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Rolling/Sliding of a Vibrating Elastic Body on an Elastic Substrate

The rolling/sliding in the presence of friction of a vibrating elastic body on an elastic substrate is studied. It is shown that the longitudinal components of the velocity of the center of mass of the body and of the resultant frictional force are not affected by the vibration process. However, the normal vibration of the body influences the slip velocities and the distribution of frictional forces. For the problem of a harmonically oscillating long cylinder rolling/sliding on the flat surface of an elastic substrate, the time histories of the width of the contact zone and the length of adhesion subzone are computed. It is shown that the local frictional forces and slip velocities oscillate, and that the set of admissible values of the external frictional forces and moments providing the rolling/sliding regime is smaller under conditions of normal vibrations than that when the cylinder does not vibrate.

1 Introduction

As has been pointed out by Ibrahim (1994), friction-induced vibrations, chatter, and squeal are serious problems in many industrial applications, such as turbine blade joints, robot joints, electric motor drives, water-lubricated bearings in ships and submarines, wheel/rail of mass transit systems, machine tool/ work piece systems and brake systems. Tolstoi (1967) studied experimentally the friction-induced vibration or stick-slip phenomena in the sliding contact initiated by normal vibration of the body. He showed that the resultant frictional force depended upon the sliding velocity and that it decreased once the sliding speed exceeded a critical value. Because of this, the horizontal velocity of the center of mass of the body oscillates with time with a frequency equal to that of normal vibrations of the body. Aronov et al. (1983, 84) tested a pin-on-disk type model and found that the measured frictional force and the velocity of the pin changed randomly with time.

The first mathematical models of machine elements under conditions of frictional/normal vibrations included a system of rigid bodies connected by one-dimensional elastic springs (e.g., see the review articles by Ibrahim, 1994). Their analysis was based on the dynamic equations with the resultant frictional force assumed to be a known function of the slip velocity. Later, Hess and Soom (1991) included in the dynamic equations of motion the normal contact reaction derived first from the Hertzian theory and then from the rough surface theory. Oden and Martins (1985) have studied the sliding of an elastic body on a substrate under conditions of normal vibration. They determined the resultant normal and frictional forces from the local time-dependent distributions of contact stresses. Martins et al. (1990) and Tworzydlo et al. (1994) have developed various models of frictional contact interaction and have analysed a number of problems involving the sliding of an elastic body in the presence of frictional and normal vibrations.

We study here the contact interaction of a vibrating body contacting the flat surface of an elastic substrate in the rolling/ sliding regime. Such regimes are characterized by the magnitude of the slip velocity being small as compared to that of the center of mass velocity. In the rolling/sliding regime, the longitudinal component of the resultant frictional force is uniquely related to the external tangential forces (Spector and Batra, 1995). We use this result to show that the longitudinal components of the velocity of the center of mass of the body and of the resultant frictional force are unaffected by the normal vibration process. However, normal vibrations do influence the slip velocities and the local distribution of frictional forces. We study, in detail, the contact interaction of a long cylinder rolling/ sliding on a substrate and also acted upon by a time-harmonic resultant contact force. The width of the contact zone and the relative length of the adhesion subzone oscillate in time and are asymmetric with respect to the no-vibration case; this asymmetry increases with the rise in the amplitude of the normal force applied to the cylinder. We also prove that the set of admissible values of the external driving moments and horizontal forces that permit the rolling/sliding regime becomes smaller when normal vibrations are imposed on the cylinder as compared to the case of no vibrations.

A reviewer of this manuscript pointed out that the problem of the contact interaction of a long cylinder on a substrate acted upon by a normal force and a harmonic total contact force has also been studied by Gross-Thebing (1993).

