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DEFORMATIONS OF AN AXIALLY LOADED THERMOVISCOPLASTIC BAR DUE TO LASER HEATING

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We study quasi-static deformations of a thermoviscoplastic thin rectangular bar subjected to a dead axial load and the upper surface heated by a laser. By assuming that strains due to elastic deformations and thermal expansion are small as compared to the plastic strains, the problem is simplified to that of simultaneously solving two coupled ordinary differential equations. One of these equations gives the evolution of the axial strain, and the other describes the evolution of the damage. It is found that after a certain time interval, whose duration depends upon the axial prestress, the strain rate increases rapidly. On a log-log plot, the time to failure decreases essentially affinely with an increase in the laser power density.

Keywords damage, one-dimensional deformations, thermoviscoplasticity

The development of inelastic stresses due to transient heating of a structure and especially a space structure has received considerable attention. For example, Weiner [1] analyzed stresses in a plate on the assumption that the heat input varies slowly. Landau et al. [2] found stresses developed during the quenching process by assuming that the material viscosity is temperature dependent. Kammash et al. [3] considered a linearly work-hardening cylinder subjected to a distributed heat source, lateral pressure, and axial loads. Parkes [4, 5] determined stresses in an elastoplastic bar due to a sudden change in its surface temperature. He delineated effects of the plate thickness, axial load, residual stresses, and the intensity of heating on the

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development of plastic zones as a function of time. These investigations are mainly focused on the analysis of creep or heat treatment.

The deformation behavior of metals including aluminum alloys at elevated temperatures and moderate strain rates (0.01–100 $\sec^{-1}$) has been recently reviewed by McQueen [6] and Evangelista and Spigarelli [7]. Nearly all metals exhibit strain hardening, strain-rate hardening, and thermal softening. Among several constitutive relations incorporating these three effects, the one proposed by Johnson and Cook [8] is often used because of the availability of values of parameters for several materials. We use it here to analyze deformations of a thin rectangular aluminum 2024 bar whose upper surface is exposed to laser heating and lower surface is insulated.

Recently, emphasis has been placed on developing an understanding of the interaction of a low- and mid-power continuous-wave laser with a prestressed structure; for example, see Chen [9, 10] and Zhao et al. [11]. The deformation and failure mechanisms of prestressed structures due to laser heating are also of interest to several industries. The transfer of energy from the laser to the structure can induce thermal softening of the material, its melting, and evaporation. Depending upon the prestress, the laser power, the duration of exposure, and the boundary conditions, the structure may deform severely and cracks may develop in it. A thorough analysis of such a problem will need to be numerical. However, considerable insight can be gained by simplifying the problem through various engineering approximations. Here we use the continuum damage mechanics approach to analyze deformations and the failure of a prestressed thin rectangular bar.

FORMULATION OF THE PROBLEM

A schematic sketch of the problem studied is shown in Figure 1. A thin rectangular bar under an axial dead load P has length L_0 and area of cross section S_0 in the reference configuration. The upper surface of the bar is subjected to laser heating and the lower surface is thermally insulated. The bar is assumed to be made of a thermo-viscoplastic material that exhibits strain hardening, strain-rate hardening, and thermal softening. Furthermore, relative changes in the dimensions of the bar caused by the temperature rise, change in volume due to the damage evolved, and elastic deformations are assumed to be negligible as compared to those induced by plastic deformations. This is reasonable since the axial plastic strain at failure is about 0.2, and approximate values of the elastic and the thermal strains are 0.007 and 0.01,

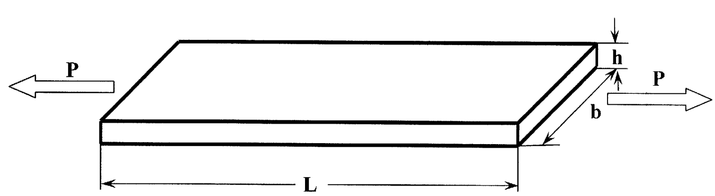


Figure 1. Schematic sketch of the problem studied.

respectively. The temperature rise due to energy dissipated because of plastic deformations has been assumed to be negligible as compared to that caused by the laser heating. This is justified since the density of plastic working is several orders of magnitude smaller than the density of laser power except when the material is about to fail.

