eq. (2) were found to strongly depend upon the value of the shape parameter c. Whereas the shape parameter in Refs. [1,2] was estimated, here techniques presented in Refs. [12,13] are employed to adaptively assign the optimum value to the shape parameter ε. However, in Refs. [12,13] we used multiquadrics and inverse multiquadrics as basis functions. In Ref. [14], we used the cross validation algorithm to find the optimized shape parameter coupled with radial basis functions of compact support given by Eq. (1) to study static deformations and vibrations of a plate made of an isotropic material. Here we extend the concept to study vibrations of anisotropic plates. Because of a large number of non-zero material elasticities for anisotropic materials, the coupling among terms is different from that in an isotropic material. It is thus interesting to investigate if the collocation method employing compactly supported radial basis functions with an adaptively optimized shape parameter will accurately predict the ten lowest frequencies of a thick anisotropic plate.

We note that the use of compactly supported radial basis functions instead of globally supported functions (such as multiquadrics) is motivated and justified by the fact that for the eigenproblem at hand the global functions suffer from too much ill-conditioning and, therefore, are unstable and produce ill-shaped eigenvectors. The compactly supported functions, on the other hand, are stable, produce good eigenmodes and are convergent as we use more nodes. Together with the shape parameter optimization, the use of compactly supported functions provides, in our opinion, the best choice so far for analysis of natural vibrations of plates based on radial basis functions.

Based on a large variety of problems studied, we believe that globally or compactly supported functions work well for static problems, but for eigenproblems one should definitively use compactly supported radial basis functions.

2. Results

2.1. Orthotropic materials

We assume that the plate material is Aragonite for which [1]

$$[D] = \begin{bmatrix} 160 & 37.3 & 1.72 & 0 & 0 & 0 \\ & 86.87 & 1.72 & 0 & 0 & 0 \\ & & 84.81 & 0 & 0 & 0 \\ \text{sym.} & & & 25.58 & 0 & 0 \\ & & & & 42.68 & 0 \\ & & & & 0 & 42.06 \end{bmatrix} \text{GPa.} \tag{3}$$

Table 1
For different aspect ratios, first 10 non-dimensional natural frequencies of a simply supported orthotropic square plate