2 Formulation of the Problem

We study the motion of a linear elastic body in contact with the flat surface of a linear elastic substrate and assume that the typical dimension of the contact area is much smaller than the characteristic dimension of the body. The kinematics of body points away from the vicinity of the contact zone are described by assuming that the body is absolutely rigid. The spatial position of the center of mass of the body is determined by two components of its velocity in the horizontal plane and its vertical coordinate with respect to the flat surface of the substrate. The motion of the body about its center of mass is determined by two components of its angular velocity (e.g., see Fig. 1). These kinematical assumptions are reasonable for describing the motion of railroad wheels and roller bearings when the slip velocity has longitudinal, lateral and spin components.

We assume that the body vibrates in the vertical direction; this normal vibration could be due to the dynamic oscillatory change of the normal approach of the body and the substrate due to their elastic deformations caused by the external normal force and the normal component of contact reaction. These vibrations could also be caused by the prescribed harmonic

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Fig. 1 Rolling/sliding of a body on a substrate

motion in the vertical direction of the center of mass of the body or the vertical component of the contact reaction.

We assume that the body and the substrate have originally a point contact, and the regime of rolling/sliding interaction, defined by

$$|\mathbf{s}| \le V_c \tag{1}$$

is realized. Here V_c is the typical speed of the center of mass of the body, and the slip velocity s is determined from

$$\mathbf{s} = \mathbf{v} + \frac{d}{dt} \left(\mathbf{u}^{b} - \mathbf{u}^{s} \right)$$
$$= \mathbf{v} - V_{x} \frac{\partial}{\partial x} \left(\mathbf{u}^{b} - \mathbf{u}^{s} \right) + \frac{\partial}{\partial t} \left(\mathbf{u}^{b} - \mathbf{u}^{s} \right) \quad (2)$$

The last two terms in Eq. (2) account for the elastic deformations of the body and the substrate. The first term in Eq. (2) is determined by the slip velocity of the body assumed as absolutely rigid and its components are given by

$$v_x = V_x - \Omega_y R_x - \Omega_z y, \quad v_y = V_y + \Omega_z x. \tag{3}$$

The boundary conditions at the contact surface are

$$p = 0 \quad \text{if} \quad w^b \ge (w^s + \delta - f), \tag{4}$$

$$p \ge 0 \quad \text{if} \quad w^b = (w^s + \delta - f), \tag{5}$$

Nomenclature

- A = contact area
- E^b , E^s = Young's moduli for the material of the body and the substrate
- G^b , G^s = shear moduli for the material of the body and the substrate
 - E' = reduced Young's modulus
 - for the body/substrate pair
 - f = function describing the undeformed surface of the body
- \mathbf{T}^{ex} , \mathbf{M}^{ex} = externally applied force and moment
 - l_c = characteristic dimension of the contact area
 - l = half width of the contact zone between the cylinder and the substrate
 - $l_a = x$ -coordinate of the edge of the adhesion subzone in the problem of a cylinder rolling/ sliding on a flat surface of a substrate
 - M = mass of the body

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- $I_y = mass moment of inertia of the body about y-axis$
- p = normal reaction force at a point within the contact area

$$|\tau| \le \mu p \quad \text{if} \quad |\mathbf{s}| = 0, \tag{6}$$

$$\tau = \mu p \mathbf{s} / |\mathbf{s}| \quad \text{if} \quad |\mathbf{s}| > 0 \tag{7}$$

The boundary conditions (4) and (5) hold at points on the free surface and in the contact area respectively. Conditions (6) and (7) describe, respectively, the adhesion and slip subareas. These boundary conditions can be formulated in terms of contact stresses by using

$$w^{b} = A^{b}(p) + A^{b}(\boldsymbol{\tau}), \qquad (8)$$

$$w^{s} = A^{s}(p) + A^{s}(\boldsymbol{\tau}), \qquad (9)$$

$$\mathbf{u}^{b} = \mathbf{B}^{b}(p) + \mathbf{B}^{b}(\boldsymbol{\tau}), \qquad (10)$$

$$\mathbf{u}^{s} = \mathbf{B}^{s}(p) + \mathbf{B}^{s}(\boldsymbol{\tau}); \qquad (11)$$

the expressions for the integral operators A^s , A^b etc. are given in the Appendix.

