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VIBRATION OF THERMALLY POST-BUCKLED ORTHOTROPIC CIRCULAR PLATES

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Axisymmetric vibrations of a statically buckled polar orthotropic circular plate due to uniform temperature rise have been studied numerically. Effects of geometric nonlinearities have been incorporated into the problem formulation. The problem is challenging because the buckled configuration is unknown a priori. By assuming that the amplitude of plate's vibration and the additional strains induced in it are infinitesimal, and its response harmonic, the non-linear partial differential equations are reduced to two sets of coupled ordinary differential equations; one for the thermal post-buckling, and the other for linear vibrations of the plate superimposed upon the post-buckled configuration. The plate's boundary is taken to be either clamped or simply supported but restrained from moving in the radial direction. The two sets of coupled boundary value problems are solved numerically by a shooting method. The dependence of the first three frequencies upon the temperature rise, for both pre-buckled and post-buckled plates, have been computed, and characteristic curves of the frequency versus temperature rise for different values of material anisotropy parameters are plotted. It is found that the three lowest frequencies of the pre-buckled plate decrease with an increase in the temperature, but those of a buckled plate increase monotonically with the temperature rise. The fundamental frequency of the deformed plate approaches zero at the onset of buckling.

INTRODUCTION

Buckling and vibration of plates due to thermal loads are important considerations in the design and analysis of engineering structures in such diverse

fields as aerospace, ocean and nuclear engineering, electronics, and oil refineries. Accordingly, static and dynamic responses of structures exposed to thermal environments have been studied by many investigators; e.g., see [1–15]. Even though there are extensive investigations on the buckling of plates, there are very few works on the dynamic response of post-buckled configurations of plates buckled due to thermal loads. In general, a thermally buckled structure can support additional load in the post-buckled regime. However, the vibrational characteristics of a buckled plate will differ from those of an unbuckled plate. The vibration response of both flat and curved panels subjected to thermomechanical loads has been studied by Librescu et al. [16–18], who have also considered effects of geometric non-linearity, initial imperfections, and supports partially immovable in the longitudinal direction. Li and Zhou [5, 19] used the shooting method to analyze nonlinear vibrations of uniformly heated orthotropic circular/annular plates. They found that the fundamental frequency of nonlinear vibrations increases with an increase in the amplitude of vibrations but decreases with an increase in the temperature. Lee and Lee [21] examined the vibration of thermally buckled composite plate by using the first-order shear deformation plate theory. Ng [22] investigated theoretically and experimentally nonlinear acoustic response of thermally buckled rectangular plates. Oh et al. [23] employed the finite element method to study vibrations of a buckled thermopiezoelectric composite plate. Liu and Huang [24] studied nonlinear vibrations of free composite laminated plates subjected to uniform temperature changes, and computed the dependence of the fundamental frequency and the amplitude of vibration upon the temperature change. Recently, Park et al. [25] have given the temperature-frequency relationship for both pre- and post-buckled states of thermally loaded composite plates containing shape memory alloy fibers.

Here we employ the von Karman plate theory and the Hamilton principle to study axisymmetric vibrations of a thermally loaded polar orthotropic circular plate with boundary immovably constrained. By expressing the solution of governing equations as the sum of two parts [16–18, 20], one pertaining to static thermal buckling deformations and the other to infinitesimal free vibrations superimposed upon the buckled state, linear equations for vibrations of a buckled plate are derived. The coupled nonlinear differential equations for thermal post-buckling, and linear equations for free vibration are solved simultaneously with an efficient shooting method [5, 19, 20]. Characteristic relations between natural frequencies, the temperature rise, and the material anisotropy are depicted graphically.

GOVERNING EQUATIONS FOR VON KARMAN'S PLATE THEORY

Consider a thin polar orthotropic annular circular plate of radius R, constant thickness h, and a cylindrical coordinate system (r, θ, z) with the origin at the plate centroid, and the mid-plane given by $z = 0$. The plate boundary is assumed to be either simply supported or clamped. Assume that a steady and uniform temperature field T is applied to the natural state of the plate. We study free transverse vibrations of the heated plate that may have buckled due to the temperature rise. Using von Karman's plate theory that accounts for geometric non-linearities, and Hamilton's principle [29], we obtain following equations of motion and boundary conditions in

terms of non-dimensional variables [5, 19]:

$$\frac{\partial^2 U}{\partial x^2} + \frac{1}{x}\frac{\partial U}{\partial x} - \frac{k}{x^2}U + \frac{\partial W}{\partial x}\frac{\partial^2 W}{\partial x^2} + \frac{1-v_\theta}{2x}\left(\frac{\partial W}{\partial x}\right)^2 = \frac{\lambda}{12\delta^2}\left[(1-\mu)\frac{\Theta}{x} + \frac{d\Theta}{dx}\right] \quad (1)$$

$$\frac{\partial^4 W}{\partial x^4} + \frac{2}{x}\frac{\partial^3 W}{\partial x^3} - \frac{k}{x^2}\frac{\partial^2 W}{\partial x^2} + \frac{k}{x^3}\frac{\partial W}{\partial x} + \lambda\left(\frac{\partial^2 W}{\partial x^2} + \frac{1}{x}\frac{\partial W}{\partial x}\right) + \frac{\partial^2 W}{\partial \tau^2}$$

$$= 12\delta^2\frac{1}{x}\frac{\partial}{\partial x}\left[x\left(\frac{\partial U}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{v_\theta}{x}U\right)\frac{\partial W}{\partial x}\right] \quad (2)$$

$$U = 0 \quad \frac{\partial W}{\partial x} = 0 \quad \frac{\partial^3 W}{\partial x^3} + \frac{1}{x}\frac{\partial^2 W}{\partial x^2} = 0 \quad \text{at } x = \Delta x \quad (3)$$

$$U = 0 \quad W = 0 \quad \frac{\partial W}{\partial x} + K\frac{\partial^2 W}{\partial x^2} = 0 \quad \text{at } x = 1 \quad (4)$$

