

Transient hydroelastic analysis of sandwich beams subjected to slamming in water

M.C. Ray*, R.C. Batra

Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, MC 0219 Blacksburg, VA 24061, USA



ARTICLE INFO

Article history:

Received 21 February 2012

Received in revised form

4 June 2013

Accepted 4 June 2013

Keywords:

Transient deformations

Sandwich hull

Hashin's failure criteria

Coupled fluid-structure interaction

Sloshing frequencies

ABSTRACT

This work deals with the transient hydroelastic analysis of a sandwich beam which represents a boat hull. The beam is subjected to slamming pressure while it enters into water with constant vertically downward velocity. A coupled hydroelastic finite element model is developed using higher order shear and normal deformation theories for the faces and the core of the beam and the velocity potential theory for the fluid. Transient responses of the beam for transverse deflection and stresses are studied. Dynamic failure analysis has been carried out to investigate the initiation and cause of the failure of the beam due to slamming load.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Because of high bending stiffness to weight and strength to weight ratios, sandwich structures with fiber reinforced composite faces have become potential candidates for boat hulls and submersible vehicles [1]. When a boat or marine vessel sails at high speed, the part of the bottom face of the vessel emerges out of the water and reenters into the water. This induces large impact force at the bottom of the boat hull. Such force is generally called slamming force. This slamming force can attain very high peak value within a very short duration and cause the boat to undergo transient vibrations leading to damage initiation at the bottom of the hull. A good account of research has been carried out on the slamming of bottom hulls of marine vessels. For example, Bishop et al. [2] and Belik et al. [3] investigated the response of beam like ship structures due to slamming pressure. Lee and Leonard [4] carried out a finite element analysis of structures floating or moored in a wave in the time domain. Broderick and Leonard [5] investigated the nonlinear interaction between fluid-filled membranes and ocean waves using boundary element model for the fluid and finite element model for the membrane structure. Lin and Ho [6] presented numerical and experimental analysis for the two-dimensional water entry of a wedge into initially calm water. Rassiot and Mansor [7] presented a method to determine the hull bending moment. Faltinsen [8]

theoretically studied the effect of hydroelasticity on ship slamming by developing a hydroelastic beam model. Landa et al. [9] carried out an analytical study to investigate the effect of slamming pressure on the interlaminar behavior of ship panels made of composite materials. Mei et al. [10] presented the analytical solutions for the water impact of general two-dimensional bodies entering into initially calm water. Lu et al. [11] and Xiao and Batra [25] carried out an hydroelastic analysis of beam subjected to water impact employing boundary element method for the fluid and finite element method for the structure. Battistin and Iafrati [12] estimated the hydrodynamic loads acting on the two-dimensional and axisymmetric bodies entering into the water using boundary element method. Sun and Faltinsen [13] presented a boundary element method to simulate the water impact of horizontal circular cylinders. Korobkin et al. [14] developed a finite element model for hydroelastic analysis of beam utilizing the Wagner theory of water impact. Greco et al. [15] theoretically studied the bottom slamming of a very large floating structure. Qin and Batra [16] developed a hydroelastic model for investigating the fluid-structure interaction during slamming of sandwich composite hulls.

Here we investigate the transient hydroelastic response of a sandwich beam which corresponds to one half of a symmetric boat hull subjected to a slamming pressure. A coupled hydroelastic finite element model has been developed using higher order shear and normal deformation theories for each layer of the beam and the velocity potential theory for the fluid. Failure analysis is also carried out to ascertain the initiation, location and cause of the failure of the boat hull due to water impact.

* Corresponding author.

E-mail addresses: mcray@mech.iitkgp.ernet.in (M.C. Ray), rbatra@vt.edu (R.C. Batra).

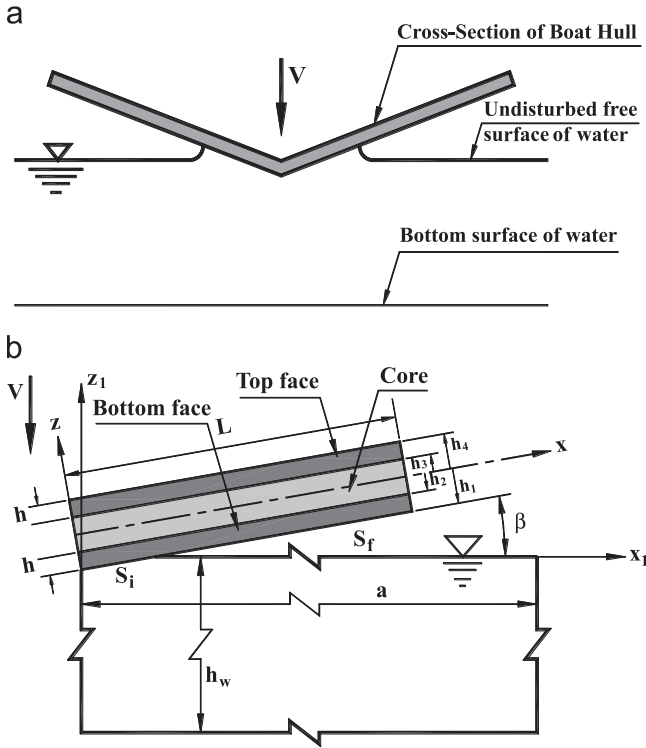


Fig. 1. Schematic diagram of a boat hull entering into water with constant vertical velocity: (a) vertical cross-section of the coupled hull and water; (b) coordinate systems and geometrical features of one half of the hull coupled with water for finite element model.

2. Problem description and basic equations

Fig. 1 illustrates a two-dimensional cross-section of the boat hull entering into the water with a constant vertical velocity. As shown in this figure, the hull is a sandwich structure having symmetry about the vertical plane. Two Cartesian coordinate systems are used to model the coupled system. The origin of the coordinate system (x_1, z_1) for modeling the fluid is located on the surface of the undisturbed fluid such that lines $x_1 = 0$ and $x_1 = a$ represent vertical boundaries of the fluid domain while lines $z_1 = 0$ and $z_1 = -h_w$ represent the top undisturbed surface and the bottom surface of the fluid, respectively. The origin of the coordinate system (x, z) for modeling the beam is at the mid-plane of the beam such that lines $x = 0, L$ represent ends of the beam. The angle β as shown in Fig. 1(b) is called the dead rise angle of the boat hull. The depth of the water, the free water surface and the interface between the bottom wetted surface of the beam and the water are denoted, respectively, by h_w , S_f and S_i . The top and the bottom face sheets of the sandwich beam are composed of a unidirectional fiber-reinforced composite while the core of the beam is made of a foam being modeled as an isotropic homogeneous material. The thickness of the top face sheet, the core and the bottom face sheet equal, respectively, h , $2h_c$ and h .

High-order shear and normal deformation theory (HSNDT) proposed by Lo et al. [17] is augmented for each face and the core of the beam to model kinematics of deformations of the beam while maintaining the continuity of displacements across the two interfaces between the core and the face sheets. Thus, the x -displacements u_t , u_c and u_b of a point in the top face sheet, the core and the bottom face sheet, respectively, are given by

$$u_t = u_0 + h_c l_x + h_c^2 m_x + h_c^3 n_x + (z - h_c) \theta_x + (z^2 - h_c^2) \alpha_x + (z^3 - h_c^3) \beta_x \quad (1)$$

$$u_c = u_0 + z l_x + z^2 m_x + z^3 n_x \quad (2)$$

$$u_b = u_0 - h_c l_x + h_c^2 m_x - h_c^3 n_x + (z + h_c) \phi_x + (z^2 - h_c^2) \gamma_x + (z^3 + h_c^3) \lambda_x \quad (3)$$

Here u_0 represents the generalized x -displacement of a point on the reference plane ($z = 0$), l_x is the first order rotation of the normal to the mid-plane with respect to the x -axis, and variables $m_x, n_x, \theta_x, \alpha_x, \beta_x, \phi_x, \gamma_x$ and λ_x represent the higher order rotations of the normal. Superscripts and subscripts t , c , and b designate the top face sheet, the core and the bottom face sheet, respectively. Substitution of $z = h_c$ in Eq. (1) and $z = -h_c$ in Eq. (3) yields $u_t = u_c = u_b$ ensuring the continuity of x -displacements at two interfaces. The transverse or the z -displacements w_t , w_c and w_b of a point in the top face sheet, the core and the bottom face sheet, respectively, are assumed to be given by

$$w_t = w_0 + h_c l_z + h_c^2 m_z + (z - h_c) \alpha_z + (z^2 - h_c^2) \beta_z \quad (4)$$

$$w_c = w_0 + z l_z + z^2 m_z \quad (5)$$

$$w_b = w_0 - h_c l_z + h_c^2 m_z + (z + h_c) \gamma_z + (z^2 - h_c^2) \lambda_z \quad (6)$$

in which w_0 is the z -displacement of a point on the mid-surface of the beam.