The dynamics of the body is described by three sets of equations. The first one giving the vertical position of its center of mass is

$$M\ddot{\delta} = P^{ex} + P(\delta) \tag{12}$$

and determines normal vibration of the body. If the vibration is determined by the prescribed harmonic oscillation of the center of mass of the body, then Eq. (12) is replaced by

$$\delta = \delta_0 + D \sin \left(\omega_{\delta} t + \phi_{\delta} \right). \tag{13a}$$

However, if the vibration is determined by the prescribed harmonic oscillation of the normal contact reaction, then

$$P = P_0 + C \sin(\omega_p t + \phi_p). \tag{13b}$$

The second set of equations describes the variation of the longitudinal component of the center of mass velocity and the component Ω_y of the angular velocity responsible for the rolling process:

- force $p_c = \text{characteristic contact pressure}$
- R = radius of the cylinder

 P^{ex} = externally applied normal

- R_x = radius of curvature in the *xz*plane of the surface of the
 - body at the point of initial contact
- $\mathbf{s}(s_x, s_y) =$ the slip velocity
 - T_x , T_y = components of the resultant frictional force
 - t = time
 - T_c^v = typical time-period of the dynamic process of the motion of the body
- \mathbf{u}^{b} , \mathbf{u}^{s} = tangential displacements of the body and the substrate $\mathbf{v}(v_{x}, v_{y})$ = the slip velocity when the
- $(v_x, v_y) =$ the snp velocity when the body is regarded as absolutely rigid
- w^b , w^s = normal components of the displacements of the body and the substrate
- ν^{b} , ν^{s} = Poisson's ratios for the materials of the body and the substrate
- η = characteristic of the distribution of mass in the body $\tau(\tau_x, \tau_y) = \text{local frictional forces}$ μ = coefficient of friction $\omega =$ frequency of vibration δ = normal approach of the body and the substrate $\Omega(\Omega_{v}, \Omega_{z}) =$ angular velocity of the body s = slip velocity V_c = speed of the center of mass of the body (x, y, z) = coordinates of a point with respect to a rectangular cartesian coordinate system $b^* = \text{dimensionless half-length}$ of the adhesion subzone $C^* =$ amplitude of the nondimensional sinusoidal contact force $M^* =$ nondimensional external moment acting on the cylinder
 - *l** = dimensionless half-width of the contact zone

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$$M\dot{V}_x = -T_x + T_x^{ex}, \tag{14}$$

$$I_{y}\dot{\Omega}_{y} = R_{x}T_{x} + M_{y}^{ex} \tag{15}$$

The third set of equations determines V_y and Ω_z . We assume that

$$|V_{\mathbf{y}}| \ll |V_{\mathbf{x}}|, \quad |\Omega_{\mathbf{z}}| \ll |\Omega_{\mathbf{y}}| \tag{16}$$

which is a reasonable approximation in rolling/sliding of wheels and roller bearings. Thus we can neglect acceleration terms in the corresponding dynamic equations and obtain

$$T_{\rm v} = F_{\rm v}^{ex},\tag{17}$$

$$M_z = M_z^{ex}.$$
 (18)

In some cases, the prescribed ratios V_y/V_x (lateral slip velocity divided by the rolling velocity) and/or $(\Omega_z l)/V_x$ (spin velocity divided by the rolling velocity) are used instead of Eqs. (17) and (18).