The non-dimensional variables are defined as

$$(x, U, W) = (r, u, w)/R \quad \tau = (t/R^2)(D/\rho h)^{1/2} \quad \delta = R/h \quad (5a)$$

$$\lambda = 12(1 + v_\theta\beta)\delta^2\alpha_r T_0 \quad \mu = (v_\theta + \beta k)/(1 + \beta v_\theta) \quad (5b)$$

$$k = E_\theta/E_r = v_{\theta r}/v_{r\theta} \quad \beta = \alpha_\theta/\alpha_r \quad v_\theta = v_{\theta r} \quad (5c)$$

Here $u(r, t)$ and $w(r, t)$ denote, respectively, the radial and the transverse displacements of a point on the mid-surface of the plate, t the time, E_r and E_θ the elastic moduli in the radial and the circumferential directions respectively, $v_{\theta r}$ and $v_{r\theta}$ Poisson's ratios, α_r and α_θ the coefficients of thermal expansion in the radial and the circumferential directions respectively, $D = E_r h^3/[12(1 - v_{\theta r}v_{r\theta})]$ the flexural rigidity of the plate, ρ the mass density of plate's material, and the constant K defines boundary conditions; $K = 0$ and $1/v_\theta$ correspond to the clamped and the simply supported edges, respectively. Δx is a small positive real number, and T_0 is the reference temperature so that the temperature rise in the plate can be expressed as $T(r) = T_0\Theta(x)$. Note that Young's modulus in the radial direction equals that in the thickness direction but differs from that in the circumferential direction. For an isotropic plate, $k = \beta = \mu = 1$; parameters k and β characterize plate's anisotropy.

EQUATIONS FOR INFINITESIMAL DEFORMATIONS SUPERIMPOSED UPON A BUCKLED PLATE

We focus on studying steady-state vibrations corresponding to infinitesimal deformations superimposed upon a quasi-statically buckled plate. We seek solutions of Eqs. (1)–(4) of the form

$$U(x, \tau) = U_0(x) + U_s(x) + U_d(x, \tau) \quad (6)$$

$$W(x, \tau) = W_s(x) + W_d(x, \tau) \quad (7)$$

[6–9], where U_0, the radial displacement of a point on the mid-surface of the pre-buckled plate, is the solution of the following boundary-value problem [5]:

$$\frac{\partial^2 U_0}{\partial x^2} + \frac{1}{x}\frac{\partial U_0}{\partial x} - \frac{k}{x^2}U_0 = \frac{\lambda}{12\delta^2}\left[(1-\mu)\frac{\Theta}{x} + \frac{d\Theta}{dx}\right] \tag{8}$$

$$U_0(0) = U_0(1) = 0 \tag{9}$$

Displacements U_s and W_s of the thermally post-buckled plate are solutions of the boundary-value problem defined by Eqs. (10)–(13).

$$\frac{d^2 U_s}{dx^2} + \frac{1}{x}\frac{dU_s}{dx} - \frac{k}{x^2}U_s + \frac{dW_s}{dx}\frac{d^2 W_s}{dx^2} + \frac{1-v_\theta}{2x}\left(\frac{dW_s}{dx}\right)^2 = 0 \tag{10}$$

$$\frac{d^4 W_s}{dx^4} + \frac{2}{x}\frac{d^3 W_s}{dx^3} - \frac{k}{x^2}\frac{d^2 W_s}{dx^2} + \frac{k}{x^3}\frac{dW_s}{dx} + \lambda\Theta\left(\frac{d^2 W_s}{dx^2} + \frac{1}{x}\frac{dW_s}{dx}\right) + \lambda\frac{d\Theta}{dx}\frac{dW_s}{dx}$$
$$= 12\delta^2\frac{1}{x}\frac{d}{dx}\left[x\left(\frac{dU_0}{dx} + \frac{dU_s}{dx} + \frac{1}{2}\left(\frac{dW_s}{dx}\right)^2 + \frac{v_\theta}{x}(U_0 + U_s)\right)\frac{dW_s}{dx}\right] \tag{11}$$

$$U_s = 0 \quad \frac{dW_s}{dx} = 0 \quad \frac{d^3 W_s}{dx^3} + \frac{1}{x}\frac{d^2 W_s}{dx^2} = 0 \quad \text{at } x = \Delta x \tag{12}$$

$$U_s = 0 \quad W_s = 0 \quad \frac{dW_s}{dx} + K\frac{d^2 W_s}{dx^2} = 0 \quad \text{at } x = 1 \tag{13}$$

Infinitesimal displacements $U_d(x, \tau)$ and $W_d(x, \tau)$ are superimposed on the thermally post-buckled configuration of the plate.