We assume that the heat flux supplied by the laser is independent of time t and is uniformly distributed on the upper surface, $z = 0$, of the bar. Also, the heat is assumed to flow only in the thickness direction. The evolution of temperature, T, in the bar is governed by

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} \qquad 0 < z < h,\ t > 0 \tag{1}$$

$$k \frac{\partial T}{\partial z}\bigg|_{z=h} = 0 \tag{2}$$

$$k \frac{\partial T}{\partial z}\bigg|_{z=0} = F_0 = \beta I_0 \tag{3}$$

$$T(z,0) = T_r = 293\ \mathrm{K} \tag{4}$$

Here c is the specific heat, ρ the mass density, k the thermal conductivity, h the bar thickness, I_0 the laser intensity, and β the fraction of the laser output absorbed by the bar. The solution of these equations, taken from Carlsaw and Jaeger [12], is

$$T(z,t) - T(z,0) = \frac{F_0 \bar{k}}{h} t + \frac{F_0 h}{\bar{k}} \left\{ \frac{3(h-z)^2 - h^2}{6h^2} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-\frac{\bar{k} n^2 \pi^2 t}{h^2}} \cos \frac{n\pi(h-z)}{h} \right\} \tag{5}$$

where $\bar{k} = k/\rho c$ is the diffusivity. For a 2.5-mm-thick aluminum 2024 bar, $\bar{k}\pi^2/h^2 = 77.6\,\mathrm{sec}^{-1}$, so for $t \geq 0.04\,\mathrm{sec}$, the exponential term on the right-hand side of Eq. (5) makes a negligible contribution. For a thicker bar, this term will become small at a later time. The difference between the temperatures of the top and the bottom surfaces of the bar is given by

$$\Delta(t) = T(0,t) - T(h,t) = \frac{F_0 h}{\bar{k}} \left[\frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-\frac{\bar{k}\pi^2 (2n-1)^2 t}{h^2}} \right] \tag{6}$$

We let $T_a(t) = (T(0,t) + T(h,t))/2$ and $\delta = \frac{\Delta}{T_a}$. T_a equals the average of the temperatures of the top and the bottom surfaces of the bar, Δ the difference in the temperatures of these two surfaces, and δ the relative temperature difference. Figure 2 depicts the evolution of δ with the change in the intensity of the laser, and for a given laser intensity. Figure 3 shows the evolution of δ for bars of different thicknesses. It is clear that δ is usually less than 10%. If the insulation boundary condition (2) were replaced by that corresponding to free convection, then the

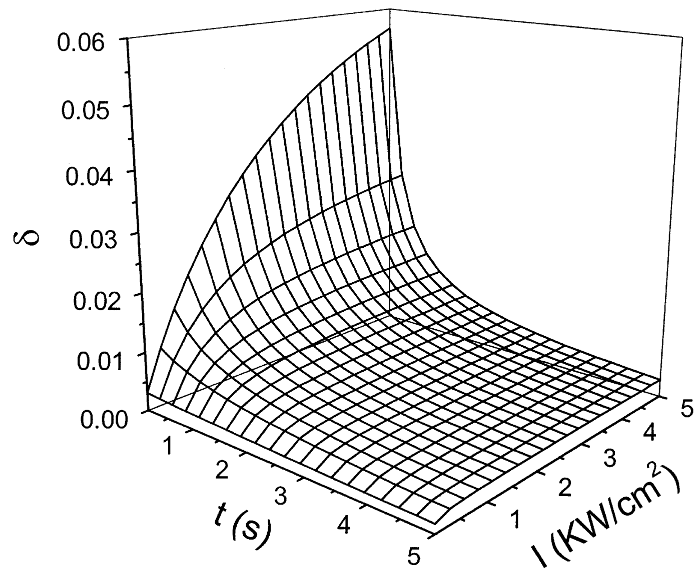


Figure 2. Evolution of the relative temperature difference in a 2.5-mm-thick aluminum bar for different values of the laser power density.