3 Analysis of the Problem

We nondimensionalize variables in the same way as was done by Spector and Batra (1995). The dynamic Eqs. (14) and (15) imply that the characteristic time T_c is given by

$$T_c = (MR_x/p_c l_c^2)^{1/2}$$
(19)

However, the characteristic time T_c^v of the vibration process described by Eq. (12) or Eq. (13) is given by

$$T_c^v = \omega^{-1} \tag{20}$$

We assume that

$$T_c^{\nu} \sim T_c. \tag{21}$$

Following the reasoning of Spector and Batra (1995) we can prove that for $t \sim T_c$ the local derivative in Eq. (2) is of the order

$$\epsilon = l_c / R_x \ll 1. \tag{22}$$

Combining dynamic Eqs. (14) and (15) and neglecting terms proportional to $v_x^0 = V_x - \Omega_y R_x \sim \epsilon V_x$, we obtain the relationship

$$T_{x} = \frac{1}{\eta + 1} T_{x}^{ex} - \frac{\eta}{\eta + 1} \frac{M_{x}^{ex}}{R_{x}}, \quad \eta = \frac{MR_{x}^{2}}{I_{y}}$$
(23)

between the longitudinal component of the resultant frictional force and the external force and moment. From Eq. (23) we can conclude that the component T_x of the resultant frictional force is unaffected by the harmonic vibrations of the body in the vertical direction if the rolling/sliding regime is realized. From Eqs. (14), (15), and (23) we conclude that kinematic quantities V_x and Ω_y are also unaffected by the normal vibration process. From boundary conditions (4)–(7) and equations of motion (12) or (13), we see that the contact stresses, slip velocities, contact area, and slip/adhesion subareas change due to the vibrations of the center of mass of the body. Components V_y and Ω_z , characterizing the slip velocities, are affected by the vibration process if they are determined by Eqs. (17) and (18).

We give below quantitative measures of these quantities when the body is a cylinder.

4 Rolling/Sliding of a Vibrating Cylinder

4a Analysis of Contact Characteristics. We study the motion of a long elastic cylinder under the action of the constant driving moment M^{ex} . We assume that because of normal vibration of the moving cylinder, the resultant normal contact force per unit length of the cylinder varies sinusoidally according to

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Fig. 2 Rolling/sliding of a cylinder under the action of a driving moment

$$P = P_o + C\sin\omega t. \tag{24}$$

We note that the dependence of the normal contact force P on the normal approach δ of the elastic cylinder and the elastic half-space is nearly linear (e.g., see Johnson, 1985; Harris, 1991). Thus Eq. (24) is close to the particular solution of Eq. (12) if P^{ex} = const. We neglect the weak effect of frictional forces on the normal contact characteristics (Johnson, 1985; Kalker, 1990); there is no such effect when the cylinder and the substrate are made of the same material. We study the motion of the cylinder in the xz-plane where z-direction is normal to the flat surface of the substrate and the cylinder rolls in the x-direction (cf. Fig. 2). As discussed above, only spatial derivatives can be kept in the expression for the slip velocity, thus time can be considered as a parameter in the boundaryvalue problem for the contact stresses. Accordingly, we can use the Carter-Poritsky (Johnson, 1985; Kalker, 1990) solution of the contact problem for the long cylinder; this solution gives

$$\tau_{x} = \begin{cases} -\mu p_{o} \left(1 - \left(\frac{x}{l}\right)^{2}\right)^{1/2} & \text{if } -l \leq x \leq l_{a}, \\ -\mu p_{o} \left[\left(1 - \left(\frac{x}{l}\right)^{2}\right)^{1/2} \\ -\frac{l-d}{l} \left(1 - \left(\frac{x-d}{l-d}\right)^{2}\right)^{1/2} \right] & \text{if } l_{a} \leq x \leq l, \end{cases}$$
(25)

$$s_{x} = \begin{cases} \frac{8\mu p_{\sigma} V_{x}}{\mathrm{E}'} \left(\frac{l-d}{l}\right) \left[\left(\frac{x-d}{l-d}\right)^{2} - 1 \right]^{1/2} & \text{if } -l \leq x \leq l_{a}, \\ 0 & \text{if } l_{a} \leq x \leq l, \end{cases}$$

$$(26)$$

$$v_x^0 = -\epsilon \mu V_x \left[1 - \left(1 + \frac{T_x}{\mu P} \right)^{1/2} \right], \qquad (27)$$