By substituting from Eqs. (6) and (7) into Eqs. (1)–(4), using Eqs. (10)–(13), and neglecting terms non-linear $U_d(x, \tau)$ and $W_d(x, \tau)$, we obtain following equations for the determination of U_d and W_d:

$$\frac{\partial^2 U_d}{\partial x^2} + \frac{1}{x}\frac{\partial U_d}{\partial x} - \frac{k}{x^2}U_d + \frac{dW_s}{dx}\frac{\partial^2 W_d}{\partial x^2} + \left(\frac{d^2 W_s}{dx^2} + \frac{1-v_\theta}{x}\frac{dW_s}{dx}\right)\frac{\partial W_d}{\partial x} = 0 \tag{14}$$

$$\frac{\partial^4 W_d}{\partial x^4} + \frac{2}{x}\frac{\partial^3 W_d}{\partial x^3} - \frac{k}{x^2}\frac{\partial^2 W_d}{\partial x^2} + \frac{k}{x^3}\frac{\partial W_d}{\partial x} + \lambda\Theta\left(\frac{\partial^2 W_d}{\partial x^2} + \frac{1}{x}\frac{\partial W_d}{\partial x}\right) + \lambda\frac{\partial\Theta}{\partial x}\frac{\partial W_d}{\partial x} + \frac{\partial^2 W_d}{\partial \tau^2}$$
$$= \frac{12\delta^2}{x}\frac{\partial}{\partial x}x\left[\left(\frac{dU_0}{dx} + \frac{dU_s}{dx} + \frac{1}{2}\left(\frac{dW_s}{dx}\right)^2 + \frac{v_\theta}{x}(U_0 + U_s)\right)\frac{\partial W_d}{\partial x}\right.$$
$$\left. + \left(\frac{\partial U_d}{\partial x} + \frac{dW_s}{dx}\frac{\partial W_d}{\partial x} + \frac{v_\theta}{x}U_d\right)\frac{dW_s}{dx}\right] \tag{15}$$

$$U_d = 0 \quad \frac{\partial W_d}{\partial x} = 0 \quad \frac{\partial^3 W_d}{\partial x^3} + \frac{1}{x}\frac{\partial^2 W_d}{\partial x^2} = 0 \quad \text{at } x = \Delta x \tag{16}$$

$$U_d = 0 \quad W_d = 0 \quad \frac{\partial W_d}{\partial x} + K\frac{\partial^2 W_d}{\partial x^2} = 0 \quad \text{at } x = 1 \tag{17}$$

If the plate is not buckled, i.e., $U_s(x) = W_s(x) \equiv 0$, then Eqs. (14)–(17) govern linear vibrations of a pre-buckled plate [5].

We assume that Eqs. (14)–(17) have a solution of the form

$$U_d(x, \tau) = \xi(x) \cos \omega\tau \quad W_d(x, \tau) = \eta(x) \cos \omega\tau \tag{18}$$

where ω is a natural frequency of the plate. Substitution from Eq. (18) into Eqs. (13)–(17) yields the following ordinary differential equations for the amplitudes $\xi(x)$ and $\eta(x)$.

$$\frac{d^2\xi}{dx^2} + \frac{1}{x}\frac{d\xi}{dx} - \frac{k}{x^2}\xi + \frac{dW_s}{dx}\frac{d^2\eta}{dx^2} + \frac{d\eta}{dx}\frac{d^2W_s}{dx^2} + \frac{1 - v_\theta}{x}\frac{dW_s}{dx}\frac{d\eta}{dx} = 0 \tag{19}$$

$$\frac{d^4\eta}{dx^4} + \frac{2}{x}\frac{d^3\eta}{dx^3} - \frac{k}{x^2}\frac{d^2\eta}{dx^2} + \frac{k}{x^3}\frac{d\eta}{dx} + \lambda\Theta\left(\frac{d^2\eta}{dx^2} + \frac{1}{x}\frac{d\eta}{dx}\right) + \lambda\frac{d\Theta}{dx}\frac{d\eta}{dx} - \omega^2\eta$$

$$= \frac{12\delta^2}{x}\frac{d}{dx}x\left[\left(\frac{dU_0}{dx} + \frac{dU_s}{dx} + \frac{1}{2}\left(\frac{dW_s}{dx}\right)^2 + \frac{v_\theta}{x}(U_0 + U_s)\right)\frac{d\eta}{dx}\right.$$

$$\left. + \left(\frac{d\xi}{dx} + \frac{dW_s}{dx}\frac{d\eta}{dx} + \frac{v_\theta}{x}\xi\right)\frac{dW_s}{dx}\right] \tag{20}$$

$$\xi = 0 \quad \frac{d\eta}{dx} = 0 \quad \frac{d^3\eta}{dx^3} + \frac{1}{x}\frac{d^2\eta}{dx^2} = 0 \quad \text{at } x = \Delta x \tag{21}$$

$$\xi = 0 \quad \eta = 0 \quad \frac{d\eta}{dx} + K\frac{d^2\eta}{dx^2} = 0 \quad \text{at } x = 1 \tag{22}$$

We note that Eqs. (19)–(22) are the same as those of free vibration of a polar orthotropic circular plate with initial deformations U_s and W_s. However, here initial deformations are unknown, and are to be determined by solving coupled nonlinear Eqs. (10)–(13).