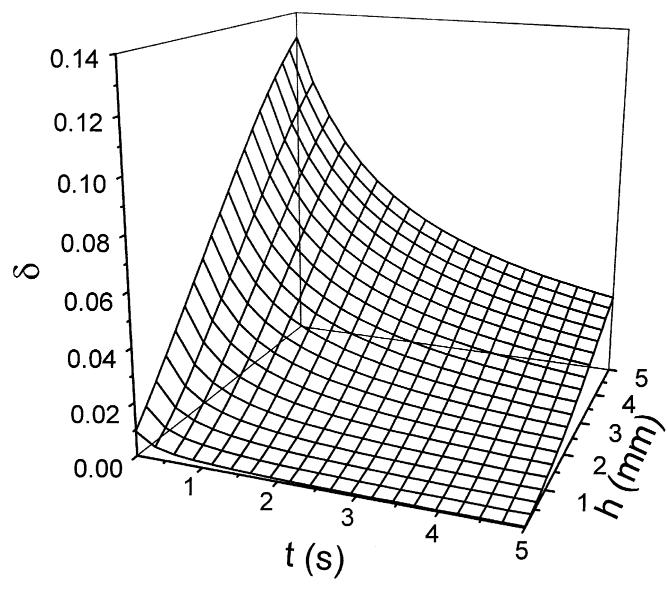


Figure 3. Evolution of the relative temperature difference in an aluminum bar for $I_0 = 10^7\,\mathrm{W/m^2}$ and different thicknesses of the bar.

maximum value of δ will be higher than 0.1. Henceforth, when studying the evolution of the axial stress, axial strain, and resulting damage, we ignore through-the-thickness variation of temperature. We thus take the temperature of the bar at any time to be uniform and given by

$$T(t) = T(0) + \frac{\beta I_0}{\rho c h} t \tag{7}$$

Equation (7) is obtained by equating the heat input to that needed to raise the temperature of the body. Alternatively, $k\frac{\partial^2 T}{\partial z^2}$ in Eq. (1) can be approximated by $\frac{\beta I_0}{h}$ and the resulting equation integrated with respect to time. For a 2.5-mm-thick aluminum 2024 bar and $\beta = 0.25$,

$$T(t) = T(0) + 41.26 \times 10^{-6} I_0 t \tag{8}$$

where I_0 is in $\mathrm{W/m^2}$, t in seconds, and T in Kelvin.

We assume that mechanical deformations of the bar are isochoric and slow, so inertia effects are negligible. Thus, for equilibrium,

$$P = \sigma_0(1 - D_0)S_0 = \sigma(1 - D)S \tag{9}$$

where σ is the axial stress, D the damage parameter, S the area of cross section, and subscript zero signifies the value of the quantity in the reference configuration. Here we followed Lemaitre [13] and assumed that the effective area of cross section of the damaged material equals $(1 - D)$ times the geometric area of cross section. In order for viscoplastic deformations to be isochoric,

$$L_0 S_0 = LS \tag{10}$$

Recall that the logarithmic axial strain or the true axial strain, ε, is given by

$$\varepsilon = \ln\frac{L}{L_0} = \ln\frac{S_0}{S} \tag{11}$$

Substituting in Eq. (11) for $\frac{S_0}{S}$ from Eq. (9) we obtain

$$\varepsilon = \ln\frac{\sigma(1 - D)}{\sigma_0(1 - D_0)} \quad \text{or} \quad \sigma = \frac{\sigma_0(1 - D_0)}{(1 - D)} e^{\varepsilon} \tag{12}$$