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
1	Batra et al. [3]	0.0477*	0.1721*	0.3407*	0.5304*	0.7295*
		(0.0474)	(0.1694)	(0.3320)	(0.5134)	(0.7034)
	Present 9 × 9	0.0478	0.1725	0.3406	0.5294	0.7267
	11 × 11	0.0478	0.1724	0.3406	0.5293	0.7266
	15 × 15	0.0478	0.1724	0.3406	0.5293	0.7266
	Ref. [11]	0.0477	0.1725	0.3410	0.5300	0.7273
2	Batra et al. [3]	0.1021*	0.3221	0.4832	0.6443	0.8054
		(0.1033)	[0.3222]	[0.4833]	[0.6444]	[0.8055]
	Present 9 × 9	0.1028	0.3221	0.4831	0.6442	0.8052
	11 × 11	0.1029	0.3221	0.4832	0.6443	0.8053
	15 × 15	0.1029	0.3221	0.4832	0.6443	0.8054
	Ref. [11]	0.1031				
3	Batra et al. [3]	0.1227*	0.3221	0.4832	0.6443	0.8054
		(0.1188)	[0.3222]	[0.4833]	[0.6444]	[0.8055]
	Present 9 × 9	0.1225	0.3221	0.4832	0.6442	0.8053
	11 × 11	0.1228	0.3221	0.4832	0.6443	0.8054
	15 × 15	0.1228	0.3221	0.4832	0.6443	0.8054
	Ref. [11]	0.1232				
4	Batra et al. [3]	0.1611	0.3372*	0.6198*	0.8666	1.0823
		[0.1611]	(0.3476)		(0.8667)	(1.0824)
	Present 9 × 9	0.1610	0.3380	0.6185	0.8786	1.0983
	11 × 11	0.1611	0. 3381	0.6186	0.8782	1.0977
	15 × 15	0.1611	0.3381	0.6186	0.8781	1.0976
	Ref. [11]		0.3387	0.6195		
5	Batra et al. [3]	0.1611	0.4012*	0.6504	0.9158*	1.2144*
		[0.1611]	(0.3707)	(0.6504)		
	Present 9 × 9	0.1611	0.4005	0.6590	0.9102	1.2027
	11 × 11	0.1611	0.4007	0.6586	0.9103	1.2028
	15 × 15	0.1611	0.4007	0.6586	0.9103	1.2027
	Ref. [11]		0.4018		0.9113	1.2039
6	Batra et al. [3]	0.1721*	0.4338	0.7318*	1.0756*	1.4214*
		[0.1694]	(0.4338)			
	Present 9 × 9	0.1729	0.4393	0.7293	1.0694	1.4095
	11 × 11	0.1725	0.4391	0.7295	1.0696	1.4098
	15 × 15	0.1724	0.4390	0.7294	1.0695	1.4097
	Ref. [11]	0.1728		0.7310	1.0714	1.4112
7	Batra et al. [3]	0.1828*	0.5304*	0.9324*	1.2886	1.4924
		(0.1888)	(0.5134)			
	Present 9 × 9	0.1833	0.5300	0.9279	1.2878	1.4991
	11 × 11	0.1843	0.5294	0.9272	1.2884	1.4991
	15 × 15	0.1842	0.5293	0.9270	1.2886	1.4991
	Ref. [11]	0.1728	0.5305	0.9288		
8	Batra et al. [3]	0.2169	0.5508*	0.9566*	1.2886	1.6107
		(0.2170)				[1.6110]
	Present 9 × 9	0.2197	0.5492	0.9484	1.2881	1.6097
	11 × 11	0.2195	0.5510	0.9508	1.2885	1.6105
	15 × 15	0.2195	0.5508	0.9505	1.2886	1.6107
	Ref. [11]		0.5517	0.9514		
9	Batra et al. [3]	0.2327*	0.6443	0.9664	1.3409*	1.6107
			[0.6444]	[0.9666]		[1.6110]
	Present 9 × 9	0.2309	0.6439	0.9658	1.3292	1.6102
	11 × 11	0.2327	0.6442	0.9663	1.3285	1.6106

Table 1 (*continued*)

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
	15 × 15	0.2326	0.6443	0.9664	1.3283	1.6107
	Ref. [11]	0.2335			1.3303	
10	Batra et al. [3]	0.2459*	0.6443	0.9664	1.3668*	1.7119
		(0.2475)	[0.6444]	[0.9666]		
	Present 9 × 9	0.2477	0.6441	0.9661	1.3481	1.7267
	11 × 11	0.2471	0.6443	0.9664	1.3511	1.7260
	15 × 15	0.2467	0.6443	0.9664	1.3507	1.7258
	Ref. [11]	0.2473			1.3514	

Exact frequencies from Ref. [4] are listed in parentheses, and those from Ref. [5] in square brackets. Frequencies of flexural modes are marked with *. Frequencies from Ref. [11] are for the 15 × 15 uniformly distributed collocation points.

Table 2
For different aspect ratios, first 10 non-dimensional natural frequencies of a clamped orthotopic square plate