NUMERICAL RESULTS AND DISCUSSIONS

Henceforth, we assume that the temperature rise in the plate is uniform, i.e., $\Theta(x) \equiv 1$. In this case, Eqs. (8)–(9) have the solution

$$U_0(x) = \begin{cases} \dfrac{B}{1 - k}\left(\dfrac{(\Delta x - \Delta x^{-\sqrt{k}})x^{\sqrt{k}} - (\Delta x - \Delta x^{\sqrt{k}})x^{-\sqrt{k}}}{\Delta x^{-\sqrt{k}} - \Delta x^{\sqrt{k}}} + x\right) & \text{for } k \neq 1 \\ 0 & \text{for } k = 1. \end{cases} \tag{23}$$

where $B = (1 - \mu)\lambda/(12\delta^2)$. The limiting values of both sides of Eq. (23) as $\Delta x \to 0$ give the following solution for a circular plate

$$U_0(x) = \begin{cases} \dfrac{B}{1 - k}\left(x - x^{\sqrt{k}}\right) & \text{for } k \neq 1 \\ 0 & \text{for } k = 1 \end{cases} \tag{24}$$

It is hard to find a closed-form solution of Eqs. (19)–(22) because of the difficulty in finding an analytical solution of the coupled nonlinear Eqs. (10)–(13).

Table 1 Thermal load parameter, λ/λ_{cr}, versus the maximum deflection, f, of a post-buckled isotropic circular plate

	0.2	0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
					$f = w_s(0)/h$				
				Simply supported					
Present	1.0730	1.2930	1.6637	2.1906	2.8816	5.3962	9.2027	14.058	21.763
Ref. [3]	1.0730	1.2930	1.6637	2.1907	2.8818	—	—	—	—
				Clamped					
Present	1.0205	1.0835	1.1887	1.3366	1.5278	2.2013	1.1724	4.4726	6.1426
Ref. [3]	1.0209	1.0839	1.1890	1.3367	1.5277	—	—	—	—

Accordingly, we employ the shooting method, as was also done in [19, 20], to obtain the post-buckled configuration, vibration modes, and the corresponding natural frequencies. The shooting method replaces the two-point boundary-value problem by a sequence of initial-value problems. Thus, unknown values of functions at the initial point, $x = \Delta x$, are estimated to start the computations. These are iterated upon with modified values obtained by the secant method until at the final point, $x = 1$, prescribed boundary conditions are satisfied; details of this approach can be found in [5, 30].

In solving the problem numerically, we set $\delta = R/h = 20$, and Poisson's ratio $\nu_\theta = 0.3$. The singularity at $\Delta x = 0$ for a circular plate is avoided by taking $\Delta x = 0.001$. The accuracy of the numerical algorithm has been established by computing results for a few problems, and comparing them with those available in the literature. We have compared, in Table 1, presently computed values of the thermal load parameter, λ/λ_{cr}, where λ_{cr} is the non-dimensional critical temperature rise, and the corresponding maximum post-buckling deflection, $f = w_s(0)/h \approx W_s(\Delta x)\delta$, for both simply supported and clamped isotropic circular plates with those obtained in [3] by the finite element method. It is evident that for both simply supported and clamped plates presently computed results agree very well with those of Raju and Rao [3]. For centroidal deflection equal to three times the plate thickness, the thermal load for a simply supported plate equals nearly 3.55 times that for an identical clamped plate. However, for the centroidal deflection equal to 0.2h, the thermal load for the simply supported plate is nearly 5% more than that for the clamped plate. In Table 2, a comparison is made of the presently found fundamental frequency, ω_1, of a polar orthotropic circular plate with that computed by the Ritz method in [27]. For both simply supported and clamped circular plates with three values 0.75, 1.0 and 10.0 of the rigidity ratio k, the presently computed fundamental

Table 2 Fundamental frequency, ω_1, versus rigidity ratio, k, for an unheated circular plate; values from Ref. [27] are given in parentheses

	0.75	1.0	10.0
		$k = E_\theta/E_r$	
Simply supported	4.5418 (4.5418)	4.9352 (4.9351)	11.286 (11.286)
Clamped	9.8058 (9.8056)	10.216 (0.216)	16.862 (16.862)

Table 3 First three natural frequencies of an unheated isotropic circular plate

	Simply supported			Clamped		
Ref.	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
26	4.9350	29.720	74.156	10.216	39.771	89.103
28	4.9351	29.720	74.156	10.216	39.771	89.104
Present	4.9352	29.720	74.156	10.216	39.772	89.105