For a uniaxial stress state, the Johnson–Cook thermoviscoplastic relation [8] reduces to

$$\sigma = (A + B\varepsilon^n)\left(1 + C\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right)(1 - T_*^m) \tag{13}$$

where $T_* = \frac{T - T_r}{T_m - T_r}$; A, B, C, n, and m are material parameters; T_r and T_m are, respectively, the room temperature and the melting temperature of the material;

and $\dot{\varepsilon}_0$ is the reference strain rate. Substituting for σ from Eq. (12) into Eq. (13) gives the following equation for the evolution of the axial strain ε caused by the temperature rise

$$(1 - D_0)\sigma_0 e^{\varepsilon} = (1 - D)(A + B\varepsilon^n)\left(1 + C \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right)(1 - T_*^m) \tag{14}$$

We now postulate that the damage, D, evolves according to the relation

$$\dot{D} = \frac{\sigma_{eq}^2}{\alpha\sigma_y^2}\left[\frac{2}{3}(1 + v) + 3(1 - 2v)\left(\frac{\sigma_m}{\sigma_{eq}}\right)^2\right]\dot{\varepsilon} \tag{15}$$

where $\sigma_{eq} = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{1/2}$, $s_{ij} = \sigma_{ij} - \sigma_m\delta_{ij}$ is the deviatoric stress, $\sigma_m = \sigma_{ii}/3$ is the hydrostatic pressure, v is Poisson's ratio, σ_y equals the yield stress of the material in a quasi-static uniaxial stress test, δ_{ij} is the Kronecker delta, and α is a constant. For $\sigma_{ij} = \sigma\delta_{i1}\delta_{j1}$ Eq. (15) reduces to

$$\dot{D} = \frac{1}{\alpha}\frac{\sigma^2}{\sigma_y^2}\dot{\varepsilon} \tag{16}$$

which when combined with Eq. (12) gives

$$\dot{D} = \frac{1}{\alpha}\frac{(1 - D_0)^2}{(1 - D)^2}\frac{\sigma_0^2}{\sigma_y^2}e^{2\varepsilon}\dot{\varepsilon} \tag{17}$$

Equations (14) and (17) are solved simultaneously for ε and D under the initial conditions $\varepsilon(0) = 0$, $D(0) = D_0$ by the fourth-order Runge–Kutta method. We can thus determine the evolution of the plastic strain and the damage induced in the bar because of the laser heating. Note that Eq. (17) can be integrated to obtain a relationship between D and ε.

RESULTS AND DISCUSSION

Unless otherwise noted, we compute results for a 2.5-mm-thick aluminum 2024 bar. Values of material parameters are

$$A = 265\,\text{MPa} \qquad B = 426\,\text{MPa} \qquad n = 0.34 \qquad C = 0.015 \qquad m = 1.0$$

$$\alpha = 0.6 \qquad v = \frac{1}{3} \qquad T_r = 293\,\text{K} \qquad T_m = 775\,\text{K} \qquad \rho = 2770\,\text{kg/m}^3$$

$$c = 875\,\text{J/kg K} \qquad \dot{\varepsilon}_0 = \frac{1}{\text{s}} \qquad k = 119\,\text{W/mK} \qquad \sigma_y = 265\,\text{MPa}$$

We set $D_0 = 10^{-5}$, $\beta = 0.25$, and assume that the bar has failed when the damage parameter D equals 0.3. Substituting from Eq. (8) into Eq. (14) and solving for $\dot{\varepsilon}$ we get

$$\dot{\varepsilon} = e^{\gamma} \quad \text{where } \gamma = \frac{1}{C}\left[\frac{(1-D_0)\sigma_0 e^{\varepsilon}}{(1-D)(A+B\varepsilon^n)(1-8.56\times10^{-8}I_0 t)} - 1\right] \tag{18}$$

Because $C = 0.015$, the strain rate and hence the damage rate (cf. Eq. (17)) begin to increase very rapidly once the term in brackets on the right-hand side of Eq. (18) is close to C. For $\sigma_0 = 0.1A$ and $I_0 = 10^7\ \mathrm{W/m^2}$, γ will be nearly 1 at $t \simeq 1$ sec, and the bar will fail approximately one second after the laser power is switched on.