and implies that there is one adhesion subzone ahead of the contact zone and one slip subzone behind it. In Eqs. (25) - (27), l is the half-width of the contact zone, p_0 is the maximum pressure, l_a is the x-coordinate of the edge of the adhesion subzone, d is the x-coordinate of its center, and

$$l = \left(\frac{4PR}{\pi E'}\right)^{1/2},$$
 (28)

$$p_0 = \left(\frac{PE'}{\pi R}\right)^{1/2},\tag{29}$$

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Fig. 3 Evolution of the contact zone during one time period of vibration of the cylinder for different values of the applied moment

$$l_a = 2d - l, \quad d = l\left(1 - \left(1 + \frac{T_x}{\mu P}\right)^{1/2}\right),$$
 (30)

$$\frac{2}{E'} = \frac{1 - (\nu^b)^2}{E^b} + \frac{1 - (\nu^s)^2}{E^s},$$
 (31)

and *R* is the radius of the cylinder. Substituting from Eqs. (23) and (24) into Eq. (28) and recalling that $T^{ex} = 0$, we obtain the following for the quantitative effects of the vibration of the cylinder.

$$l = (4(P_0 + C\sin\omega t)R/\pi E')^{1/2}, \qquad (32)$$

$$d = l \left[1 - \left(1 - \frac{M^{ex} \eta}{(1+\eta)R(P_0 + C\sin\omega t)} \right)^{1/2} \right].$$
(33)

The dimensionless half-width of the contact zone and halflength of the adhesion subzone are given by

$$l^* = (1 + C^* \sin \omega t)^{1/2}, \qquad (34)$$

$$b^* = 1 - \frac{d}{l} = \left(1 - \frac{M^*(1 - C^*)}{1 + C^* \sin \omega t}\right)^{1/2},$$
 (35)

where

$$C^* = C/P_0, \tag{36}$$

$$0 \le M^* = \frac{M^{e_x} \eta}{(1+\eta) \mu R(P_0 - C)} \le 1.$$
(37)

The upper limit in inequality (37) corresponds to the case of full slip, and the lower one to full adhesion inside the contact zone.



Fig. 4 Evolution of the contact zone during one time period of vibration of the cylinder for different amplitudes of vibration

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Fig. 5 Evolution of the adhesion subzone during one period of vibration of the cylinder

We depict results for the time interval $0 \le \omega t \le 2\pi$ that illustrate the evolution of the contact characteristics because of normal vibrations of the cylinder. Figure 3 shows the evolution of the adhesion subzone for three different values of the external moment and for the fixed level of vibration characterized by $C^* = 0.5$; in Fig. 4 we plot the evolution of the adhesion subzone for three different values of C^* . We have also plotted results for the no vibration case $(C^* = 0, M^* = 0.16)$; the value of M^* is such that these results can be compared with those for the vibrating cylinder with $C^* = 0.8$, $M^* = 0.8$ because of the same value of the product $M^*(1 - C^*)$ and hence the same value of the dimensional driving moment. Figure 5 depicts the evolution of the half-width of the contact zone for three levels of vibration. For fixed values of the applied moment and the vibration level, we have plotted in Fig. 6 the distribution of the dimensionless frictional force at four instants of time, namely, $\omega t = 0$, $\pi/2$, $4\pi/3$ and $3\pi/2$; it follows from Eqs. (32) and (33) that the distributions of the frictional force at $\omega t = \pi$ and 2π coincide with that at $\omega t = 0$. For $\omega t = 0, \pi/2$ 2, $4\pi/3$ and $3\pi/2$, the distributions of the slip velocities are exhibited in Fig. 7.