(a) Simply supported
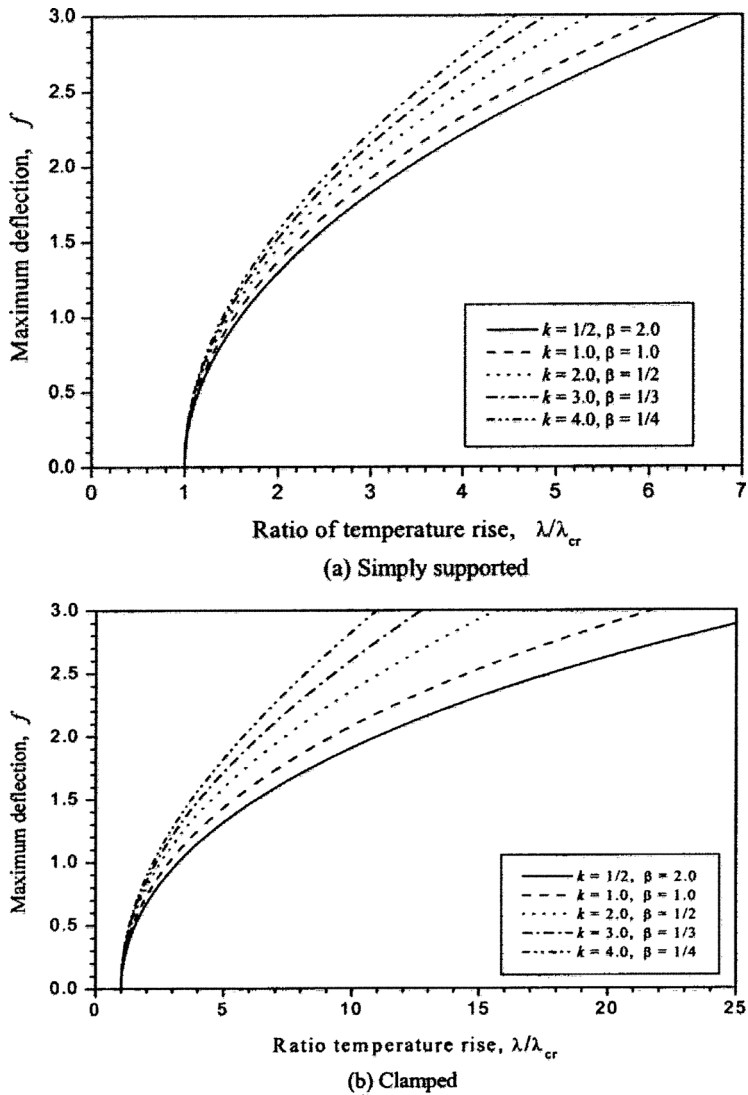
(b) Clamped

Figure 1 For specified values of k and β, post-buckled equilibrium paths of a circular plate.
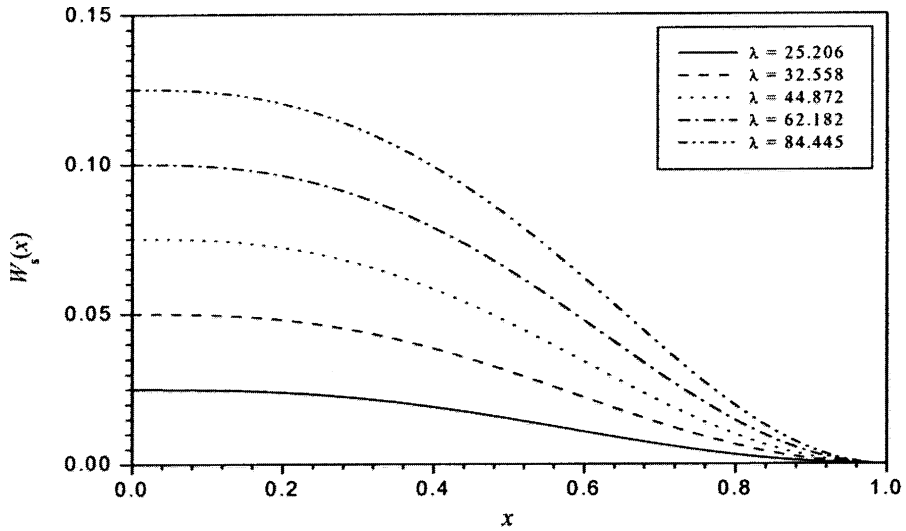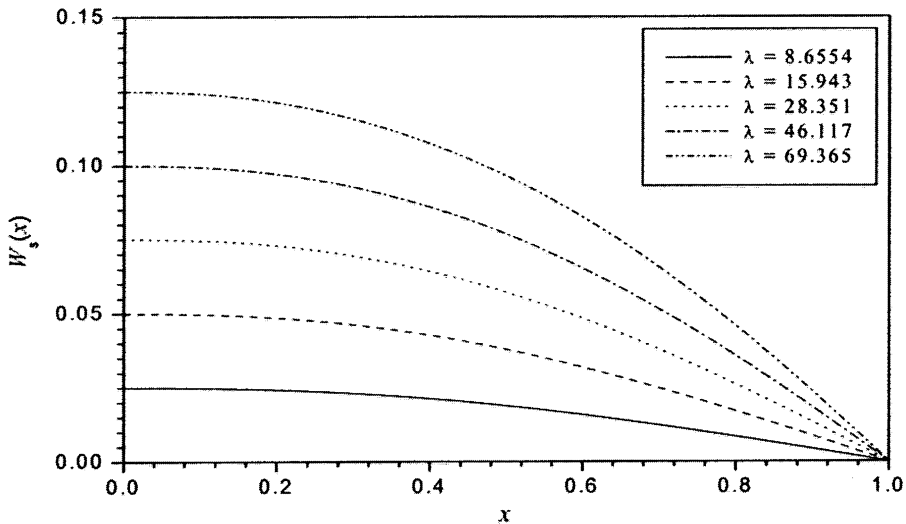
(a) Clamped ($k = 3.0$, $\beta = 1/3$)

(b) Simply supported ($k = 2.0$, $\beta = 0.5$)

Figure 2 Thermally post-buckled configurations of circular plates for different values of temperature rise.