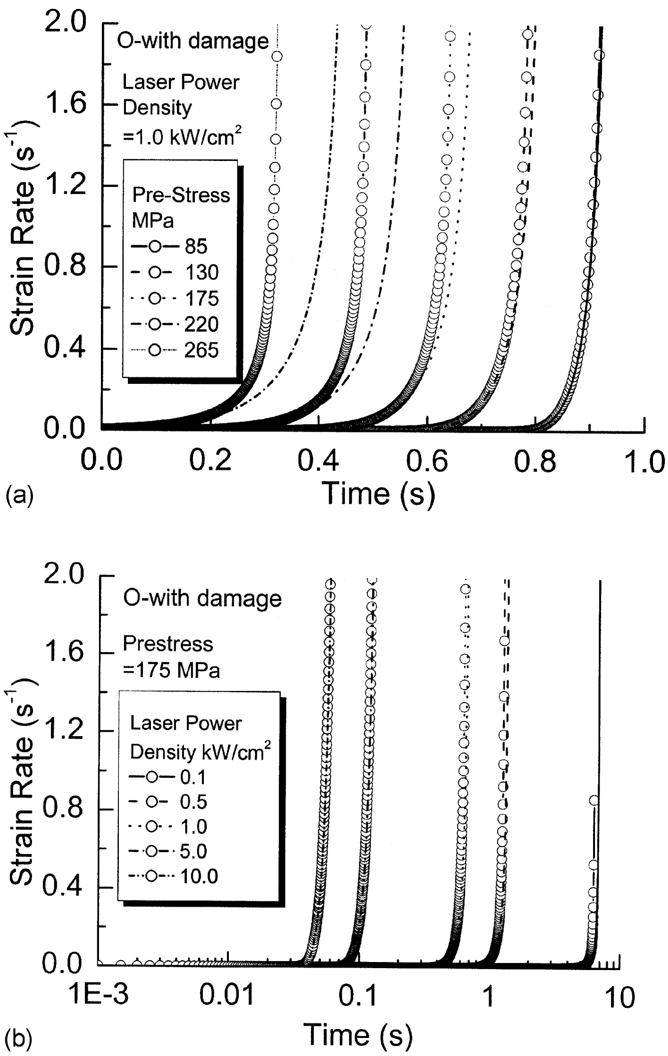
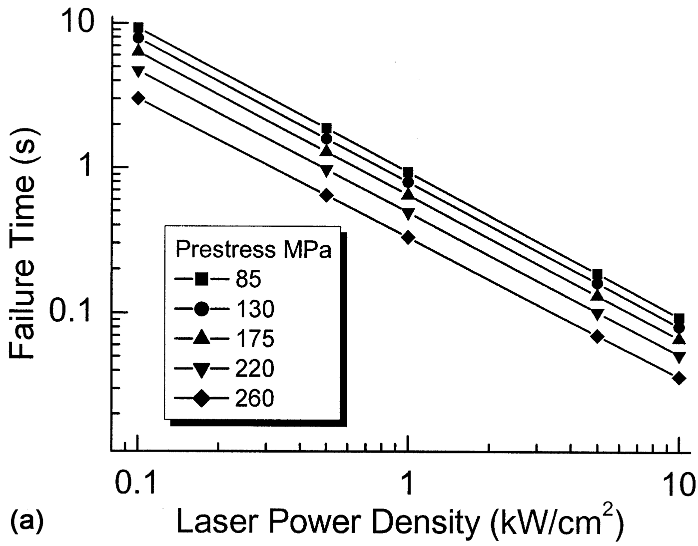


Figure 4. The evolution of the axial plastic strain rate with and without consideration of the damage for different values of (a) the prestress and (b) the laser power density.