For brevity, we group the generalized displacements into the following two vectors:

$$\{d_t\} = [u_0 \quad w_0]^T \text{ and } \{d_r\} = [l_x \quad m_x \quad n_x \quad \theta_x \quad \alpha_x \quad \beta_x \quad \phi_x \quad \gamma_x \quad \lambda_x \quad l_z \quad m_z \quad \gamma_z \quad \lambda_z \quad \alpha_z \quad \beta_z]^T \quad (7)$$

In order to implement the selective integration rule for computing the element stiffness matrices corresponding to the transverse shear deformations, the strain at a point in the beam is grouped into the following two strain vectors $\{\epsilon_n^i\}$ and ϵ_s^i :

$$\{\epsilon_n^i\} = [\epsilon_x^i \quad \epsilon_z^i]^T \text{ and } \epsilon_s^i = \epsilon_{xz}^i; \quad i = t, c, b \quad (8)$$

in which ϵ_x and ϵ_z are normal strains along the x - and the z -directions, respectively, and ϵ_{xz} is the transverse shear strain. Using displacement fields (1)–(6) and the linear strain–displacement relations, strain vectors $\{\epsilon_n^t\}$, $\{\epsilon_n^b\}$, $\{\epsilon_n^c\}$, ϵ_s^t , ϵ_s^b and ϵ_s^c can be expressed as

$$\{\epsilon_n^t\} = \{\epsilon_{bt}\} + [Z] \{\epsilon_{br}^t\}, \quad \{\epsilon_n^b\} = \{\epsilon_{bt}\} + [Z] \{\epsilon_{br}^b\} \text{ and } \{\epsilon_n^c\} = \{\epsilon_{bt}\} + [Z] \{\epsilon_{br}^c\} \quad (9)$$

$$\epsilon_s^t = \epsilon_{st} + [Z] \{\epsilon_{sr}^t\}, \quad \epsilon_s^b = \epsilon_{st} + [Z] \{\epsilon_{sr}^b\} \text{ and } \epsilon_s^c = \epsilon_{st} + [Z] \{\epsilon_{sr}^c\} \quad (10)$$

Matrices appearing in Eqs. (9) and (10) are defined in Appendix A while the generalized strain vectors are given by

$$\begin{aligned} \epsilon_{bt} &= \left[\frac{\partial u_0}{\partial x} \quad 0 \right]^T, \quad \{\epsilon_{br}^t\} = \left[\frac{\partial l_x}{\partial x} \quad \frac{\partial m_x}{\partial x} \quad \frac{\partial n_x}{\partial x} \quad \frac{\partial \theta_x}{\partial x} \quad \frac{\partial \alpha_x}{\partial x} \quad \frac{\partial \beta_x}{\partial x} \quad \alpha_z \quad \beta_z \right]^T, \\ \{\epsilon_{br}^b\} &= \left[\frac{\partial l_x}{\partial x} \quad \frac{\partial m_x}{\partial x} \quad \frac{\partial n_x}{\partial x} \quad \frac{\partial \phi_x}{\partial x} \quad \frac{\partial \gamma_x}{\partial x} \quad \frac{\partial \lambda_x}{\partial x} \quad \gamma_z \quad \lambda_z \right]^T, \\ \{\epsilon_{br}^c\} &= \left[\frac{\partial l_x}{\partial x} \quad \frac{\partial m_x}{\partial x} \quad \frac{\partial n_x}{\partial x} \quad l_z \quad m_z \right]^T, \quad \epsilon_{st} = \frac{\partial w_0}{\partial x}, \\ \{\epsilon_{sr}^t\} &= \left[\theta_x \quad \alpha_x \quad \beta_x \quad \frac{\partial l_z}{\partial x} \quad \frac{\partial m_z}{\partial x} \quad \frac{\partial \alpha_z}{\partial x} \quad \frac{\partial \beta_z}{\partial x} \right]^T, \\ \{\epsilon_{sr}^b\} &= \left[l_x \quad m_x \quad n_x \quad \frac{\partial l_z}{\partial x} \quad \frac{\partial m_z}{\partial x} \right]^T, \\ \{\epsilon_{sr}^c\} &= \left[\phi_x \quad \gamma_x \quad \lambda_x \quad \frac{\partial l_z}{\partial x} \quad \frac{\partial m_z}{\partial x} \quad \frac{\partial \gamma_z}{\partial x} \quad \frac{\partial \lambda_z}{\partial x} \right]^T \end{aligned} \quad (11)$$

Similar to strain vectors given by Eq. (8), stresses at a point in the beam are described by the following two stress vectors:

$$\{\sigma_n^i\} = [\sigma_x^i \quad \sigma_z^i]^T \text{ and } \sigma_s^i = \sigma_{xz}^i, \quad i = t, c, b \quad (12)$$

where σ_x and σ_z are the normal stresses on the x - and the z -planes, respectively, and σ_{xz} is the transverse shear stress. Assuming the material of the face sheets and the core to be linear elastic,

their constitutive relations are

$$\{\epsilon_n^i\} = [C_n^i]\{\epsilon_n^t\} \quad \text{and} \quad \sigma_s^i = C_{55}^i \epsilon_{xz}^i, \quad i = t, c, b \quad (13)$$

in which the elastic coefficient matrix $[C_n^i]$ is given by

$$[C_n^i] = \begin{bmatrix} C_{11}^i & C_{13}^i \\ C_{13}^i & C_{33}^i \end{bmatrix} \quad (14)$$

Hamilton's principle

$$\int_{t_1}^{t_2} (\delta T_p - \delta T_k) dt = 0 \quad (15)$$

is employed to derive equations governing deformations of motion of the beam. In Eq. (15) T_p and T_k are the total potential and the total kinetic energies of the beam, respectively, δ is the variational operator, and t_1 and t_2 are the starting and the ending times. The first variations of the total potential and the total kinetic energies of the beam can be expressed as

$$\delta T_p = b \int_0^L \left[\int_{h_1}^{h_2} (\delta \{\epsilon_n^t\})^T \{\sigma_n^t\} + \delta \epsilon_s^t \sigma_s^t dz + \int_{h_2}^{h_3} (\delta \{\epsilon_n^b\})^T \{\sigma_n^b\} + \delta \epsilon_s^b \sigma_s^b dz \right. \\ \left. + \int_{h_3}^{h_4} (\delta \{\epsilon_n^c\})^T \{\sigma_n^c\} + \delta \epsilon_s^c \sigma_s^c dz - p \delta w_b \right] dx \quad (16)$$

$$\delta T_k = b(\rho^t h + 2\rho^c h_c + \rho^b h) \int_0^L \delta \{\dot{d}_t\}^T \{\dot{d}_t\} dx \quad (17)$$

in which ρ^i denotes the mass per unit length of the i -th layer of the beam while p is the externally applied pressure acting normal to the bottom surface of the beam. Note that in Eq. (16), $h_1 = -(h + h_c)$, $h_2 = -h_c$, $h_3 = h_c$ and $h_4 = h + h_c$. Here, we have neglected effects of rotary inertia which is a reasonable approximation for $(h + h_c) \ll L$. Also, a dot over a variable represents the differentiation of the variable with respect to time.

3. The finite element model of the beam

The beam is discretized by three noded quadratic isoparametric beam elements of length L_e . Following Eq. (7), the generalized displacement vectors, associated with the i -th ($i = 1, 2, 3$) node of an element can be written as

$$\{d_{ii}\} = [u_{oi} \quad w_{oi}]^T \quad \text{and} \\ \{d_{ri}\} = [l_{xi} \quad m_{xi} \quad n_{xi} \quad \theta_{xi} \quad \alpha_{xi} \quad \beta_{xi} \quad \phi_{xi} \quad \gamma_{xi} \quad \lambda_{xi} \quad l_{zi} \quad m_{zi} \quad \gamma_{zi} \quad \lambda_{zi} \quad \alpha_{zi} \quad \beta_{zi}]^T. \quad (18)$$

The generalized displacement vector at a point within the element can be expressed in terms of the generalized nodal displacement vectors $\{d_t^e\}$ and $\{d_r^e\}$ by

$$\{d_t\} = [N_t]\{d_t^e\} \quad \text{and} \quad \{d_r\} = [N_r]\{d_r^e\} \quad (19)$$

in which

$$[N_t] = [N_{t1} \quad N_{t2} \quad N_{t3}]^T, \quad [N_r] = [N_{r1} \quad N_{r2} \quad N_{r3}]^T, \\ N_{ti} = n_i I_t, \quad N_{ri} = n_i I_r,$$

$$\{d_t^e\} = \begin{bmatrix} \{d_{t1}\}^T & \{d_{t2}\}^T & \{d_{t3}\}^T \end{bmatrix}^T, \\ \{d_r^e\} = \begin{bmatrix} \{d_{r1}\}^T & \{d_{r2}\}^T & \{d_{r3}\}^T \end{bmatrix}^T. \quad (20)$$