frequency agrees well with that computed by the Ritz method. For an increase in the rigidity ratio from 1 to 10, the fundamental frequency of a simply supported circular plate increases from 4.94 to 11.29 but that of a clamped circular plate increases from 10.22 to 16.86. It thus appears that the rigidity ratio has less noticeable effect on the fundamental frequency of a clamped circular plate, but a large effect on the fundamental frequency of a simply supported plate. For both simply supported and clamped unheated isotropic circular plates, the first three presently computed lowest
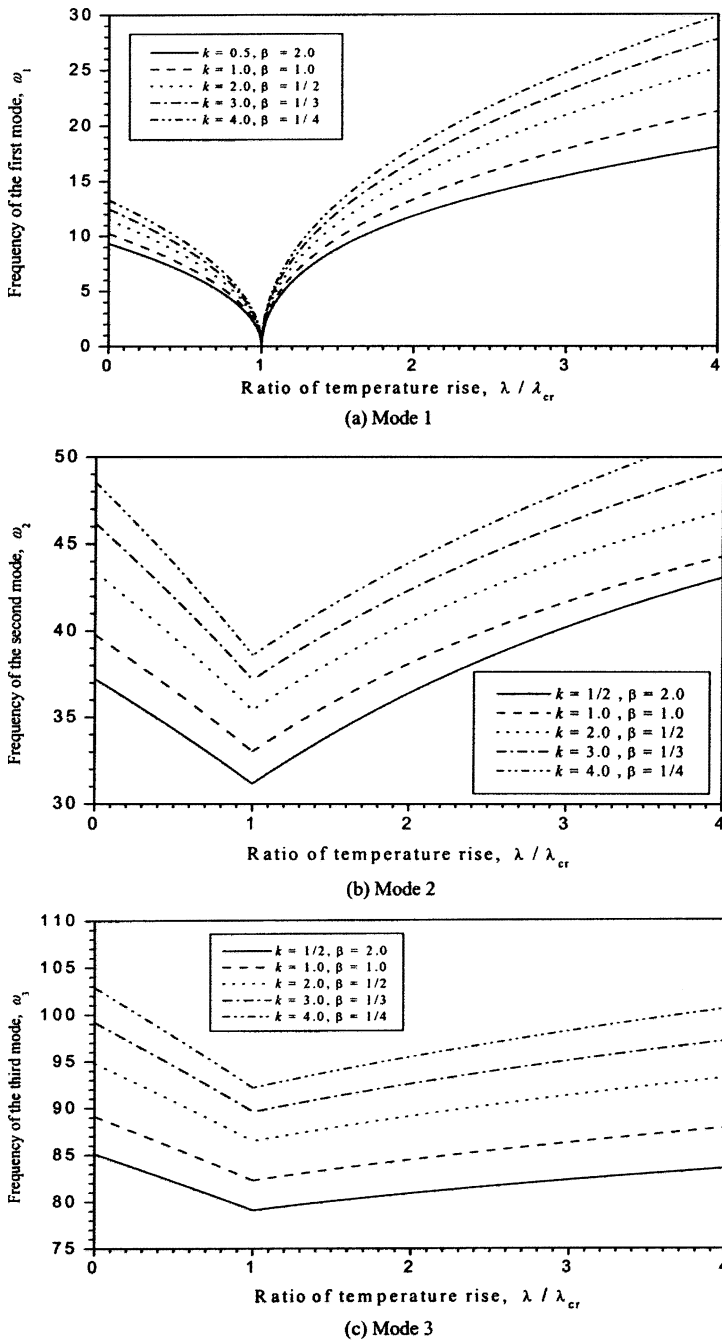
Figure 3 For specified values of k and β, first three natural frequencies versus the temperature rise of a clamped plate in pre-buckled and post-buckled regions.
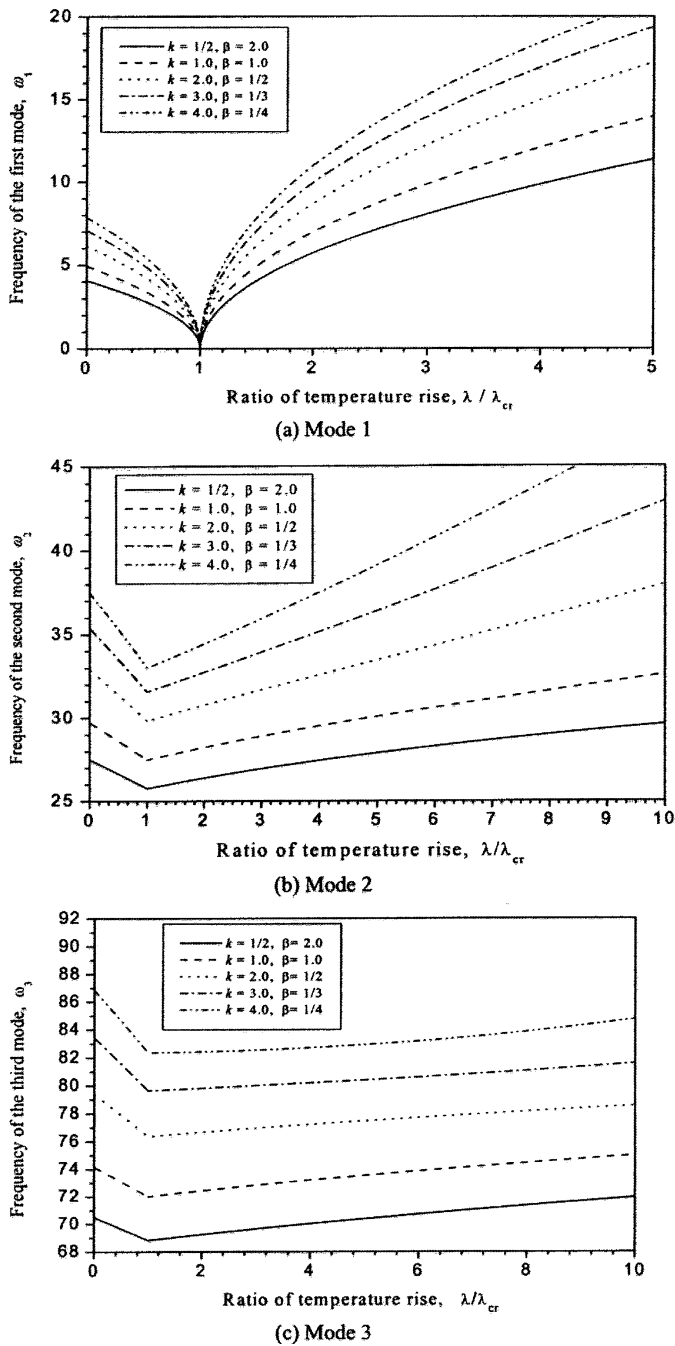
(a) Mode 1