Figure 4*a* exhibits, for different values of the prestress σ_0, the evolution of the axial plastic strain rate for $I_0 = 10^7\,\mathrm{W/m^2}$ and $D_0 = 10^{-5}$. Results have been computed with and without the consideration of the damage. It is clear that for each value of the axial prestress considered, the strain rate increases very slowly in the beginning and then increases very rapidly. The time when the material starts deforming quickly decreases with an increase in the axial prestress. Also, the rapid



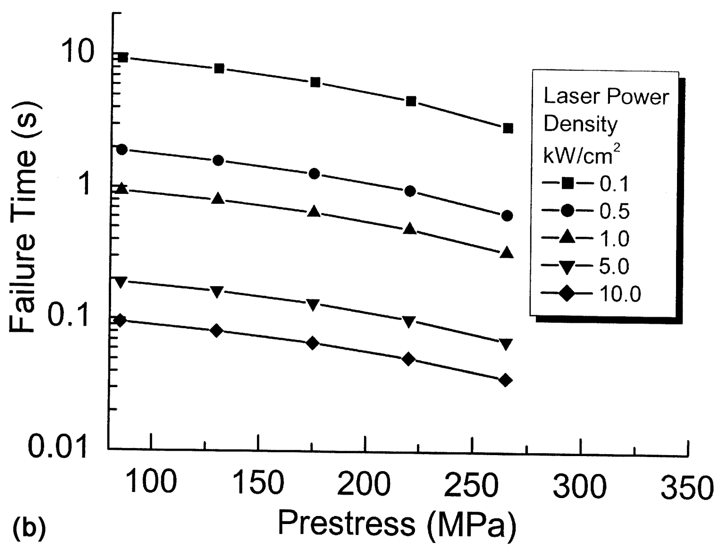
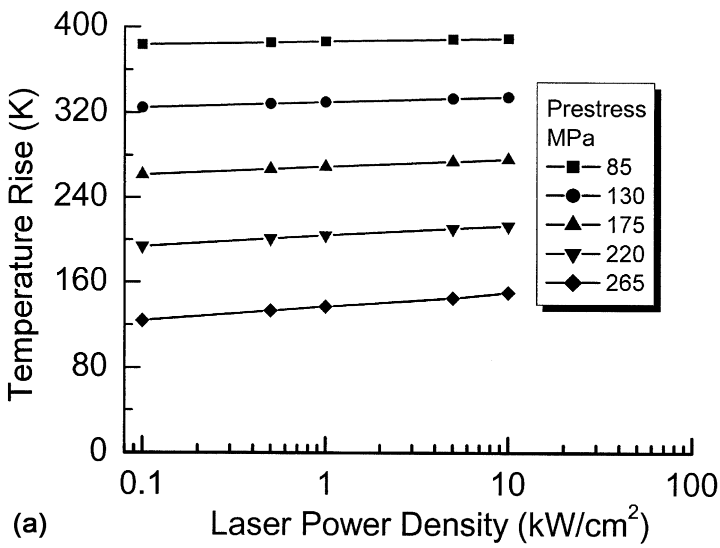


Figure 5. Effect on the time to failure of (*a*) the laser power density and (*b*) the prestress.

increase of the axial strain rate occurs earlier when the evolution of damage is considered as compared to that when the damage evolution is ignored; the difference between the two times increases with an increase in the prestress. For a fixed value of the axial prestress, as shown in Figure 4*b*, the time when the axial strain rate begins to increase swiftly decreases with an increase in the laser power density and is virtually the same whether or not the damage evolution is considered.