I_t and I_r are (2×2) and (15×15) identity matrices, respectively, and n_i is the shape function of the i -th node written in natural coordinates. Using relations (9)–(11) and (19), the strains at a point are given by

$$\{\epsilon_n^t\} = [B_{tb}]\{d_t^e\} + [Z_2][B_{rb}^t]\{d_r^e\}, \quad \{\epsilon_n^b\} = [B_{tb}]\{d_t^e\} + [Z_3][B_{rb}^b]\{d_r^e\},$$

$$\{\epsilon_n^c\} = [B_{tb}]\{d_t^e\} + [Z_4][B_{rb}^c]\{d_r^e\}, \quad \epsilon_s^t = [B_{ts}]\{d_t^e\} + [Z_5][B_{rs}^t]\{d_r^e\}, \\ \epsilon_s^b = [B_{ts}]\{d_t^e\} + [Z_6][B_{rs}^b]\{d_r^e\} \quad \text{and} \quad \epsilon_s^c = [B_{ts}]\{d_t^e\} + [Z_7][B_{rs}^c]\{d_r^e\} \quad (21)$$

in which the strain–displacement matrices $[B_{tb}]$, $[B_{rb}]$, $[B_{ts}]$ and $[B_{rs}]$ are given by

$$[B_{tb}] = [B_{tb1} \quad B_{tb2} \quad B_{tb3}], \quad [B_{rb}^i] = [B_{rb1}^i \quad B_{rb2}^i \quad B_{rb3}^i], \\ [B_{ts}] = [B_{ts1} \quad B_{ts2} \quad B_{ts3}], \quad [B_{rs}^i] = [B_{rs1}^i \quad B_{rs2}^i \quad B_{rs3}^i] \quad (22)$$

The submatrices B_{tbj} , B_{rbj}^i , B_{tsj} and B_{rsj}^i ($i = t, c, b$; $j = 1, 2, 3$) appearing in Eq. (22) are given in Appendix A. On substitution from Eqs. (13) and (21) into Eqs. (16) and (17) and subsequently, using Eq. (15) we obtain the following equations of motion at the element level:

$$[M^e]\{\ddot{d}_t^e\} + [K_{tt}^e]\{d_t^e\} + [K_{tr}^e]\{d_r^e\} = \{F_t^e\} \quad (23)$$

$$[K_{tr}^e]^T \{d_t^e\} + [K_{rr}^e]\{d_r^e\} = \{F_r^e\} \quad (24)$$

The element mass matrix $[M^e]$, the element stiffness matrices $[K_{tt}^e]$, $[K_{tr}^e]$, $[K_{rr}^e]$, and the element load vector $\{F^e\}$ are given by

$$[M^e] = (\rho^t h + 2\rho^c h_c + \rho^b h) \int_0^{L_e} [N_t]^T [N_t] dx, \quad [K_{tt}^e] = [K_{tt}^e] + [K_{ts}^e], \\ [K_{tr}^e] = [K_{tr}^e] + [K_{ts}^e], \quad [K_{rr}^e] = [K_{rr}^e] + [K_{rs}^e], \\ \{F_t^e\} = \int_0^{L_e} p(x) [N_t]^T [0 \quad 1]^T dx \\ \text{and} \quad \{F_r^e\} = \int_0^{L_e} p(x) [N_r]^T [\bar{Z}]^T dx \quad (25)$$

Explicit expressions for the matrices in Eq. (25) are given in Appendix A. It should be noted that the stiffness matrices associated with the transverse shear strains are derived separately from the stiffness matrices for the normal strains. Thus the former can be evaluated by using a lower-order integration rule than that employed to evaluate the latter to avoid the shear locking problem for thin beams. The element equations of motion are assembled to obtain the following global equations of motion:

$$[M]\{\ddot{X}\} + [K_{tt}]\{X\} + [K_{tr}]\{X_r\} = \{F_t\} \quad (26)$$

and

$$[K_{tr}]\{X\} + [K_{rr}]\{X_r\} = \{F_r\} \quad (27)$$

where $[M]$ is the global mass matrix, $[K_{tt}]$, $[K_{tr}]$ and $[K_{rr}]$ are the global stiffness matrices while $\{F_t\}$ and $\{F_r\}$ are the global nodal force vectors. The transverse stresses computed by the constitutive equations may not be accurate and continuous at the interface between two layers because of dissimilar material properties. Batra and Xiao [26,27] have used a layer-wise third-order shear and normal deformable theory (TSNDT) and shown that transverse shear and normal stresses computed from the 3-D constitutive relations are accurate. Here, the transverse stresses across the thickness of the beam are computed by integrating the governing equations of motions with respect to z as follows:

$$\sigma_{xz}^i = - \int \left(\frac{\partial \sigma_x^i}{\partial x} - \rho^i \ddot{u}^i \right) dz + C_{xz}^i(x), \\ \sigma_z^i = - \int \left(\frac{\partial \sigma_{xz}^i}{\partial x} - \rho^i \ddot{u}^i \right) dz + C_z^i(x), \quad i = t, c, b \quad (28)$$

where C_{xz}^i and C_z^i are to be evaluated by satisfying boundary conditions that σ_{xz} vanishes at points on the top and the bottom surfaces of the beam, σ_z vanishes at points on the top surface of the beam and continuities of transverse stresses at the interfaces. Thus the transverse stresses at a point in the bottom face sheet,

the core and the top face sheet are given by

$$\begin{aligned}
 \sigma_{xz}^b &= - \begin{bmatrix} C_{11}^b & C_{13}^b \end{bmatrix} (z[B_1]\{d_t^e\} + [Z_8][B_2]\{d_r^e\}) + z\rho^c \ddot{u}_0 + C_{xz}^1, \\
 \sigma_{xz}^c &= - \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} (z[B_1]\{d_t^e\} + [Z_{10}][B_3]\{d_r^e\}) + z\rho^c \ddot{u}_0 + C_{xz}^2, \\
 \sigma_{xz}^t &= - \begin{bmatrix} C_{11}^t & C_{13}^t \end{bmatrix} (z[B_1]\{d_t^e\} + [Z_{15}][B_4]\{d_r^e\}) + z\rho^t \ddot{u}_0 + C_{xz}^3, \\
 \sigma_z^t &= \begin{bmatrix} C_{11}^t & C_{13}^t \end{bmatrix} [Z_{16}][B_6]\{d_r^e\} - \frac{1}{2} z^2 \rho^t \begin{bmatrix} 1 & 0 \end{bmatrix} \\
 &\quad \times [B_{tb}]\{\ddot{d}_t^e\} - z \frac{\partial C_{xz}^3}{\partial x} + \rho^t z \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + C_z^1, \\
 \sigma_z^c &= \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} [Z_{18}][B_7]\{d_r^e\} - \frac{1}{2} z^2 \rho^c \begin{bmatrix} 1 & 0 \end{bmatrix} \\
 &\quad \times [B_{tb}]\{\ddot{d}_t^e\} - z \frac{\partial C_{xz}^2}{\partial x} + \rho^c z \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + C_z^2, \\
 \sigma_z^b &= \begin{bmatrix} C_{11}^b & C_{13}^b \end{bmatrix} [Z_{21}][B_8]\{d_r^e\} - \frac{1}{2} z^2 \rho^b \begin{bmatrix} 1 & 0 \end{bmatrix} \\
 &\quad \times [B_{tb}]\{\ddot{d}_t^e\} - z \frac{\partial C_{xz}^1}{\partial x} + \rho^b z \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + C_z^3, \quad (29)
 \end{aligned}$$