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
1	Batra et al. [3]	0.0804*	0.2563*	0.4593*	0.6674*	0.8755*
	15 × 15	0.0806	0.2553	0.4548	0.6569	0.8570
	Ref. [11]	0.0804	0.2563	0.4593	0.6674	0.8755
2	Batra et al. [3]	0.1379*	0.4053*	0.6943*	0.9850*	1.2749*
	15 × 15	0.1384	0.4027	0.6842	0.9642	1.2411
	Ref. [11]	0.1387	0.4030	0.6846	0.9646	0.2416
3	Batra et al. [3]	0.1650*	0.4770*	0.8097*	1.0886	1.3606
	15 × 15	0.1647	0.4729	0.7970	1.1006	1.3757
	Ref. [11]	0.1650	0.4731	0.7972	0.9646	1.2416
4	Batra et al. [3]	0.2120*	0.5442	0.8164	1.1441*	1.4788*
	15 × 15	0.2117	0.5503	0.8254	1.1192	1.4393
	Ref. [11]	0.2123			1.1195	1.4395
5	Batra et al. [13]	0.2193*	0.5921*	0.9930*	1.3631	1.7040
	15 × 15	0.2199	0.5864	0.9759	1.3636	1.7054
	Ref. [11]	0.2200	0.5870	0.9766	0.1195	0.4395
6	Batra et al. [3]	0.2721	0.6011*	1.0015*	1.3965	1.7937
	15 × 15	0.2751	0.5953	0.9832	1.3643	1.7493
	Ref. [11]		0.5951	0.9823	1.3646	
7	Batra et al. [3]	0.2775*	0.6814	1.0222	1.4058*	1.8010*
	15 × 15	0.2765	0.6821	1.0232	1.3708	1.7569
	Ref. [11]	0.2766			1.3703	1.7504
8	Batra et al. [3]	0.2830*	0.7178	1.0765	1.4351	1.8121*
	15 × 15	0.2826	0.7214	1.0821	1.4428	1.8035
	Ref. [11]	0.2833				1.7563
9	Batra et al. [3]	0.3145*	0.7469*	1.2354*	1.6683*	1.8740*
	15 × 15	0.3140	0.7383	1.2125	1.6800	1.8972
	Ref. [11]	0.3140	0.7379	1.2118	1.6801	1.8981
10	Batra et al. [3]	0.3175*	0.7561*	1.2466*	1.7282*	2.2113
	15 × 15	0.3170	0.7472	1.2219	1.6865	2.1590
	Ref. [11]	0.3171	0.7478	1.2226	1.6865	

Frequencies of flexural modes are marked with *.

Results for simply supported and clamped boundary conditions, presented in Tables 1 and 2 are in terms of the non-dimensional frequency $\bar{\varpi}$

$$\bar{\varpi} = \omega h \sqrt{\frac{\rho}{D_{11}}}, \tag{4}$$

where ω, h and ρ are, respectively, the dimensional frequency, the plate thickness, and the mass density of the plate material. The aspect ratio, $\bar{h}$, of a square plate is defined as

$$\bar{h} = h/a, \tag{5}$$

where a is the length of a side of the plate. An asterisk on the value of a frequency signifies that the corresponding mode of vibration is flexural. We compute results with 9, 11 and 15 collocation points uniformly spaced in the x- and the y-directions of the square plate of non-dimensional side one, and compare present results with the finite element solution of Batra et al. [3], and the exact solution of Ref. [4]. It is clear that the presently computed frequencies match very well with those reported in Refs. [3,4].

We have also listed in Tables 1 and 2 the corresponding frequencies from Ref. [11] computed with the 15×15 collocation points. It is evident that the use of multiquadratic radial basis functions given by Eq. (2)

Table 3
For different aspect ratios, first 10 non-dimensional natural frequencies of a simply supported monoclinic square plate

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
1	Batra et al. [3]	0.0527	0.1972	0.4058	0.6545	0.9036
	15×15	0.0531	0.1990	0.4103	0.6632	0.9084
	Ref. [11]	0.0527	0.1989	0.4107	0.6638	
2	Batra et al. [3]	0.1241	0.3627	0.5439	0.7251	0.9064
	15×15	0.1277	0.3634	0.5450	0.7267	0.9084
	Ref. [11]	0.1279				
3	Batra et al. [3]	0.1424	0.3628	0.5441	0.7253	0.9299
	15×15	0.1431	0.3634	0.5450	0.7267	0.9415
	Ref. [11]	0.1434				
4	Batra et al. [3]	0.1814	0.4441	0.8745	1.2999	1.6280
	15×15	0.1817	0.4559	0.8966	1.3496	1.7476
	Ref. [11]		0.4574	0.8988	1.3505	
5	Batra et al. [3]	0.1814	0.4780	0.8887	1.3494	1.7819
	15×15	0.1817	0.4827	0.9024	1.3948	1.8059
	Ref. [11]		0.4838	0.9043	1.3989	1.8062
6	Batra et al. [3]	0.1971	0.6539	0.9979	1.3587	1.7939
	15×15	0.1990	0.6632	1.0486	1.3948	1.8168
	Ref. [11]	0.1992	0.6651			
7	Batra et al. [3]	0.2423	0.6662	1.0855	1.4467	1.8064
	15×15	0.2511	0.6991	1.0901	1.4534	1.8168
	Ref. [11]	0.2526				
8	Batra et al. [3]	0.2782	0.7245	1.0865	1.4472	1.8810
	15×15	0.2808	0.7267	1.0901	1.4534	1.9231
	Ref. [11]	0.2817				
9	Batra et al. [3]	0.3004	0.7249	1.2129	1.7281	2.1418
	15×15	0.3065	0.7267	1.2359	1.7445	2.1806
	Ref. [11]	0.3079		1.2393		
10	Batra et al. [3]	0.3211	0.8124	1.3003	1.8056	2.2511
	15×15	0.3237	0.8372	1.3084	1.8522	2.2971
	Ref. [11]	0.3247	0.8403		1.8566	