(b) Mode 2

(c) Mode 3

Figure 4 For prescribed values of k and β, first three natural frequencies versus temperature rise of a simply supported plate in pre-buckled and post-buckled regions.

Table 4 Critical temperature rise parameter λ_{cr} for given values of (k, β)

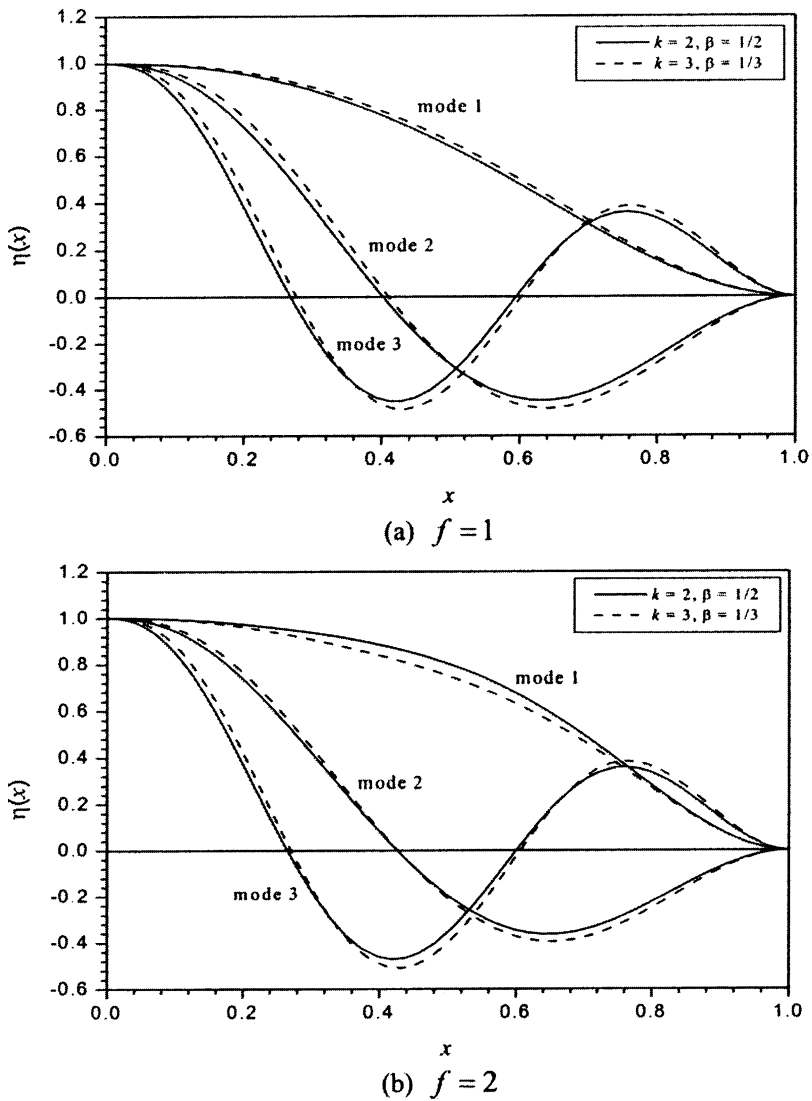|	(k, β)						
	$(1/3, 3)$	$(1/2, 2)$	$(1, 1)$	$(2, 1/2)$	$(3, 1/3)$	$(4, 1/4)$	$(5, 1/5)$
Simply supported (SS)	2.6648	3.0640	4.1978	6.2574	8.1581	9.9591	11.694
Clamped (C)	10.916	11.988	14.682	19.044	22.763	26.162	29.254
Ratio (C/SS)	4.1	3.9	3.5	3.04	2.79	2.63	2.5