where

$$\begin{aligned}
 C_{xz}^1 &= \begin{bmatrix} C_{11}^b & C_{13}^b \end{bmatrix} (h_1[B_1]\{d_t^e\} + [Z_9][B_2]\{d_r^e\}) - h_1 \rho^b \ddot{u}_0, \\
 C_{xz}^2 &= \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} (h_2[B_1]\{d_t^e\} + [Z_{12}][B_3]\{d_r^e\}) - \begin{bmatrix} C_{11}^b & C_{13}^b \end{bmatrix} \\
 &\quad \times (h_2[B_1]\{d_t^e\} + [Z_{11}][B_2]\{d_r^e\}) + h_2(\rho^b - \rho^c) \ddot{u}_0 + C_{xz}^1, \\
 C_{xz}^3 &= \begin{bmatrix} C_{11}^t & C_{13}^t \end{bmatrix} (h_3[B_1]\{d_t^e\} + [Z_{15}][B_4]\{d_r^e\}) - \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} \\
 &\quad \times (h_3[B_1]\{d_t^e\} + [Z_{14}][B_3]\{d_r^e\}) + h_3(\rho^c - \rho^t) \ddot{u}_0 + C_{xz}^2, \\
 C_z^1 &= - \begin{bmatrix} C_{11}^t & C_{13}^t \end{bmatrix} [Z_{17}][B_6]\{d_r^e\} + \frac{1}{2} h_4^2 \rho^t \\
 &\quad \times \begin{bmatrix} 1 & 0 \end{bmatrix} [B_{tb}]\{\ddot{d}_t^e\} + h_4 \frac{\partial C_{xz}^3}{\partial x} - \rho^t h_4 \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} \\
 C_z^2 &= - \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} [Z_{19}][B_7]\{d_r^e\} + \frac{1}{2} h_c^2 \rho^c \\
 &\quad \times \begin{bmatrix} 1 & 0 \end{bmatrix} [B_{tb}]\{\ddot{d}_t^e\} + h_c \frac{\partial C_{xz}^2}{\partial x} - \rho^c h_c \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + \begin{bmatrix} C_{11}^t & C_{13}^t \end{bmatrix} \\
 &\quad \times [Z_{20}][B_6]\{d_r^e\} - \frac{1}{2} h_c^2 \rho^t \begin{bmatrix} 1 & 0 \end{bmatrix} [B_{tb}]\{\ddot{d}_t^e\} - h_c \frac{\partial C_{xz}^3}{\partial x} + \rho^t h_c \\
 &\quad \times \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + C_z^1, \\
 C_z^3 &= - \begin{bmatrix} C_{11}^b & C_{13}^b \end{bmatrix} \times [Z_{22}][B_6]\{d_r^e\} + \frac{1}{2} h_c^2 \rho^b \begin{bmatrix} 1 & 0 \end{bmatrix} [B_{tb}]\{\ddot{d}_t^e\} \\
 &\quad - h_c \frac{\partial C_{xz}^2}{\partial x} + \rho^b h_c \times \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + \begin{bmatrix} C_{11}^c & C_{13}^c \end{bmatrix} \\
 &\quad \times [Z_{23}][B_7]\{d_r^e\} - \frac{1}{2} h_c^2 \rho^c \begin{bmatrix} 1 & 0 \end{bmatrix} [B_{tb}]\{\ddot{d}_t^e\} + h_c \frac{\partial C_{xz}^1}{\partial x} - \rho^c h_c \\
 &\quad \times \begin{bmatrix} 1 & 0 \end{bmatrix} [N_t]\{\ddot{d}_t^e\} + C_z^2
 \end{aligned}$$

$$\begin{aligned}
 [B_1] &= \begin{bmatrix} B_{11} & B_{12} & B_{13} \end{bmatrix}, \\
 [B_2] &= \begin{bmatrix} B_{21} & B_{22} & B_{23} \end{bmatrix}, \\
 [B_3] &= \begin{bmatrix} B_{31} & B_{32} & B_{33} \end{bmatrix}, \\
 [B_4] &= \begin{bmatrix} B_{41} & B_{42} & B_{43} \end{bmatrix}, \\
 [B_6] &= \begin{bmatrix} B_{61} & B_{62} & B_{63} \end{bmatrix}, \\
 [B_7] &= \begin{bmatrix} B_{71} & B_{72} & B_{73} \end{bmatrix}, \\
 [B_8] &= \begin{bmatrix} B_{81} & B_{82} & B_{83} \end{bmatrix}
 \end{aligned}$$

(30)

Expressions for the submatrices B_{1i} , B_{2i} , B_{3i} , B_{4i} , B_{6i} , B_{7i} and B_{8i} ($i=1, 2$ and 3) and $[Z_\alpha]$ ($\alpha=2, 3, \dots, 23$) are given in Appendix A.

4. Finite element model of the fluid domain

Assuming that the fluid is incompressible and inviscid, and its deformations are irrotational the two-dimensional governing

equation of the fluid is given by [4]

$$[\nabla]^T [\nabla \phi(x_1, z_1)] = 0 \quad (31)$$

where ϕ is the velocity potential at any point in the fluid domain and $[\nabla] = [\partial/\partial x_1 \quad \partial/\partial z_1]^T$. At the top free surface, the linearized dynamic free surface condition is given by [4]

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z_1} = 0 \quad (32)$$

The boundary conditions associated with the governing Eq. (31) are [4,11]:

$$\frac{\partial \phi}{\partial x_1} = 0 \text{ at } x_1 = 0 \text{ and } a; \quad \frac{\partial \phi}{\partial z_1} = 0 \text{ at } z_1 = -h_w \text{ and } \frac{\partial \phi}{\partial z} = -V \cos \beta + \dot{w} \text{ on } S_i \quad (33)$$

The elevation of free surface $\eta(x_1, 0)$ in terms of the velocity potential is given by [18]

$$\eta(x_1, 0) = -\frac{1}{g} \frac{\partial \phi}{\partial t}(x_1, 0) \quad (34)$$

where g is the gravitational constant.

The functional which yields the above governing equation and boundary conditions can be written as

$$\Pi_f = \frac{1}{2} \int_{t_1}^{t_2} \left[\int_{\Omega} [\nabla \phi]^T [\nabla \phi] d\Omega - \frac{1}{g} \int_{S_f} \left(\frac{\partial \phi}{\partial t} \right)^2 dS_f - 2 \int_{S_i} \phi (V \cos \beta + \dot{w}) dS_i \right] dt \quad (35)$$

The fluid domain is discretized by two-dimensional four noded isoparametric elements. The velocity potential at any point within a typical finite element of the fluid domain can be expressed as

$$\phi = [N_\phi] \{\phi^e\} \quad (36)$$

where $[N_\phi]$ is the shape function matrix and $\{\phi^e\}$ is the nodal potential degrees of freedom of the element. Substitution of Eq. (36) into Eq. (35) yields the functional for the fluid finite element as follows:

$$\begin{aligned}
 \Pi_f^e &= \frac{1}{2} b \int_{t_1}^{t_2} \left[\int_0^{a^e} \int_0^{h_w^e} \{\phi^e\}^T ([\nabla][N_\phi])^T ([\nabla][N_\phi]) \{\phi^e\} \right. \\
 &\quad \left. dx_1 dz_1 - \frac{1}{g} \int_0^{a^e} \{\dot{\phi}^e\}^T [N_\phi]^T [N_\phi] \Big|_{z_1=0} \{\dot{\phi}^e\} dx_1 \right. \\
 &\quad \left. - 2 \int_0^{a^e} \{\phi^e\}^T [N_\phi]^T (-V \cos \beta + [0 \quad 1][N_t]\{\dot{d}_t^e\} + [\bar{Z}][N_r]\{\dot{d}_r^e\}) dx_1 \right] dt \quad (37)
 \end{aligned}$$

where a^e , b and h_w^e are the length, the width and the height of the fluid element, respectively. Extremization of Π_f^e (i.e. $\delta \Pi_f^e = 0$) leads to the derivation of the following governing finite element equations of the fluid domain:

$$[M_f^e] \{\ddot{\phi}^e\} + [K_f^e] \{\phi^e\} - [R_{fst}^e] \{\dot{d}_t^e\} - [R_{fsr}^e] \{\dot{d}_r^e\} = \{F_\phi^e\} \quad (38)$$

where

$$\begin{aligned}
 [M_f^e] &= \frac{1}{g} \int_0^{a^e} [N_\phi]^T [N_\phi] \Big|_{z_1=0} dx_1, \\
 [K_f^e] &= \int_0^{a^e} \int_0^{h_w^e} ([\nabla][N_\phi])^T ([\nabla][N_\phi]) dx_1 dz_1, \\
 \{R_{fst}^e\} &= \int_0^{a^e} [N_\phi]^T [0 \quad 1][N_t] dx, \quad \{R_{fsr}^e\} = \int_0^{a^e} [N_\phi]^T [\bar{Z}][N_r] dx, \\
 \text{and } \{F_\phi^e\} &= \int_0^{a^e} [N_\phi]^T V \cos \beta \Big|_{z=h_1} dx_1 \quad (39)
 \end{aligned}$$

The elemental governing finite element equations given by (38) are now assembled over the entire fluid space to derive the following global set of equations governing the fluid deformations:

$$[M_f] \{\ddot{\Phi}\} + [K_f] \{\Phi\} - [R_{fst}] \{\dot{X}\} - [R_{fsr}] \{\dot{X}_r\} = \{F_\Phi\} \quad (40)$$

where $[M_f]$ and $[K_f]$ are the global mass and the global stiffness matrices of the fluid, $[R_{fst}]$ and $[R_{fsr}]$ are the global fluid–structure coupling matrices, $\{F_\phi\}$ is the global nodal fluid loading vector and $\{\Phi\}$ is the global nodal velocity potential vector.