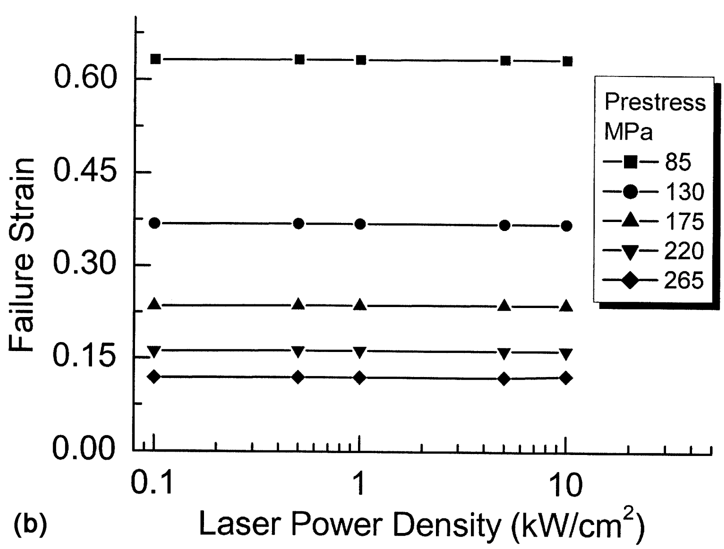


Figure 6. The dependence upon the laser power density at the instance of failure of (*a*) the temperature rise and (*b*) the axial strain.

In Figure 5*a*, we have plotted, on a log-log scale, the time to failure, t_f, as a function of the laser power density. It is evident that $t_f \propto \frac{1}{I_0^a}$ and the value of a is essentially independent of the prestress. As shown in Figure 5*b*, the time to failure decreases exponentially with an increase in the prestress σ_0; note the logarithmic scale on the vertical axis. For different values of the axial prestress, the temperature

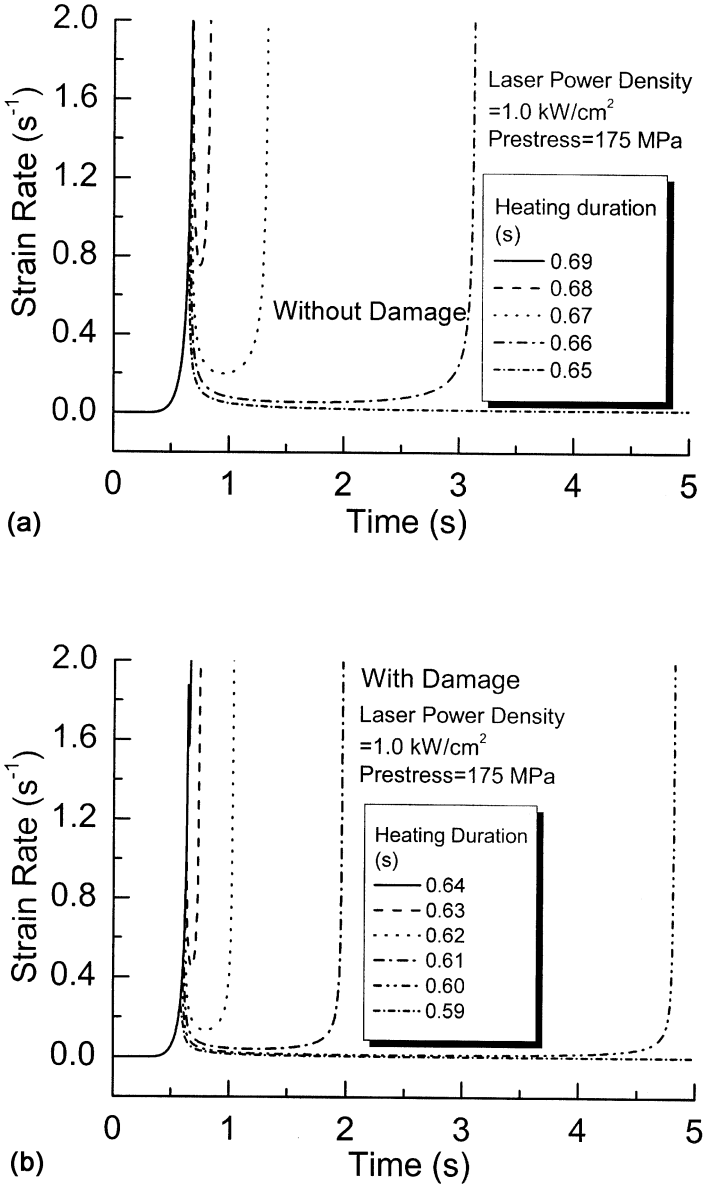


Figure 7. The evolution of the plastic strain rate for different time durations of the laser power density (*a*) without damage evolution and (*b*) with damage evolution.

rise at the instant of the material failure as a function of the laser power density is plotted in Figure 6*a*; corresponding values of the failure axial strain are plotted in Figure 6*b*. Whereas the failure axial strain does not depend upon the laser power density, the temperature at the instant of failure exhibits a strong dependence upon both the laser power density and the prestress.