Frequencies of flexural modes are marked with *.

misses some natural frequencies. Frequencies missed vary with the aspect ratio of the plate. However, all of the first 10 frequencies are accurately captured when basis functions given by Eq. (1) are employed.

2.2. Monoclinic materials

For a monoclinic material we have

$$[D] = \begin{bmatrix} 86.74 & -8.25 & 27.15 & -3.66 & 0 & 0 \\ & 129.77 & -7.42 & 5.7 & 0 & 0 \\ & & 102.83 & 9.92 & 0 & 0 \\ \text{sym.} & & & 38.61 & 0 & 0 \\ & & & & 68.81 & 2.53 \\ & & & & 0 & 29.01 \end{bmatrix} \text{GPa}, \quad (6)$$

and $\rho = 2649\,\text{kg/m}^3$. Computed natural frequencies for simply supported and clamped plates are listed in Tables 3 and 4, respectively.

Table 4
For different aspect ratios, first 10 non-dimensional natural frequencies of a clamped monoclinic square plate

N	Source	$\bar{h} = 0.1$	$\bar{h} = 0.2$	$\bar{h} = 0.3$	$\bar{h} = 0.4$	$\bar{h} = 0.5$
1	Batra et al. [3]	0.0993*	0.3382*	0.6405*	0.9694*	1.3091*
	15 × 15	0.1009	0.3432	0.6479	0.9769	1.3148
	Ref. [11]	0.1012	0.3435	0.6481	0.9771	1.3150
2	Batra et al. [3]	0.1835*	0.6012*	1.0465*	1.4010	1.7518
	15 × 15	0.1886	0.6058	1.0521	1.4484	1.8105
	Ref. [11]	0.1894	0.6059	1.0519		
3	Batra et al. [3]	0.2005*	0.6061*	1.0501	1.5028*	1.9593*
	15 × 15	0.2021	0.6180	1.0863	1.5081	1.9659
	Ref. [11]	0.2025	0.6185		1.5079	1.9656
4	Batra et al. [3]	0.2633*	0.6994	1.1119*	1.6295*	2.0888
	15 × 15	0.2669	0.7242	1.1273	1.6481	2.1195
	Ref. [11]	0.2680		1.1274	1.6480	
5	Batra et al. [3]	0.3133*	0.8000*	1.2602	1.6766	2.1274*
	15 × 15	0.3230	0.8097	1.2717	1.6956	2.1530
	Ref.[11]	0.3240	0.8105			2.1529
6	Batra et al. [3]	0.3393*	0.8408	1.3865	1.8479	2.3086
	15 × 15	0.3424	0.8478	1.4179	1.8492	2.3678
	Ref. [11]	0.3424				
7	Batra et al. [3]	0.3492	0.9244	1.4035*	2.0116*	2.6005*
	15 × 15	0.3621	0.9404	1.4207	2.0325	2.6342
	Ref. [11]			1.4185	2.0329	2.6346
8	Batra et al. [3]	0.3746*	0.9336*	1.5608*	2.1029	2.6146
	15 × 15	0.3823	0.9471	1.5717	2.1291	2.6613
	Ref. [11]	0.3839	0.9395	1.5702		
9	Batra et al. [3]	0.3875*	0.9751*	1.5828	2.1942*	2.8229*
	15 × 15	0.3914	0.9946	1.5968	2.2129	2.8583
	Ref. [11]	0.3923	0.9941		2.2110	2.8558
10	Batra et al. [3]	0.4210	1.0557*	1.7128*	2.4169*	2.8245*
	15 × 15	0.4239	1.0645	1.7429	2.4918	3.0460
	Ref. [11]		1.0937	1.7413	2.4895	3.0409