5. Coupled fluid–structure model

The linearized expression for the hydrodynamic pressure acting at the wetted bottom surface of the sandwich beam is given by [16]

$$p = -\rho_f \frac{\partial \phi}{\partial t} \quad (41)$$

where ρ_f is the density of the fluid. Using (41) in the expression for elemental load vectors of the beam given by (25), the elemental slamming load at the bottom surface of the sandwich beam can be expressed as

$$\{F_t^e\} = -\rho_f [R_{fst}^e]^T \{\dot{\phi}^e\} \text{ and } \{F_r^e\} = -\rho_f [R_{fsr}^e]^T \{\dot{\phi}^e\} \quad (42)$$

where $[R_{fst}^e]$ and $[R_{fsr}^e]$ are defined in Appendix A.

Substituting Eq. (42) into Eqs. (23) and (24) and then combining the resulting global equations with Eq. (40), the global coupled fluid–structure equations can be obtained as

$$\begin{bmatrix} [M] & [O_{tr}] & [O_{t\phi}] \\ [O_{tr}]^T & [O_{rr}] & [O_{r\phi}] \\ [O_{t\phi}]^T & [O_{r\phi}]^T & [M_f] \end{bmatrix} \begin{Bmatrix} \{\ddot{X}\} \\ \{\ddot{X}_r\} \\ \{\ddot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [O_{tt}] & [O_{tr}] & \rho_f [R_{fst}]^T \\ [O_{tr}]^T & [O_{rr}] & \rho_f [R_{fsr}]^T \\ -[R_{fst}] & -[R_{fsr}] & [O_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{X\} \\ \{X_r\} \\ \{\Phi\} \end{Bmatrix} = \begin{Bmatrix} \{O\} \\ \{O_r\} \\ \{F_\phi\} \end{Bmatrix} \quad (43)$$

in which $[O_{tt}]$, $[O_{tr}]$, $[O_{t\phi}]$, $[O_{rr}]$, $[O_{r\phi}]$ and $[O_{\phi\phi}]$ are null matrices of appropriate sizes while $\{O\}$ and $\{O_r\}$ are appropriate null column vectors.

6. Failure criteria

The stress-based Hashin's criteria [19] are used to determine whether or not a material point of the beam has failed and the corresponding failure mode. According to these criteria, a material point is considered to have failed if the following conditions are satisfied:

Fiber failure $\sigma_x^i/X_t^i \geq 1$ or $|\sigma_x^i|/X_c^i \geq 1$, $i = t$ and b

Matrix tensile or shear failure : $(\sigma_z^i/Y_t^i)^2 + (\sigma_{xz}^i/S^i)^2 \geq 1$, $i = t$ and b

Matrix compressive failure : $\{(Y_c^i/2S^i)^2 - 1\} |\sigma_z^i|/Y_c^i + (\sigma_z^i/2S^i)^2 + (\sigma_{xz}^i/S^i)^2 \geq 1$, $i = t$ and b

Delamination failure : $(\bar{\sigma}_z^i/Z_t^i)^2 + (\bar{\sigma}_{xz}^i/\bar{S}^i)^2 \geq 1$, $i = t$ and b

Core compression : $|\sigma_z^c|/Z_c^c \geq 1$ (44)

where X_t^i , Y_t^i (X_c^i , Y_c^i) are the tensile (the compressive) strengths in the x -, and the z -directions, respectively, and S^i is the transverse shear strength of the materials of the different layers of the beam as denoted by the superscript 'i'. Z_t^i and $\bar{S}^i$ are the interfacial strengths and Z_c^c is the strength of the core in compression.

7. Results and discussions

We compute results for a sandwich beam with the 15 mm thick top and the 15 mm thick bottom face sheets composed of a layer of

unidirectional transversely isotropic T300/5208 graphite/epoxy composite [20] while the 20 mm thick core of the beam is made of polyurethane foam which is treated as an isotropic material [21]. The fibers in the face sheets are aligned along the x -axis, and values of material parameters are listed in Table 1. The length of the beam is considered as 1 m. The number of three noded isoparametric bar elements used for discretizing the beam is taken as 30. We verify the accuracy of the present finite element model of the beam by computing natural frequencies of a cantilever sandwich beam studied by Banerjee and Sobey [22]; the two sets of results listed in Table 2 are in excellent agreement with each other. We have also analyzed deformations of the simply supported composite beam ($0^\circ/90^\circ/0^\circ$), studied by Pagano [23] and compared in Fig. 2 the through the thickness variations of the transverse shear stress. It is clear that the present approach gives very accurate values of the transverse shear stress. To verify the accuracy of the finite element formulation of the fluid domain, the first few slosh frequencies of a 2-D fluid continuum ($a = 2$ m, $h_w = 1$ m) contained in a rigid rectangular tank have been computed and compared with exact solutions available in Ref. [18]. Table 3 illustrates that such two sets of frequencies are in excellent agreement with each other. The Newmark implicit unconditionally stable integration method is employed to compute the hydrodynamic and coupled hydroelastic responses in the time domain. The fluid domain is discretized by 60 four noded two-dimensional isoparametric elements along its length while 40 such elements are used along its depth. Numerical responses are computed by considering the length and the depth of water as 10 m and 6 m, respectively. For further verification of the accuracy of the present finite element model of the fluid domain, the hydrodynamic pressures at the wetted surfaces of rigid V-shaped hull entering into water with constant vertical velocity have been computed. Figs. 3 and 4 illustrate the comparison of such slamming pressures with those obtained by Zhao and Faltinsen [24] when values of the dead rise angle β are 10° and 20° , respectively. It may be observed that the present model also fairly accurately computes the slamming pressure. The normal and tangential velocities at the interface between the water and the rigid hull have been computed and illustrated in Fig. 5. It may be observed from this figure that the normal velocity of the rigid hull is equal to that of the water at the interface between the water and the rigid hull ensuring further that the present finite element model accurately estimates the responses.

For computing hydroelastic responses, the end $x = L$ of the beam is considered to be clamped while at the other end ($x = 0$) the following boundary conditions are imposed:

$$u_0 = l_x = m_x = n_x = \theta_x = \alpha_x = \beta_x = \phi_x = \gamma_x = \lambda_x = 0 \quad (45)$$

Unless otherwise mentioned, it is assumed that the beam enters into the water with constant downward vertical velocity until the wetted length of the beam equals one half of its length after which the beam is assumed to have zero rigid body velocity and undergo transient vibrations. First the effect of flexible beam impacting the water is studied and illustrated in Fig. 6. It may be observed that the slamming pressure at the interface between the water and flexible hull is reduced as compared to that with rigid hull. This may be attributed to the deformation of the hull. Fig. 7 illustrates variation of the transverse displacement at the end (0, 0) of the beam with time for different values of dead rise angles of the beam while the value of the vertically downward water-entry velocity of the beam is 1.5 m/s. It may be observed from this figure that the beam is set into transient vibrations after an initial time during which the beam is wetted. The amplitude of vibration decreases with the increase in the value of the dead rise angle. As expected, Fig. 8 illustrates that for a particular value of the dead rise angle, the amplitude of transverse vibrations of the beam due

Table 1

Material properties of face sheets and the core of the beam.

Materials	C_{11} (GPa)	C_{13} (GPa)	C_{33} (GPa)	C_{55} (GPa)	X_t (MPa)	$Y_t = Y_c$ $= Z_t = Z_c$ (MPa)	X_c (MPa)	S (MPa)	ρ (kg/m ³)
T300/5208 [20]	134.68	4.73	14.38	5.7	1515	43.8	1697	86.9	1620
Core [21]	3.61	1.94	3.61	.0672	8.48	8.48	8.48	4.59	320

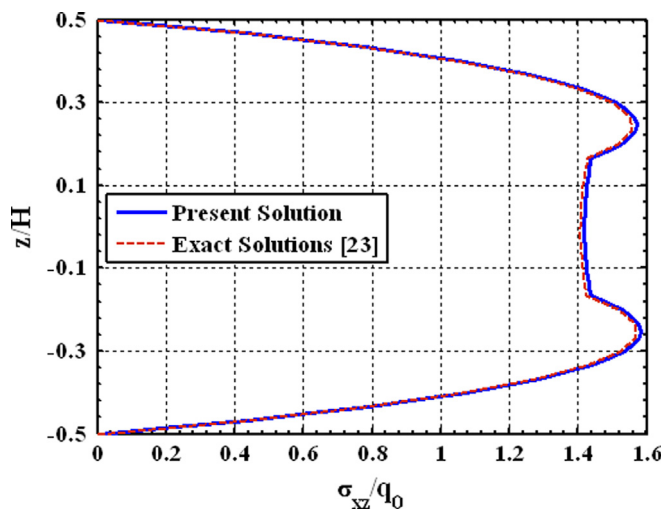
Table 2

Comparison of natural frequencies (Hz) of a sandwich beam with the existing results.