(a) $f = 1$

(b) $f = 2$

Figure 5 Mode shapes of a clamped plate in the post-buckled region.
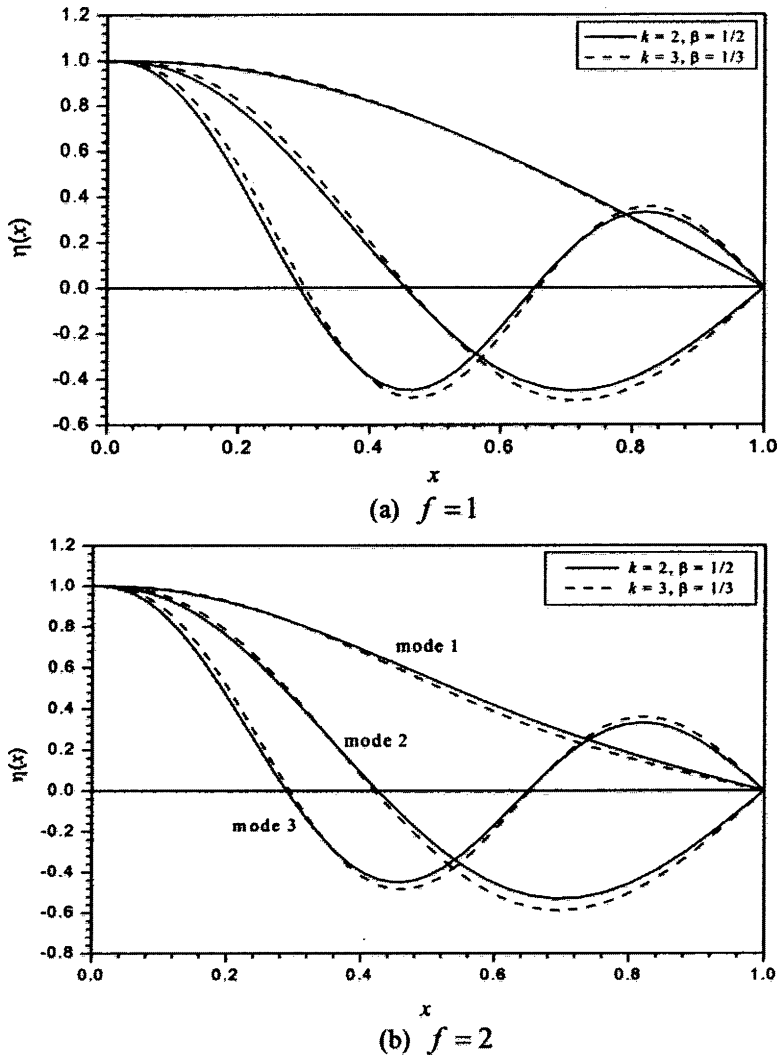
Figure 6 Mode shapes of a simply supported plate in the post-buckled region.

natural frequencies are compared with those of [28] in Table 3. The close agreement between the two sets of frequencies further confirms that the shooting method used herein gives accurate results.

Post-buckled paths in terms of the maximum deflection f and the thermal load parameter λ/λ_{cr} for some specified values of k and β are plotted in Figure 1 for both clamped and simply supported edges. These reveal that results for the two boundary conditions are qualitatively similar to each other. For a fixed value of the temperature rise, λ, the post-buckling maximum deflection f increases with an increase in the value of k and a simultaneous decrease in the value of β. This is because a higher value of k and a lower value of β decrease plate's flexibility in

the radial direction. Figure 2 depicts thermally post-buckled configurations of both simply supported and clamped plates for different values of temperature rise.

By simultaneously solving boundary-value problems (10)–(13) and (19)–(22) we get frequencies and the corresponding mode shapes of a thermally post-buckled orthotropic circular plate with either simply supported or clamped boundary. Figures 3 and 4 exhibit the dependence of the first three natural frequencies upon the thermal load parameter, λ/λ_{cr}. For determining the relationship between ω and λ in Figures 3 and 4, values of the critical temperature rise corresponding to values of k and β used to plot results in these figures are given in Table 4. For all values of k and β considered here, the critical temperature for a clamped plate is higher than that for an identical simply supported plate, the ratio of the former to the latter varies from 4.1 to 2.5. From results plotted in Figures 3 and 4, we conclude that the first three frequencies of a pre-buckled plate decrease with an increase in the temperature. As expected, the fundamental frequency approaches zero at the buckling point. For a buckled plate, frequencies are increasing functions of λ, implying thereby that a buckled plate can support additional load without failure. Furthermore, we see that the characteristic curves are continuous but not differentiable at $\lambda = \lambda_{cr}$. Results plotted in Figure 1 suggest that this is a bifurcation point through which the plate goes into the secondary equilibrium path (post-buckling region) from the initial plane equilibrium state. Pre-buckled and post-buckled equilibrium configurations of the plate are totally different. The first three mode shapes of vibration of a post-buckled plate are plotted in Figures 5 and 6; it is clear that the mode shapes are virtually unaffected by the material property parameters k and β.

CONCLUSIONS

We have analyzed free vibrations of a pre-buckled and a thermally post-buckled polar orthotropic circular plate with boundary either clamped or simply supported but constrained from moving radially. The dependence of the first three natural frequencies upon the temperature rise for both pre-buckled and post-buckled plates has been obtained by solving the nonlinear governing equations with a shooting method. Effects of the temperature rise, the degree of anisotropy, and the boundary conditions on plate's frequencies have been delineated. For a plate in an unbuckled equilibrium configuration, the first three frequencies decease with an increase in the temperature. However, for a buckled plate, these frequencies increase monotonically with an increase in the temperature. Thus, a buckled plate can carry additional load when geometric nonlinearities are considered. The buckling of a deformed plate is synonymous with its first frequency approaching zero. The frequency vs. thermal load curve is continuous but not differentiable at the buckling temperature, and the sign of its slope changes abruptly at the buckling temperature.

ACKNOWLEDGMENTS

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