Frequencies of flexural modes are marked with *.

2.3. Hexagonal materials

The beryllium crystal belongs to the close-packed hexagonal system. It has an axis of symmetry such that a rotation of the crystal through 60° about that axis brings the space lattice into coincidence with its original configuration. The mass density of beryllium equals $1850\,\mathrm{kg/m^3}$, and the elastic constants are

$$[D] = \begin{bmatrix} 298.2 & 27.7 & 11.0 & 0 & 0 & 0 \\ & 298.2 & 11.0 & 0 & 0 & 0 \\ & & 340.8 & 0 & 0 & 0 \\ \text{sym.} & & & 165.5 & 0 & 0 \\ & & & & 165.5 & 0 \\ & & & & 0 & 135.3 \end{bmatrix} \text{GPa} \tag{7}$$

Computed natural frequencies of simply supported and clamped plates are listed in Tables 5 and 6, respectively.

Table 5
For different aspect ratios, first 10 non-dimensional natural frequencies of a simply supported hexagonal square plate

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
1	Batra et al. [3]	0.0555*	0.2076*	0.4264*	0.6857*	0.9681*
	15 × 15	0.0556	0.2081	0.4281	0.6899	0.9765
	Ref. [11]	0.0552	0.2080	0.4285	0.6907	0.9776
2	Batra et al. [3]	0.1340*	0.4230	0.6343	0.8453	1.0558
	15 × 15	0.1342	0.4232	0.6348	0.8465	1.0581
	Ref. [11]	0.1345				
3	Batra et al. [3]	0.1340*	0.4230	0.6343	0.8453	1.0558
	15 × 15	0.1342	0.4232	0.6348	0.8465	1.0581
	Ref. [11]	0.1345				
4	Batra et al. [3]	0.2076*	0.4662*	0.8940*	1.1935	1.4898
	15 × 15	0.2081	0.4683	0.9012	1.1969	1.4961
	Ref. [11]	0.2083	0.4696	0.9036		
5	Batra et al. [3]	0.2116	0.4662*	0.8940*	1.3599*	1.8379*
	15 × 15	0.2116	0.4683	0.9012	1.3773	1.8720
	Ref. [11]		0.4696	0.9036	1.3804	1.8756
6	Batra et al. [3]	0.2116	0.5979	0.8961	1.3599*	1.8379*
	15 × 15	0.2116	0.5984	1.2697	1.3773	1.8720
	Ref. [11]				1.3804	1.8757
7	Batra et al. [3]	0.2543*	0.6855*	1.2624*	1.6834	2.0983
	15 × 15	0.2550	0.6900	1.2697	1.6929	2.1161
	Ref. [11]	0.2562	0.6917	1.2807		
8	Batra et al. [3]	0.2543*	0.8165*	1.2654	1.6834	2.0983
	15 × 15	0.2550	0.8229	1.2697	1.6929	2.1161
	Ref. [11]	0.2563	0.8253			
9	Batra et al. [3]	0.2991	0.8165*	1.2654	1.7656	2.2003
	15 × 15	0.2992	0.8229	1.2776	1.7773	2.2216
	Ref. [11]		0.8256			
10	Batra et al. [3]	0.3214*	0.8449	1.3273	1.8659*	2.3407
	15 × 15	0.3225	0.8464	1.3330	1.8925	2.3657
	Ref. [11]	0.3236			1.9059	

Frequencies of flexural modes are marked with *.