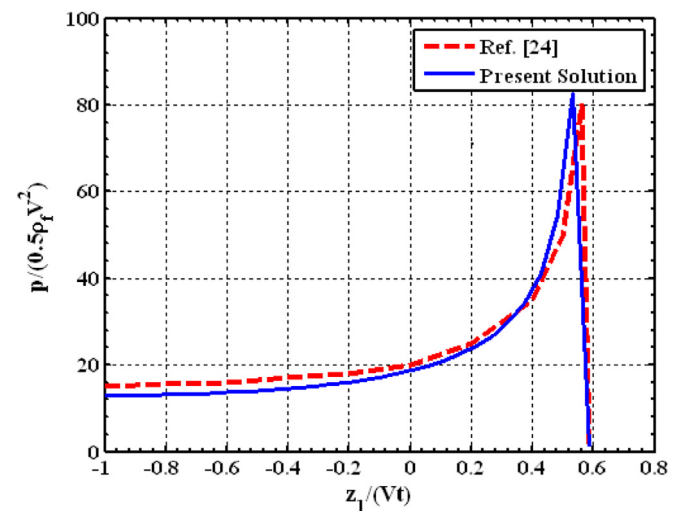
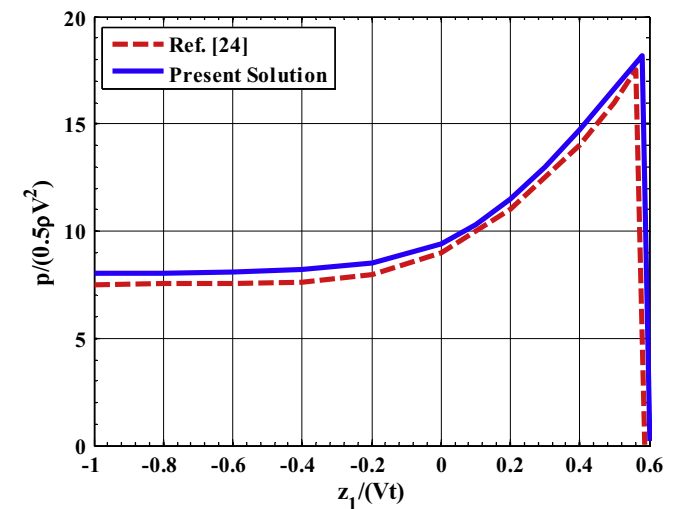
Source	1st mode (Hz)	2nd mode (Hz)	3rd mode (Hz)
Present solution	33.62	198.6	510.2
Ref. [23]	33.74	198.8	511.4

Table 3Comparison of sloshing frequencies (Hz) of two-dimensional fluid contained in a rigid tank ($a = 3$ m, $h_w = 1$ m)

Source	1st mode (rad/s)	2nd mode (rad/s)	3rd mode (Hz)
Present solution	3.7546	5.5210	6.7863
Ref. [18]	3.7594	5.5411	6.7986

**Fig. 2.** Comparison of the transverse normal stress across the thickness of a three-layered ($0^\circ/90^\circ/0^\circ$) simply supported beam of $L/H=4$ with the exact solution of Pagano [23]; H equals the beam thickness and the amplitude of the distributed sinusoidal load equals q_0 .

to coupled hydroelasticity increases with the increase in the value of the water-entry velocity of the beam (V). For a particular value of water-entry velocity ($V=1.5$ m/s), the variation with time of the distribution of axial normal stress (σ_x^b) at the bottom surface of the beam has been illustrated in Figs. 9–11 when values of β are 10° , 15° and 20° , respectively. It may be observed from these figures that since the transverse motion of the end of the beam ($x=L$) is restrained, the magnitude of the axial normal stress is maximum at this fixed end. Also, the magnitude of normal stress decreases with the increase in the value of the dead rise angle. Although not presented here, similar variation with time of the distribution of axial stress (σ_x^t) at the top surface of the top face sheet of the beam has been obtained. Figs. 12–14 demonstrate the distributions of transverse shear stress across the thickness of the beam at the fixed end of the beam for different values of β while the beam enters into water with $V=1.5$ m/s. It can be observed from these figures that the transverse shear stress is continuous along the thickness of the beam and maximum at the middle of both the top and the bottom faces of the beam. The transverse shear stress also decreases with the increase in the dead rise angle. For investigating the initiation of the failure due to slamming pressure, the failure index

**Fig. 3.** Distribution of slamming pressure over the wetted surface of a rigid hull with $\beta = 10^\circ$.**Fig. 4.** Distribution of slamming pressure over the wetted surface of a rigid hull with $\beta = 20^\circ$.

corresponding to each failure mechanism as described by Eq. (44) has been computed by varying the value of the water-entry velocity (V) of the beam. It has been found that the initiation of the

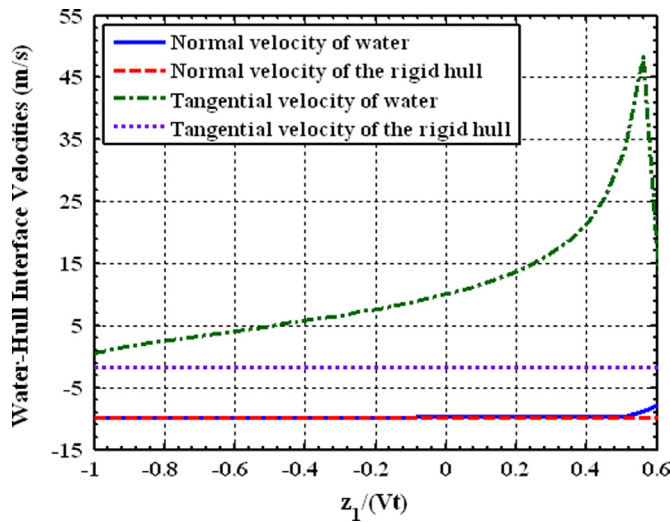


Fig. 5. Normal and tangential velocities at the interface between the water and the rigid hull.

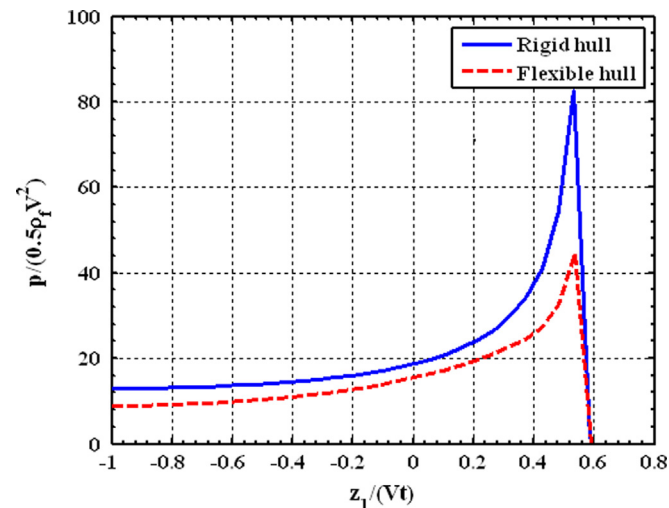


Fig. 6. Comparison of distribution of slamming pressure over the wetted surface of rigid and deformable hulls with $\beta = 10^\circ$.

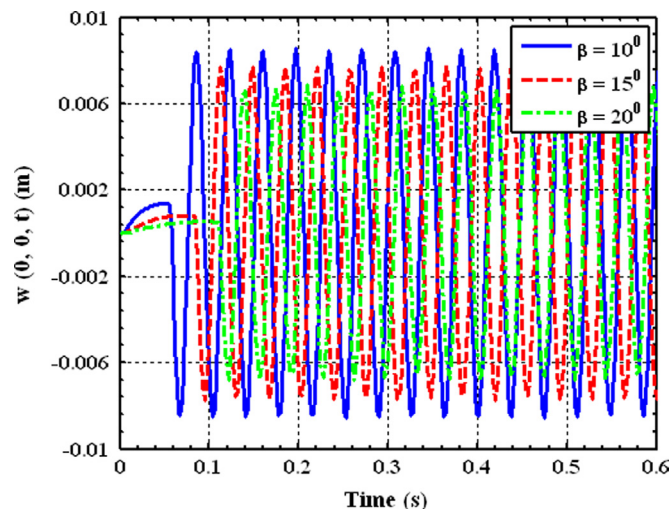


Fig. 7. Transverse deflection at the end (0, 0) of the beam for different dead rise angles of the beam ($V = 1.5$ m/s).

failure in the beam occurs first due to the core compression at that portion of the interface between the core and the bottom face which is located at the clamped end of the beam as shown in Fig. 15 for $\beta = 10^\circ$. For causing this failure the beam enters into water with $V = 1.88$ m/s and the time to cause this failure after the beam starts entering into water is 56.5 ms. It may also be observed from Fig. 15 that except at very small portion of the clamped end of the beam where damage occurs first, the failure index is negligibly small elsewhere due to core compression. For this water-entry velocity which causes core compression failure, the failure indices corresponding to other failure modes are also very small as shown in Figs. 16 and 17 based on the delamination mode of failure and the fiber failure in the bottom face sheet, respectively. Since, $Y_t = Z_t$ for the materials being considered here, the matrix tensile or shear failure criterion yields same value of the failure index. Also, although not shown here, the value of the failure index based on the matrix compressive failure criterion is also very small. Hence, in order to investigate the further load carrying capability of the beam (i.e., the ultimate failure of the beam), the stiffness coefficient of the 10% length of the core starting from the clamped end is degraded and the failure indices corresponding to different failure modes are further computed with gradually increased value of water-entry velocity of the boat. Fig. 18 illustrates such failure