4b Acceptable Values of External Forces and the Effect of Normal Vibration. We now study the motion of the cylinder on the substrate under the action of an external moment M^{ex} and horizontal force T^{ex} with the objective of finding acceptable values of M^{ex} and T^{ex} for which the rolling/sliding regime defined by inequality (1) applies. We regard the existence of an adhesion subzone, including the extreme case of its degeneration into a single point, as the criterion for the occurrence of the rolling/sliding regime. This ensures that inequality (1)



Fig. 6 Variation of the local frictional forces during one period of vibration of the cylinder

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Fig. 7 Variation of the slip velocities during one period of vibration of the cylinder

holds. To prove this, we first conclude from expression (26) for the slip velocity that

$$|s_x|/|V_x| \sim p_0/E'.$$
 (38)

Equations (28) and (29) imply that

$$\frac{p_0}{\mathbf{E}'} \sim \frac{l_c}{R} = \epsilon \ll 1. \tag{39}$$

Estimates (38) and (39) are equivalent to inequality (1).

In order for the rolling/sliding regime to exist, the resultant frictional force should satisfy the following inequality

$$-\mu P_0(1 + C^* \sin \omega t) \le T_x \le \mu P_0(1 + C^* \sin \omega t) \quad (40)$$

By using Eq. (23) which is valid for the rolling/sliding regime, we obtain the following constraints on the external resultant force and moment.

$$C^* - 1 \le \frac{T^{ex}}{(\eta + 1)\mu P_0} - \frac{\eta}{(\eta + 1)} \frac{M^{ex}}{\mu P_0 R} \le 1 - C^* \quad (41)$$

Inequalities (41) delineate vividly the effect of cylinder vibrations on the acceptable values of external forces and moments. In Fig. 8, we depict the boundaries of the acceptable values in $(T^{ex}/\mu P_0, M^{ex}/\mu P_0 R)$ space for a homogeneous cylinder with $\eta = 2$ for two levels of vibration; the case of no vibration is also shown.

5 Discussion of Results

The results plotted in Figs. 3 through 8 show the evolution with time of the kinematic characteristics and frictional forces responsible for the rolling/sliding process. It is evident that the longitudinal component of the resultant frictional force is unaffected by the normal vibration process. However, the rigid and actual slip velocities are influenced by the vibration of the body and oscillate synchronously with it. The local frictional force also oscillates synchronously with the harmonic vibrations of the body. A comparison of our results with those reported in the review article of Ibrahim (1994) for the sliding process in the presence of normal vibration reveals that the main difference between the two is in the behavior of the longitudinal component of the center of mass velocity and the resultant frictional force. In a sliding process, both of these quantities oscillate. However, in the rolling/sliding process, the longitudinal component of the frictional force is strictly determined by the external horizontal forces. Such a relationship between them is necessary for the body to have a small sliding velocity. Tolstoi (1967) has shown experimentally that some critical level of sliding must be attained before horizontal oscillatory motion begins. This supports our computed result of no oscillations of the longitudinal component of the center of mass velocity when the sliding velocities are small.

An analysis of the variation of the width of the contact zone and the length of the adhesion subzone (Figs. 3-5) reveals its asymmetric character with respect to the no vibration level. The deviation from the no vibration value is greater for the minimum value than that for the maximum value. Thus the average values of the width of the contact zone and the length of the adhesion subzone are lower than the corresponding values in the no vibration case. This asymmetry is especially noticeable for the length of the adhesion subzone. The deviations in the width of the contact zone and the length of the adhesion subzone from the no vibration case increase with an increase in the amplitude of the normal vibration of the body.

Figures 6 and 7 exhibit the variation of the local frictional forces and slip velocities during one-half cycle of vibrations of the body. During the period $0 \le \omega t \le \pi/2$, both the widths of the contact zone and the adhesion subzone are more as compared to those for the no-vibration case; they return to their values for the no-vibration case during the time $\pi/2 \le \omega t \le \pi$. Subsequently, they decrease and return to their values for the no-vibration case at $\omega t = 2\pi$. Despite the significant redistribution of the frictional forces and slip velocities during one cycle of vibration of the body, their maximum and mean values do not differ much from those for the no-vibration case.