Table 6
For different aspect ratios, first 10 non-dimensional natural frequencies of a clamped hexagonal square plate

N	Source	$\bar{h}=0.1$	$\bar{h}=0.2$	$\bar{h}=0.3$	$\bar{h}=0.4$	$\bar{h}=0.5$
1	Batra et al. [3]	0.0968*	0.3325*	0.6305*	0.9510*	1.2778*
	15 × 15	0.0969	0.3325	0.6298	0.9487	1.2732
	Ref. [11]	0.0970	0.3330	0.6304	0.9494	1.2741
2	Batra et al. [3]	0.1878*	0.5960*	1.0639*	1.4856	1.8530
	15 × 15	0.1880	0.5964	1.0643	1.4924	1.8654
	Ref. [11]	0.1887	0.5970	1.0649		
3	Batra et al. [3]	0.1878*	0.5960*	1.0639*	1.4856	1.8530
	15 × 15	0.1880	0.5964	1.0643	1.4924	1.8654
	Ref. [11]	0.1887	0.5970	1.0649		
4	Batra et al. [3]	0.2660*	0.7449	1.1160	1.5370*	1.9980*
	15 × 15	0.2664	0.7462	1.1193	1.5381	2.0021
	Ref. [11]	0.2677			1.5386	2.0027
5	Batra et al. [3]	0.3157*	0.7449	1.1160	1.5370*	1.9980*
	15 × 15	0.3162	0.7462	1.1193	1.5381	2.0021
	Ref. [11]	0.3169			1.5386	2.0027
6	Batra et al. [3]	0.3183*	0.8081*	1.4089*	1.9088	2.3772
	15 × 15	0.3189	0.8098	1.4141	1.9226	2.4033
	Ref. [11]	0.3196	0.8111	1.4154		
7	Batra et al. [3]	0.3727	0.9280*	1.4357	2.0105*	2.5981*
	15 × 15	0.3731	0.9303	1.4420	2.0237	2.6270
	Ref. [11]		0.9300		2.0252	2.6287
8	Batra et al. [3]	0.3727	0.9405*	1.5847*	2.1974	2.7334
	15 × 15	0.3731	0.9429	1.5919	2.2188	2.7735
	Ref. [11]		0.9427	1.5911		
9	Batra et al. [3]	0.3840*	0.9591	1.6125*	2.2343*	2.8658*
	15 × 15	0.3849	0.9613	1.6197	2.2539	2.9098
	Ref. [11]	0.3864		1.6189	2.2528	2.9085
10	Batra et al. [3]	0.3840*	1.1045*	1.6542	2.2810*	2.9352*
	15 × 15	0.3849	1.1094	1.6641	2.2999	2.9772
	Ref. [11]	0.3864	1.1105		2.2988	2.9759

Frequencies of flexural modes are marked with *.

3. Discussion

A review of results presented in Tables 1–6 reveals that the multiquadric basis functions coupled with the collocation method miss some frequencies of plates comprised of orthotropic, monoclinic and hexagonal materials. The number of frequencies missed is more for monoclinic and hexagonal materials than that for orthotropic materials. However, the higher-order Wendland radial basis functions used herein capture reasonably well the first 10 frequencies. The first-order shear deformation theory used herein is the same as that employed in Ref. [11]. It is shown in Ref. [15] that for an isotropic plate the first-order shear deformation theory can predict through-the-thickness modes. Qian et al. [16,17] used a meshless method employing basis functions derived by the moving least squares approximation and a local weak form of the governing equations to find frequencies, under different boundary conditions, of a thick rectangular plate made of an isotropic material.

4. Conclusions

For square plates comprised of orthotropic, monoclinic and hexagonal materials, we have listed the first 10 frequencies for different edge conditions and aspect ratios. The converged frequencies were computed with a meshless collocation method using the Wendland compact support radial basis functions and an optimal shape parameter. Computed frequencies match very well with the analytical frequencies. For each set of boundary conditions and material symmetries considered, there are non-flexural modes of vibration.

The use of Wendland compact support radial basis functions captures frequencies missed by using multiquadric basis functions.

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