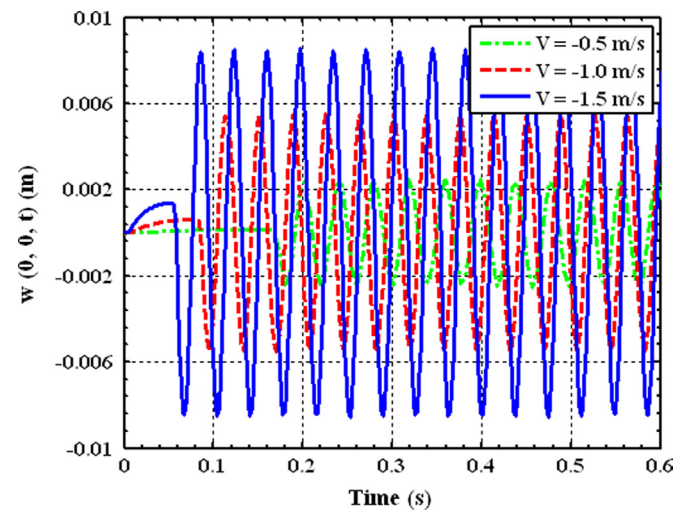


Fig. 8. Transverse deflections at the end (0, 0) of the beam with $\beta = 10^\circ$ for different values of water entry velocity.

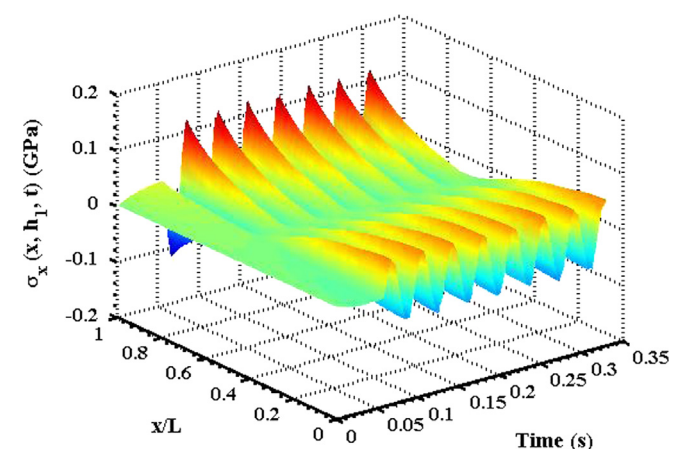


Fig. 9. Variation with time of the axial stress at the bottom surface of the bottom face sheet of the beam ($\beta = 10^\circ$, $V = 1.5$ m/s).

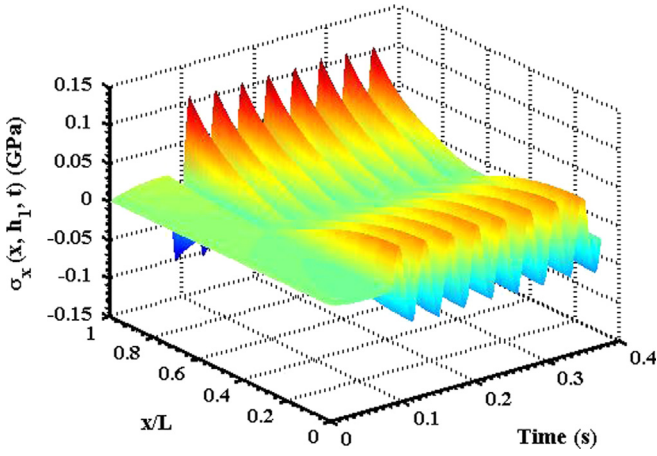


Fig. 10. Variation with time of the axial stress at the bottom surface of the bottom face sheet of the beam ($\beta = 15^\circ$, $V = 1.5$ m/s).

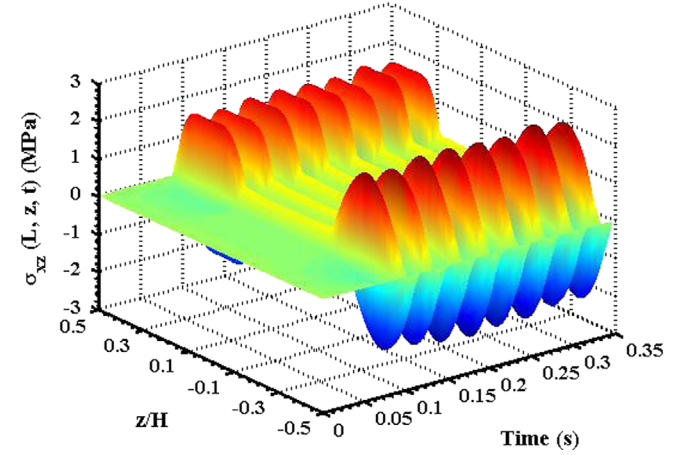


Fig. 13. Variation with time of the transverse shear stress across the thickness of the beam at its clamped end ($\beta = 15^\circ$, $V = 1.5$ m/s).

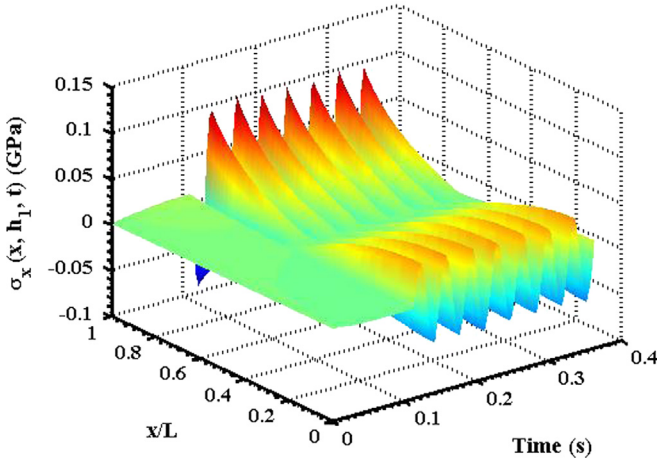


Fig. 11. Variation with time of the axial stress at the bottom surface of the bottom face sheet of the beam ($\beta = 20^\circ$, $V = 1.5$ m/s).

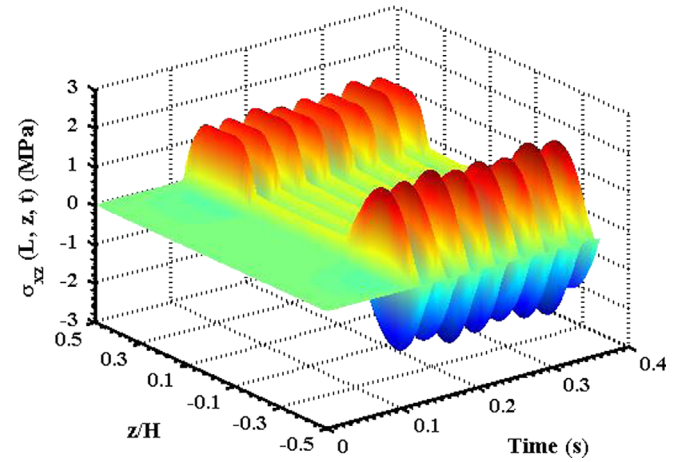


Fig. 14. Variation with time of the transverse shear stress across the thickness of the beam at its clamped end ($\beta = 20^\circ$, $V = 1.5$ m/s).

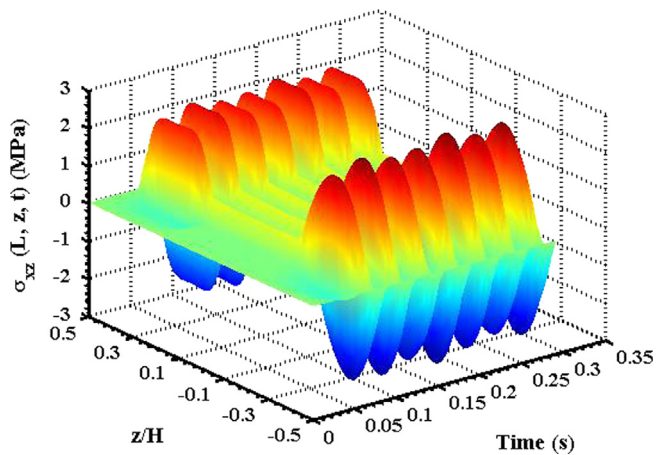


Fig. 12. Variation with time of the transverse shear stress across the thickness of the beam at its clamped end ($\beta = 10^\circ$, $V = 1.5$ m/s).