For $\sigma_0 = 175\,\text{MPa}$, Figures 7*a*,*b* exhibit the effect of the duration of laser heating with $I_0 = 1\,\text{kW/cm}^2$ on the resulting deformations. The bar is heated with the laser for a certain time t_0 and the power is turned off. Subsequently, the temperature of the body is assumed to equal that when the laser power was shut down. In view of the time to failure of 1 sec or less, this is reasonable since the heat transfer to the surrounding air is usually a slow process. Whereas the evolution of damage is considered for results plotted in Figure 7*b*, it is neglected for those plotted in Figure 7*a*. A comparison of these two sets of results reveals that subsequent to turning off the laser power the strain rate is higher when damage evolution is considered than that when it is ignored. These results suggest that the following three types of deformation modes will occur depending upon the time when the laser is turned off. For values of t_0 exceeding 0.66 sec, the bar will rupture soon after the laser is shut off. For low values of t_0 such as 0.1 sec, and 0.2 sec, the bar will not fail since the strain rate stays very small. For moderate values of t_0 it will take a while before the bar fails after the laser is turned off. These values of t_0 are material dependent and will vary with the critical value assigned to the damage. Values of the temperature, the axial strain, and the axial strain rate at the instant of turning off the laser power are listed in Table 1. It is evident that they are very sensitive to the duration t_0 of laser heating and also to the consideration of the damage.

We remark that various simplifying assumptions have reduced the solution of the initial boundary value problem to that of simultaneously solving two coupled nonlinear ordinary differential equations. Thus the computed time to failure should be regarded as approximate since the actual time will also depend upon the boundary conditions imposed at the edges of the bar/plate.

Table 1 Duration of laser heating and values of the axial strain, the axial strain rate, and the temperature rise when the laser is turned off

	Laser Heating Duration (s)	Temperature Rise (K)	Axial Strain Rate (1/s)	Axial Strain
With damage	0.59	243	0.18	0.018
	0.60	247.5	0.42	0.020
	0.61	251	0.46	0.026
	0.62	255.8	0.74	0.031
	0.63	260	1.08	0.040
	0.64	264.1	1.75	0.054
Without damage	0.65	268.2	0.91	0.045
	0.66	272.3	1.18	0.055
	0.67	276.4	1.6	0.069
	0.68	280.6	2.35	0.088
	0.69	284.7	4.08	0.12

Equations (14) and (18) reveal that if thermal softening signified by the term $(1 - T_*^m)$ is enhanced, then for the same value of the axial strain ε, $\dot{\varepsilon}$ will increase, which because of Eq. (17) will enlarge $\dot{D}$. A higher value of D gives even a larger $\dot{\varepsilon}$; cf. Eq. (18). This process feeds on itself until the material fails. It suggests that the thermal softening will play a dominant role in determining the time to failure.

CONCLUSIONS

By neglecting inertia effects, assuming that the temperature of the bar is uniform, and using a continuum damage mechanics approach, we computed the time to failure of a prestressed thin rectangular bar whose upper surface is heated by a laser and the lower surface is thermally insulated. The solution of the transient problem reveals that the temperature of the thin bar will become uniform soon after the laser is turned on. The Johnson–Cook thermoviscoplastic relation is used to model the material. It is found that the time to failure t_f and the laser power density I_0 satisfy $t_f I_0^a = \text{const}$, where a is a constant and t_f decreases exponentially with an increase in the level of the prestress. If the laser is turned off prior to the onset of failure, then depending upon when the laser is shut off, the bar may rupture quickly or slowly or not at all.

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