Results in Fig. 8 show how the vibrations of the body shrink the set of admissible values of the external force and moment for the existence of the rolling/sliding regime. Therefore, higher the amplitude of the normal vibrations of the body, the greater will be the likelihood of the system to go out of the rolling/ sliding regime.

6 Conclusions

The process of rolling/sliding of a vibrating elastic body on the flat surface of an elastic substrate is characterized by the specific form of the variations of the moving body kinematics and the contact parameters. Contrary to the case of sliding regime, the longitudinal components of the center of mass velocity and of the resultant frictional force are not affected by the normal vibrations of the body. The rigid and actual slip velocities, slip/adhesion subareas, and frictional forces oscillate synchronously with the normal vibrations of the body. For a vibrating elastic cylinder rolling/sliding on a substrate, the variation with time of the width of the contact zone and the length of the adhesion subzone is asymmetric with respect to that for the case of no vibrations, these differences increase with an increase in the amplitude of vibrations of the cylinder, and their average values are lower than those for the non-vibrating cylinder.

The local frictional forces and slip velocities are redistributed significantly during the period of vibration although their maxi-



Fig. 8 Effect of vibrations on the admissible values of the external forces and moments

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mum and average values do not differ significantly from those for the non-vibrating cylinder. The increase in the amplitude of vibrations of the cylinder decreases the set of admissible values of external forces and moments for the rolling/sliding regime to exist. Thus, an increase in the amplitude of vibrations of the cylinder will enhance the chance that the contact will not be of the rolling/sliding type.

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Appendix

$$A^{b}(p) = \frac{1-\nu^{b}}{2\pi G^{b}} \int_{A} \frac{p}{r} dx' dy',$$

$$A^{s}(p) = \pm \frac{1-\nu^{s}}{2\pi G^{s}} \int_{A} \frac{p}{r} dx' dy',$$

$$A^{b}(\tau) = -\frac{1-2\nu^{b}}{4\pi G^{b}} \int_{A} \frac{1}{r} (\tau_{x'} \cos \theta + \tau_{y'} \sin \theta) dx' dy',$$

$$A^{s}(\tau) = \frac{1-2\nu^{s}}{4\pi G^{s}} \int_{A} \frac{1}{r} (\tau_{x'} \cos \theta + \tau_{y'} \sin \theta) dx' dy',$$

$$\mathbf{B}^{b}(p) = -\frac{1-2\nu^{b}}{4\pi G^{b}} \int_{A} \frac{p}{r} \left\{ \frac{\cos \theta}{\sin \theta} \right\} dx' dy',$$

$$\mathbf{B}^{s}(p) = \frac{1-2\nu^{s}}{4\pi G^{s}} \int_{A} \frac{p}{r} \left\{ \frac{\cos \theta}{\sin \theta} \right\} dx' dy',$$

$$(\tau) = -\frac{1}{2\pi G^{b}} \int_{A} \frac{1}{r}$$

$$\times \left\{ \frac{(1-\nu^{b} \sin^{2} \theta)\tau_{x'} + \nu^{b} \sin \theta \cos \theta\tau_{y'}}{\nu^{b} \sin \theta \cos \theta\tau_{x'} + (1-\nu^{b} \cos^{2} \theta)\tau_{y'}} \right\} dx' dy'$$

$$\mathbf{B}^{s}(\boldsymbol{\tau}) = \frac{1}{2\pi G^{s}} \int_{A} \frac{1}{r}$$

$$\times \begin{cases} (1 - \nu^{s} \sin^{2} \theta) \tau_{x'} + \nu^{s} \sin \theta \cos \theta \tau_{y'} \\ \nu^{s} \sin \theta \cos \theta \tau_{x'} + (1 - \nu^{s} \cos^{2} \theta) \tau_{y'} \end{cases} dx' dy'$$

$$r^{2} = (x - x')^{2} + (y - y')^{2},$$

$$\cos \theta = (x - x')/r$$
, $\sin \theta = (y - y')/r$.

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