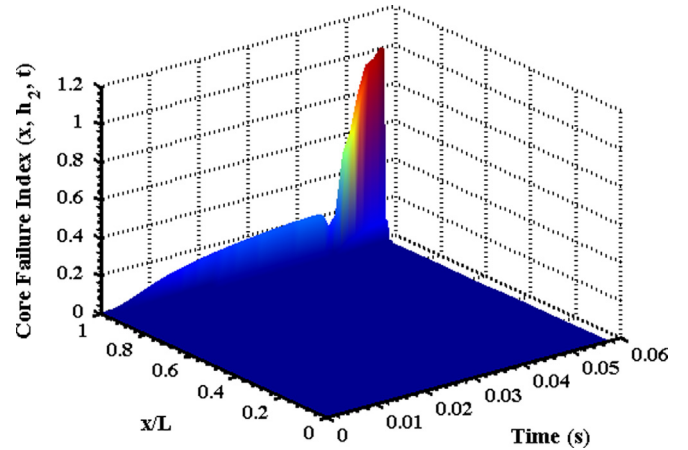


Fig. 15. Variation with time of the failure index due to core compression ($\beta = 10^\circ$, $V = 1.88$ m/s).

indices and it may be observed from this figure that the further initiation of the failure of the beam occurs again due to the core compression when the water-entry velocity of the beam is as high as 23.8 m/s. Note that this failure occurs at the end of the beam

other than the clamped end. Also, for $V = 23.8$ m/s the beam is safe as far as the delamination failure is concerned as shown in Fig. 18. Thus the ultimate failure of the boat hull with sandwich construction is also due to core compression failure. Das and Batra [28] used the commercial software LSDYNA to analyze finite

deformations of a sandwich structure due to water slamming loads and found that the deformations of the core due to transverse shear strains dominate over those due to transverse normal strains. Similar results were obtained by and Qin and Batra [16] who used, respectively, the TSNDT and the {3,2} plate theory. Values of material and geometric parameters used in this paper are different from those employed in [16].

8. Conclusions

Transient hydroelastic analysis of a sandwich beam which represents a boat hull has been performed. The beam enters into water with constant vertically downward velocity until its half of the length is wetted. Thus the beam is subjected to slamming load and undergoes transient vibrations. One end of the beam is fixed while the axial motion of the other end of the beam is restrained for approximate simulation of the boat hull. A coupled hydro-elastic finite element model is developed using layer-wise higher order shear and normal deformation theories for the face sheets and the core of the beam and the velocity potential theory for the fluid. Hydroelastic responses of the beam indicate that for a constant water-entry velocity, the amplitude of vibrations of the beam impacted by water increases with the decrease in the value of the dead rise angle while for a constant dead rise angle the amplitude of vibrations increases with the increase in the water-

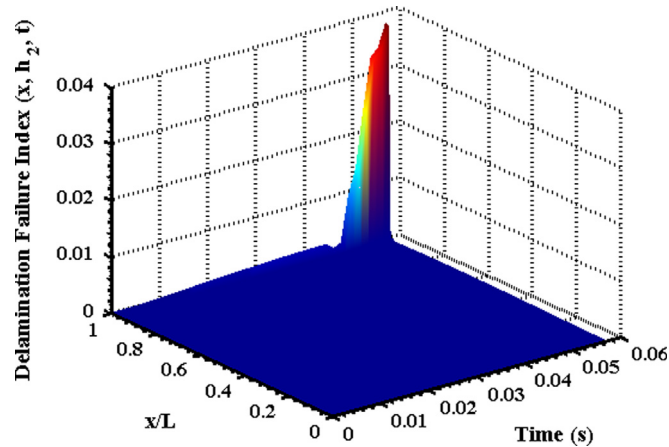


Fig. 16. Variation with time of the delamination failure index at the interface between the bottom face sheet and the core of the beam based on delamination failure criterion ($\beta = 10^\circ$, $V = -1.88$ m/s).

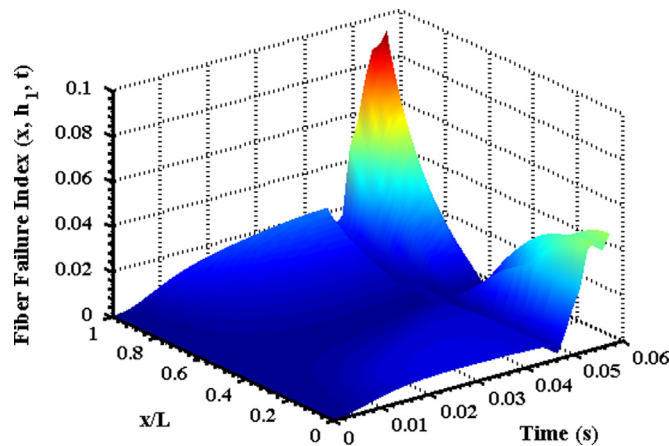


Fig. 17. Variation with time of the fiber failure index at the interface between the bottom face sheet and the core of the beam based on fiber failure criterion ($\beta = 10^\circ$, $V = 1.88$ m/s).

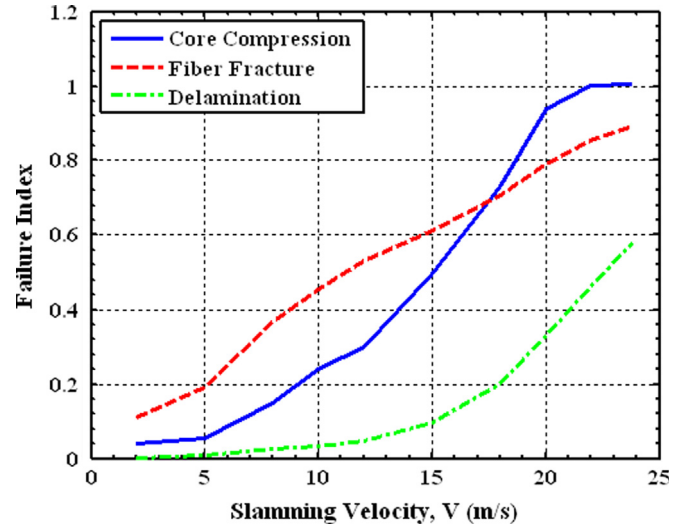


Fig. 18. Variation with time of the failure indices at the interface between the bottom face sheet and the core of the beam after 10% degradation of the core from the clamped end ($x = L$) of the beam ($\beta = 10^\circ$).

entry velocity. For a constant water-entry velocity, both axial normal stress and transverse shear stress in the beam increase with the decrease in the value of the dead rise angle. If the boat hull with sandwich construction is subjected to slamming load at the bottom surface of the bottom face sheet of the boat, the first failure occurs due to core compression at the interface between the core and the bottom face sheet and the damaged interface is located at the clamped end. The ultimate failure of the boat is also due to core compression at the interface between the core and the bottom face sheet of the boat while the location of the failure is at the end of the boat other than its clamped end.

Acknowledgment

This work was partially supported by the Office of Naval Research grant N00014-11-1-0594 to Virginia Polytechnic Institute and State University with Dr. Y.D.S. Rajapakse as the Program Manager. Views expressed in the paper are those of the authors and neither of the funding agency nor of authors' Institution.

Appendix A

Various matrices appearing in expressions for strains given by Eqs. (9) and (10) and in the expression of $\{F_r^e\}$ given by Eq. (25) are as follows:

$$\begin{aligned}
 [Z_2] &= \begin{bmatrix} h_c & h_c^2 & h_c^3 & z-h_c & z^2-h_c^2 & z^3-h_c^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2z \end{bmatrix}, \\
 [Z_3] &= \begin{bmatrix} -h_c & h_c^2 & -h_c^3 & z+h_c & z^2-h_c^2 & z^3+h_c^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\
 [Z_4] &= \begin{bmatrix} z & z^2 & z^3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2z \end{bmatrix}, \\
 [Z_5] &= \begin{bmatrix} 1 & 2z & 3z^2 & h_c & h_c^2 & z-h_c & z^2-h_c^2 \end{bmatrix}, \\
 [Z_6] &= \begin{bmatrix} 1 & 2z & 3z^2 & -h_c & h_c^2 & z+h_c & z^2-h_c^2 \end{bmatrix}, \\
 [Z_7] &= \begin{bmatrix} 1 & 2z & 3z^2 & z & z^2 \end{bmatrix}, \\
 [Z_8] &= \begin{bmatrix} -h_c z & h_c^2 z & -h_c^3 z & z^2/2 + h_c^2 z & z^3/3 - h_c^2 z & z^4/4 + h_c^3 z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & z & z^2 \end{bmatrix},
 \end{aligned}$$

[illegible]

[illegible]

